Article

Interacting Two Fluid Dark Energy Bianchi Type-I Radiating Cosmological Model in *f*(*R*) Gravity

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Abstract

In this paper, we have investigated some features of purely past decelerated to present accelerating locally rotationally symmetric Bianchi type-I cosmological model in the presence of interacting two fluids namely matter and dark energy in the framework of $f(R)$ theory of gravity. In this study we have produced the solution of field equations towards hybrid expansion law which makes the model past decelerated to present accelerating expansion. It is observed that the stable Universe has point-type singularity with an equation of state parameter showing Quintessence region and evolving towards ΛCDM which may also be established for the current accelerated expansion of the Universe.

Keywords: Bianchi type-I model, matter, dark energy, $f(R)$ gravity, cosmology.

1. Introduction

The most striking discovery of modern cosmology indicates that the current Universe is expanding but the expansion is accelerating, this behavior of the Universe is confirmed by various independent observational data like Type-Ia Supernovae (SNe-Ia) [1–4], Cosmic Microwave Background Radiation (CMBR) [5] and Large Scale Structure (LSS) [6,7]. In addition, measurements of the CMBR and the LSS strongly indicate that our Universe is dominated by a component with negative pressure, dubbed as dark energy. The dark energy model has been characterized in a conventional manner by the equation of state parameter which is not necessarily constant.

The equation of state parameter lies close to -1. If equation of state parameter would be equal to -1 it represents standard ΛCDM cosmology, if the equation of state parameter be a little bit more than -1 it denotes the quintessence dark energy model while it is less than -1 corresponds to a phantom dark energy model, whereas the possibility $\omega < -1$ ruled out by current

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cosmological data from SNe-Ia. The simplest candidate for the dark energy is a cosmological constant Λ which has a negative pressure. Many authors have analyzed the dark energy model in which the equation of state parameter has either constant or vary with expansion.

In recent years, Hooft [8] presented an interesting observation to determine the nature of dark energy in quantum gravity which is termed as holographic dark energy using principle of holographic dark energy in the context of black hole physics. Latter on Fischler and Susskind [9] have proposed a new version of the holographic principle, viz. at any time during cosmological evolution, the gravitational entropy within a closed surface should not be always larger than the particle entropy that passes through the past light-cone of that surface.

An alternative to discuss the accelerated expansion of the Universe is modified gravity which is a modification of general theory of relativity. There are various modifications of general theory of relativity namely $f\left(R,T\right)$ gravity, $f\left(T\right)$ gravity, $f\left(R\right)$ gravity etc. In $f\left(R,T\right)$ gravity, the gravitational action includes an arbitrary function of the Ricci scalar $\left(R\right)$ and trace of the stress energy momentum tensor $\left(T\right)$. Shaikh and Wankhade [10] have investigated Hypersurface-Homogeneous cosmological model in $f\left(R,T\right)$ theory of gravity with a term Λ and for solving Einstein's field equations, they have used hybrid expansion law for scale factor that yields time dependent deceleration parameter, while the same author Shaikh and Wankhade [11] have investigated non-static plane symmetric model in the presence of domain walls in the framework of same theory.

One of the classical generalizations of general theory of relativity is $f\left(R\right)$ modified gravity in which one replaces the Ricci scalar R in the Einstein-Hilbert action by an arbitrary function of R. Recently Katore and Baxi [12] have studied stability of Kaluza–Klein holographic dark energy cosmological models in this theory and used volumetric expansion law, power law and hybrid expansion law to obtain the exact solutions of field equations. Shaikh and Katore [13] have studied the Bianchi type VI_0 space-time filled with Bulk viscous in the framework of $f\left(R\right)$ gravity.

Sharif and Shamir [14] have studied the vacuum solutions of Bianchi-type I and V space-times in the framework of $f\left(R\right)$ gravity and used power law to solve the field equations. Özdemir and Aktaş [15] have investigated generalized anisotropic Universe models for magnetized strange quark matter distribution in the framework of $f\left(R\right)$ theory of gravitation and they obtained magnetic field as zero for LRS Bianchi type-I Universe model. Santhi et al. [16] have studied bulk viscous string cosmological models in $f\left(R\right)$ gravity. Katore and Shaikh [17] have discussed Bianchi III bulk viscous cosmological model in $f(R)$ gravity. Odintsov and Oikonomou [18] have studied geometric inflation and dark energy with $f(R)$ gravity. Abebe et al. [19] have studied shear-free homogeneous anisotropic cosmological models with imperfect matter sources in $f(R)$ gravity.

