**Article**

# Bianchi Type-IX Inflationary Cosmological Model with Flat Potential for Perfect Fluid Distribution in General Relativity

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## Abstract

We have investigated Bianchi type-IX inflationary cosmological model with flat potential for perfect fluid distribution in general relativity. To obtain the deterministic solution of the model, we assume that the expansion $(\theta)$ is proportional to the shear $(\sigma)$, which leads to $A = B^n$ and potential $V(\phi)$ as constant. The behavior of the model from physical and geometrical aspects is also discussed.

**Keywords:** Bianchi IX, inflationary universe, perfect fluid, cosmology.

## 1. Introduction

It is well-known that self-interacting scalar fields play a vital role in the study of inflationary cosmology. Guth [14] has discussed the inflationary universe as a possible natural explanation for the observed large scale homogeneity and near critical density (flatness) of the universal expansion. A universe with only moderate anisotropy will undergo inflation and will be rapidly isotropized. Linde [17], Albrecht and Steinhardt [2], Abbott and Wise [1], Mijic et al. [18], Rothman and Ellis [21], are some of the authors who have investigated several aspects of inflationary universe in general relativity.

Several versions of inflationary scenario have been studied by number of authors viz. La and Steinhardt [16], Earman and Mosterin [12] and Ainsworth [4].

Bali [8] discussed the significance of inflation for isotropization of universe. This inflationary scenario is also confirmed by Cosmic Microwave Background (CMB) observations (Bassett et al. [10]). In inflationary models, the universe undergoes a phase transition characterized by the evolution of Higgs field (ϕ). The inflation will take place if the potential V(ϕ) has flat region the ϕ field evolves slowly but the universe expands in an exponential way due to the vacuum field energy as suggested by Stein-Schabes [25]. The flat part of the potential is naturally associated

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with a vacuum energy which can be identified as an effective cosmological constant ($\Lambda$) and it makes the universe to enter an inflationary period.

Bianchi type-IX inflationary cosmological models play an important role for relativistic studies as these models permits not only expansion but also shear and rotation and in general, these models are anisotropic. In recent years, many researchers have taken keen interest to study these models because well-known solution like Robertson Walker space-time, the de-Sitter space-time, the Taub-Nut space-time etc. are specific case of Bianchi type-IX universe. Bianchi type-IX inflationary cosmological models in different context have been studied by number of authors viz. Sharma and Poonia [23], Henriques et al. [15], Adhav et al. [3], Sharma et al. [22], Burd and Barrow [11].

Shri Ram and Singh [24] have obtained Bianchi type-II, VIII and IX cosmological models with matter and electromagnetic fields. Bali and Goyal [9] investigated inflationary scenario in Bianchi type-V space-time with variable bulk viscosity and dark energy in radiation dominated phase. Bali and Kumari [7] have obtained Chaotic inflation in spatially homogeneous Bianchi type-V space time. Bali and Singh [5] have discussed Bianchi type-V inflationary universe with decaying vacuum energy $(\Lambda)$. Reddy et al. [19] have discussed Axially symmetric inflationary universe in general relativity. Reddy [20] have studied Bianchi Type-V inflationary universe in general relativity. Bali and Kumari [6] have studied Bianchi type-V inflationary universe with flat potential and stiff fluid distribution in general relativity. Gron and Hervik [13] investigated Einstein's general theory of relativistic with modern applications in cosmology.

In this paper, we investigate the Bianchi type-IX inflationary cosmological model with flat potential for perfect fluid distribution in general relativity. For the complete solution of the field equation, we assume that the expansion $(\theta)$ is proportional to the shear $(\sigma)$ and $V(\phi)$ is constant. The physical and geometrical aspects of the model are also discussed.

## 2. The Metric and Field Equations

We consider Bianchi type-IX space time as

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 dy^2 + \left(B^2 \sin^2 y + A^2 \cos^2 y\right) dz^2 - 2A^2 \cos y\, dx\, dz \qquad (1)$$

where A and B are function of time t alone.

