Exploration

On the Gauge-Gravity Duality in Fractal Dimensions

Ervin Goldfain*

Ronin Institute, Montclair, NJ

Abstract

Despite its technical appeal, the main drawback of the gauge-gravity duality is that it works with a negative cosmological constant, in manifest contradiction with observations. We point out here that a 2D spacetime endowed with minimal fractality acquires a positive cosmological constant upon combining the gauge-gravity duality with the dimensional reduction conjecture. When applied to the evolution of the primordial Universe, this finding hints to an unforeseen unification of Dark Energy and Dark Matter into a single framework.

Keywords: Superstring theory, AdS/CFT, dimensional reduction, minimal fractal manifold, Dark Energy, Dark Matter.

In string theory, the *gauge-gravity duality* relates the anti-de Sitter (AdS) gravitation in 5 dimensions to conformal field theory (CFT) in 4 dimensions [1-2]. *Dimensional reduction* is a conjecture developed in the context of quantum gravity and posits that, at ultrashort distances, spacetime becomes effectively two dimensional (2D) [3].

According to the gauge-gravity duality, Einstein's equations in $D=d+1$ spacetime dimensions contain a negative cosmological constant Λ and are written as:

$$R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu}+\Lambda g_{\mu\nu}=0 \tag{1}$$

in which,

$$\Lambda=-\frac{d\,(d-1)}{2R_{AdS}^{2}};\;\; d=0,1,2,... \tag{2}$$

with R_{AdS} denoting the AdS curvature radius, a parameter that can be conveniently set to unity [2].

*Correspondence: Ervin Goldfain, Ph.D., Research Scholar, Ronin Institute, Montclair, NJ 07043
E-mail: ervin.goldfain@ronininstitute.org

In a *minimal fractal spacetime* defined in 1+1 dimensions ($\mu,\nu=0,1$), the spatial dimension flows with the Renormalization Group (RG) scale and spans a continuous range of values as in [4]

$$d(\mu_{RG})=1-\varepsilon(\mu_{RG})\propto 1-O\left[m^2(\mu_{RG})/\Lambda_{UV}^2\right];\quad \varepsilon<<1 \tag{3}$$

where μ_{RG} stands for the RG scale and Λ_{UV} is the ultraviolet cutoff. In contrast with the conventional gauge-gravity duality, it follows from (2) and (3) that (1) acquires a *positive* cosmological constant, i.e.

$$\boxed{\overline{\Lambda}=\Lambda R_{AdS}^2=O(\varepsilon)>0} \tag{4}$$

There are (at least) three tacit assumptions (A1-A3) behind this derivation, as detailed below.

A1) The evolution of the early Universe is compatible with the dimensional reduction ansatz and the concept of minimal fractal spacetime emerging above the Fermi scale [4].

A2) The low level fractality implied by (3) is a reasonable approximation which enables the use of the left-hand side of (1) in its original form given in General Relativity.

A3) Once fixed by the early Universe dynamics, the cosmological constant (4) no longer flows with spacetime dimensionality, on account of the inherent long-range/memory effects associated with the differential operators on fractal spacetime [4-5]. Stated differently, if $\mu_{RG}\geq\mu_{RG}^0$ represents the range of scales for which (2) - (4) hold, the cosmological constant stays stationary ($\Lambda_0\neq\Lambda(d)$) for any $\mu_{RG}<\mu_{RG}^0$.

Retracing the steps of [6], the dimensional deviation is interpreted as an *infinite string* of component deviations defined by

$$\varepsilon=\sum_1^\infty\varepsilon_i=\frac{1}{\Lambda_{UV}^2}\sum_1^\infty m_i^2<<1 \tag{5}$$

On account of (4), (5) leads to

$$\boxed{\overline{\Lambda}\propto\sum_1^\infty\frac{m_i^2}{\Lambda_{UV}^2}} \tag{6}$$

The string of deviations ε_i acts as an infinite ensemble of scalar fields clustered into a *large-scale Cantor Dust* structure formed through topological condensation. One concludes that, on scales substantially larger than the Fermi scale, the energy content of the cosmological constant comes from the cumulative contribution of energies stored in the Cantor Dust. It is known that the ΛCDM model associates Dark Energy with the cosmological constant and the accelerated expansion of the Universe. If Dark Matter is thought of as consisting of relic clusters of Cantor Dust, the observations outlined above open an unexplored path towards unifying Dark Energy and Dark Matter into a single framework [6].

Finally, an added benefit of postulating 2D gravity in the far ultraviolet sector - matching the conditions of primordial Universe ($\Lambda_{UV} = O(M_{Pl})$) - is that Newton's constant G turns into a dimensionless parameter. This can be readily seen from the mass dimension expression [7]

$$\left[G^{(D)}\right] = G^{(4)} M^{4-D} \tag{7}$$

upon taking $D = d + 1 = 2$. As a result, unlike General Relativity in four-dimensional spacetime, 2D gravity is perturbatively renormalizable.

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