2. Some Basics of *f*(*R*) Gravity

The $f(R)$ theory of gravity is the generalization of General Theory of Relativity. The three main approaches in $f(R)$ theory of gravity are "Metric approach", "Palatini formalism" and "affine $f(R)$ gravity". In the metric approach, the connection is Levi-Civita connection and variation of the action is done with respect to the metric tensor. While, in Palatini formalism, the metric and the connection are independent of each other and variation is done for the two mentioned parameters independently. In metric-affine $f(R)$ gravity, both the metric tensor and connection are treating independently and assuming the matter action depends on the connection as well. The action for this theory is given by

$$S=\int\sqrt{-g}\left(f(R)+L_m\right)d^4x \tag{1}$$

Here $f(R)$ is a general function of the Ricci scalar, g is the determinant of the metric $g_{\mu\nu}$ and L_m is the matter Lagrangian. It is noted that this action is obtained just by replacing R by $f(R)$ in the standard Einstein-Hilbert action. The corresponding field equations from this action are found

$$F(R)R_{\mu\nu}-\frac{1}{2}f(R)g_{\mu\nu}-\nabla_\mu\nabla_\nu F(R)+g_{\mu\nu}\Box F(R)=-T_{\mu\nu}\ , \tag{2}$$

where $\Box\equiv\nabla^\mu\nabla_\mu, F(R)\equiv\frac{df(R)}{dR}$, ∇_μ is the covariant derivative and $T_{\mu\nu}$ is the standard matter energy-momentum tensor derived from Lagrangian L_m .

3. Metric and Energy Momentum Tensor

Akarsu and Kılınç [20] have studied Locally Rotationally Symmetric (LRS) Bianchi type-I models with anisotropic dark energy and constant deceleration parameter. In this study, we also consider LRS Bianchi type-I space-time,

$$ds^2=-dt^2+A^2dx^2+B^2\left(dy^2+dz^2\right)\ , \tag{3}$$

where A and B are functions of t only. Some kinematical quantities related with the metric potential for the space-time (3) are defined as follows. The spatial volume V and mean scale factor a of the Universe are respectively defined as $V = a^3 = AB^2$. The directional Hubble parameters in the directions x, y, z respectively are $H_x = \frac{\dot{A}}{A}$, $H_y = \frac{\dot{B}}{B} = H_z$ .The mean Hubble parameter, which expresses the volumetric expansion rate of the Universe, is given as $H = \frac{\dot{a}}{a} = \frac{1}{3}\frac{\dot{V}}{V} = \frac{1}{3}\left(H_x + H_y + H_z\right) = \frac{1}{3}\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right)$.

The anisotropy parameter, to discuss whether the Universe either approach isotropy or not, is defined as $\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2$. Let us introduce the dynamical scalars such as expansion scalar and the shear as $\theta = 3H$, $\sigma^2 = \frac{3}{2}\Delta H^2$ .Another important dimensionless kinematical quantity is the mean deceleration parameter, which tells whether the Universe exhibits accelerating volumetric expansion or not $q = -1 + \frac{d}{dt}\left(\frac{1}{H}\right)$.The Universe exhibits accelerating volumetric expansion, decelerating volumetric expansion and the expansion with constant rate towards respectively for $-1 \le q < 0$, $q > 0$ and $q = 0$. Let us consider that the Universe consist of matter as well as holographic dark energy such that the energy momentum tensor $T_{\mu\nu}$ is taken as

$$T_{\mu\nu} = \hat{T}_{\mu\nu} + \overline{T}_{\mu\nu} \ , \tag{4}$$

where $\hat{T}_{\mu\nu}$ and $\overline{T}_{\mu\nu}$ stand for matter and holographic dark energy respectively and are defined as $\hat{T}_{\mu\nu} = \left(p_m + \rho_m\right)u_\mu u_\nu + g_{\mu\nu}p_m$ and $\overline{T}_{\mu\nu} = \left(p_\Lambda + \rho_\Lambda\right)u_\mu u_\nu + g_{\mu\nu}p_\Lambda$, u^μ is the four velocity vector of the fluid satisfying $u^\mu = \left(0,0,0,1\right)$ and $u^\mu u_\nu = -1$, p_m and ρ_m be the pressure and energy density of the matter fluid and p_Λ and ρ_Λ be the pressure and energy density of the holographic dark energy fluid respectively.

4. Field equations and their solutions

In the presence of fluid source given in equation (4), the field equations (2) corresponding to the metric (3) lead to the following set of linearly independent differential equations

$$\left(\frac{\ddot{A}}{A} + 2\frac{\dot{A}\dot{B}}{AB}\right)F\left(R\right) + \frac{1}{2}f\left(R\right) + 2\frac{\dot{B}}{B}\dot{F} + \ddot{F} = p_m + p_\Lambda \ , \tag{5}$$

$$\left(\frac{\ddot{B}}{B}+\frac{\dot{B}^2}{B^2}+\frac{\dot{A}\dot{B}}{AB}\right)F\left(R\right)+\frac{1}{2}f\left(R\right)+\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}\right)\dot{F}+\ddot{F}=p_m+p_\Lambda, \tag{6}$$

$$\left(\frac{\ddot{A}}{A}+2\frac{\ddot{B}}{B}\right)F\left(R\right)+\frac{1}{2}f\left(R\right)+\left(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}\right)\dot{F}=-\left(\rho_m+\rho_\Lambda\right). \tag{7}$$