We assume the co-ordinates to be comoving so that

$$v^1 = 0 = v^2 = v^3,\ v^4 = 1$$

In case of gravity minimally coupled to a scalar field V(ϕ), as given by Stein-Schabes [25], we have

$$S = \int \sqrt{-g}\left[R - \frac{1}{2} g^{ij}\partial_i\phi\partial_j\phi - V(\phi)\right] \mathrm{d}^4 X \tag{2}$$

The Einstein's field equation (in gravitational units $8\pi G = c = 1$), in the case of massless scalar field $\phi$ with potential $V(\phi)$ are given by

$$R_{ij} - \frac{1}{2} R g_{ij} = -T_{ij} \tag{3}$$

with

$$T_{ij} = (\rho + p)v_i v_j + p g_{ij} + \partial_i\phi\partial_j\phi - \left[\frac{1}{2}\partial_r\phi\partial^r\phi + V(\phi)\right] g_{ij} \tag{4}$$

Here ρ is the energy density, p the isotropic pressure, ϕ is Higgs field, V the potential.

$v_i$ is the unit time like vector.

The conservation relation leads to

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}\partial^\mu\phi\right) = -\frac{dV}{d\phi} \tag{5}$$

The Einstein's field equation (3) for the line-element (1) leads to non-linear differential equations are as follows

$$\frac{2B_{44}}{B} + \frac{B_4^2}{B^2} + \frac{1}{B^2} - \frac{3A^2}{4B^4} = -p - \frac{1}{2}\phi_4^2 + V(\phi) \tag{6}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} + \frac{A^2}{4B^4} = -p - \frac{1}{2}\phi_4^2 + V(\phi) \tag{7}$$

$$\frac{2A_4 B_4}{AB} + \frac{B_4^2}{B^2} + \frac{1}{B^2} - \frac{A^2}{4B^4} = \rho + \frac{1}{2}\phi_4^2 + V(\phi) \tag{8}$$

The equation (5) for scalar field $(\phi)$ leads to

$$\phi_{44} + \left( \frac{A_4}{A} + \frac{2B_4}{B} \right) \phi_4 = -\frac{dV}{d\phi} \tag{9}$$

where suffix '4' indicates derivative with respect to time t.

## 3. Solution of Field Equations

The field equations (6) - (8) are system of three equations with unknown parameters A, B, ρ, p, ϕ. To obtain the deterministic solution, we assume the following conditions:

(i) V(ϕ) is constant.

i.e. V(ϕ) = K (10)

Equations (9) and (10) lead to

$$\phi_4 = \frac{L}{AB^2} \tag{11}$$

where L is constant of integration.

(ii) Expansion $\theta$ is proportional to the shear $\sigma$, which leads to

$$A = B^n \tag{12}$$

The scale factor R for the line-element (1) is given by

$$R^3 = B^{n+2} \tag{13}$$

From equations (6) and (7), we get

$$B_{44} + (n+1)\frac{B_4^2}{B} = \frac{1}{n-1}\left[ \frac{1}{B} - \frac{1}{B^{3-2n}} \right] \tag{14}$$

Let us consider $B_4 = f(B)$ and $B_{44} = ff'$ in equation (14) we get

$$\frac{df^2}{dB} + \frac{2(n+1)}{B} f^2 = \frac{2}{n-1}\left[ \frac{1}{B} - \frac{1}{B^{3-2n}} \right] \tag{15}$$

After integrating equation (15), we get

$$f^2 = \frac{1}{(n^2-1)} - \frac{B^{2n-2}}{2n(n-1)} + MB^{-2n-2} \qquad \{n \neq 1,-1,0\} \tag{16}$$

where M is constant of integration.

From equation (16), we have

$$\int \frac{B^{n+1}dB}{\sqrt{bB^{2n+2} - hB^{4n} + M}} = t + N \tag{17}$$

where N is constant of integration and $\frac{1}{(n^2-1)} = b$, $\frac{1}{2n(n-1)} = h$. Value of $B$ can be obtained from equation (17).

After suitable transformation of coordinates, the metric (1) becomes

$$ds^2 = -\left(\frac{T^{2n+2}}{bT^{2n+2} - hT^{4n} + M}\right)dT^2 + T^{2n}dX^2 + T^2dY^2 + \left(T^2\sin^2 Y + T^{2n}\cos^2 Y\right)dZ^2 - 2T^{2n}\cos Y\, dX\, dZ \tag{18}$$

Where $x = X$, $y = Y$, $z = Z$ and $B = T$.

## 4. Physical and Geometrical Aspects

For the model (18), the rate of Higgs field

$$\phi = L\int \frac{1}{\sqrt{bT^{2n+4} - hT^{4n+2} + MT^2}} dT + Q \tag{19}$$

Where Q is constant of integration.

For the model (18), pressure $(p)$, density $(\rho)$, the spatial volume $(R^3)$, the expansion $(\theta)$, shear $(\sigma)$, decelerating parameter $(q)$ and Hubble parameter (H) are given by

$$p = K + \left(2bn - h + \frac{3}{4}\right)\left(\frac{1}{T^{-2n+4}}\right) + M(2n+1)\left(\frac{1}{T^{2n+4}}\right) - \frac{1}{2}L^2\left(\frac{1}{T^{2n+2}}\right) \tag{20}$$

$$\rho = -K + \left[(2n+1)b+1\right]\left(\frac{1}{T^2}\right) - \left[(2n+1)h + \frac{1}{4}\right]\left(\frac{1}{T^{-2n+4}}\right) + M(2n+1)\left(\frac{1}{T^{2n+4}}\right) - \frac{1}{2}L^2\left(\frac{1}{T^{2n+2}}\right) \tag{21}$$

$$R^3 = T^{n+2} \tag{22}$$

$$\theta = (n+2)\sqrt{\frac{b}{T^2} - \frac{h}{T^{-2n+4}} + \frac{M}{T^{2n+4}}} \tag{23}$$

$$\sigma = \left(\frac{n-1}{\sqrt{3}}\right)\sqrt{\frac{b}{T^2} - \frac{h}{T^{-2n+4}} + \frac{M}{T^{2n+4}}} \tag{24}$$

$$q = -1 - \frac{3}{2(n+2)}\left[\frac{-\frac{2b}{T^2} + \frac{h(-2n+4)}{T^{-2n+4}} - \frac{M(2n+4)}{T^{2n+4}}}{\frac{b}{T^2} - \frac{h}{T^{-2n+4}} + \frac{M}{T^{2n+4}}}\right] \tag{25}$$

$$H = \left(\frac{n+2}{3}\right)\sqrt{\frac{b}{T^2} - \frac{h}{T^{-2n+4}} + \frac{M}{T^{2n+4}}} \tag{26}$$

From equations (23) and (24), we get

$$\frac{\sigma}{\theta} = \frac{n-1}{\sqrt{3}(n+2)} = \text{constant}, \ (n \neq -2) \tag{27}$$

## 5. Conclusion

The model (18) starts expanding with big-bang at $T = 0$. The expansion $(\theta)$ decreases as time increases for $-2 < n < 2$. We also observe that it approaches to zero as $T \to \infty$ and stops when $n = -2$

The Spatial Volume ($R^3$) increases as time increases (when $T \to \infty, R^3 \to \infty$) for $n > -2$. It represents inflationary scenario of universe containing massless scalar field with flat potential.

Since $\lim_{T\to\infty} \frac{\sigma}{\theta} \neq 0$, hence the model represents anisotropic space-time in general.

The Hubble parameter $(H)$ decreases as time increases for $-2 < n < 2$, in model. The energy density and pressure of model is initially large.

when $n=2$ , $T \to \infty$ then deceleration parameter tends to $-1$ , so the model represent accelerating phase of universe.

The rate of Higgs field $(\phi)$ is initially large, but decreases as time increases for $n > -\frac{1}{2}$ and constant for $T \to \infty$. The model has point type singularity at $T = 0$ .

*Received October 16, 2022; Accepted December 02, 2022*

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