Here the overhead dot denotes differentiation with respect to cosmic time t. Using power law relation between F and a, established by Johri and Desikan [21]; Uddin et al. [22] and Sharif and Shamir [14,23] in the $f\left(R\right)$ gravity which shows that $F \propto \frac{1}{a^\gamma}$. Therefore, the equation leads to $F=b_1a^{-\gamma}$, where γ is arbitrary constant and b_1 is the proportionality constant. Without loss of generality take $b_1=1$. Hence , we obtain $F=\left(AB^2\right)^{-\gamma/3}$ .The solutions to the field equations are generated by using hybrid expansion law, used by Kumar [24]; Akarsu et al. [25] and Shaikh et al. [26],

$$a=a_1t^\alpha e^{\beta t}, \tag{8}$$

where $a_1>0$ and $\alpha\geq 0,\ \beta\geq 0$ are constants. Without loss of generality taking $a_1=1$, equation (8) becomes $a=t^\alpha e^{\beta t}$ .For $\beta=0$, equation (8) leads to power law and for $\alpha=0$ it leads to exponential law. These are the special cases of hybrid expansion law for which power and exponential law of expansion recovers separately. Therefore, using equation (8), it gives

$$AB^2=t^{3\alpha}e^{3\beta t}. \tag{9}$$

Therefore, we have

$$F=t^{-\alpha\gamma}e^{-\beta t\gamma}. \tag{10}$$

Now for solving the system of equations completely, we assume that the expansion scalar is proportional to the shear scalar [Collins et al. [27] and Shamir [28]]. This gives the relation between the metric potentials as

$$A=B^m, \tag{11}$$

where $m>1$ is an arbitrary constant. Using equations (9) and (11) , we obtain

$$A=t^{3\alpha m/(m+2)}e^{3\beta tm/(m+2)} \tag{12}$$

and

$$B=t^{3\alpha/(m+2)}e^{3\beta t/(m+2)}, \tag{13}$$

From the equations (12) and (13), it is observed that the metric potentials A and B are the product of power and exponential form and increase indefinitely with the passage of time. Using equations (12) and (13) in equation (3), we get

$$ds^2=-dt^2+t^{6\alpha m/(m+2)}e^{6\beta tm/(m+2)}dx^2+t^{6\alpha/(m+2)}e^{6\beta t/(m+2)}\left(dy^2+dz^2\right). \tag{14}$$

At $t=0$, all the metric potentials in the derived model vanish, hence the model has initial singularity. Afterwards, increase indefinitely with the passage of time, which is in complete agreement with the Big-Bang model of the Universe.

5. Interacting two fluid models

In this case, the densities of holographic dark energy and matter no longer satisfy independent conservation laws, they obey instead $(\dot{\rho}_m)+3\frac{\dot{a}}{a}(\rho_m+p_m)=Q$,$(\dot{\rho}_\Lambda)+3\frac{\dot{a}}{a}(\rho_\Lambda+p_\Lambda)=-Q$. The quantity $Q(Q>0)$ expressed the interaction term between the dark energy baratropic matter components. Since we are interested to investigate the interaction between dark energy and matter, it should be noted that an ideal interaction term must be motivated from the theory of quantum gravity. In the absence of such a theory, we rely on pure dimensional basis for choosing an interaction Q. In our work we consider the interaction term in the form $Q \propto H\rho_m$. $Q=3Hk\rho_m$, where k is a coupling coefficient which can be considered as a constant or variable parameter of redshift.

(1) Physical Properties of Model

The Ricci Scalar and function of the Ricci scalar of the model are found to be

$$R=\frac{6\alpha}{t^2}-\frac{18(m^2+2m+3)}{(m+2)^2}\left(\beta+\frac{\alpha}{t}\right)^2$$

and

$$f(R)=\frac{6\alpha}{t^2}-\frac{18(m^2+2m+3)}{(m+2)^2}\left(\beta+\frac{\alpha}{t}\right)^2+b\left\{\frac{6\alpha}{t^2}-\frac{18(m^2+2m+3)}{(m+2)^2}\left(\beta+\frac{\alpha}{t}\right)^2\right\}^n,$$

where b and n are arbitrary constants.

The energy density of matter fluid for interacting case as

$$\rho_m=c_2t^{-3\alpha(1+\omega_m-k)}e^{-3\beta t(1+\omega_m-k)}, \tag{15}$$

where c_2 be the constant of integration, $\omega_m=p_m/\rho_m$ is equation of state parameter of matter and considered as constant [29]. We find some other parameters of dark fluid / energy in interacting two fluid models such as: Energy density of holographic dark energy,

$$\rho_\Lambda = \left\{ \begin{array}{l} \frac{3}{(m+2)^2}\left(\beta+\frac{\alpha}{t}\right)^2 \left\{\left[\gamma(m+2)^2 - 3(m^2+2)\right] t^{-\alpha\gamma} e^{-\beta t\gamma} + 3(m^2+2m+3)\right\} \\ +\frac{3\alpha}{t^2}\left(t^{-\alpha\gamma}e^{-\beta t\gamma}-1\right) - \frac{b}{2}\left\{\frac{6\alpha}{t^2} - \frac{18(m^2+2m+3)}{(m+2)^2}\left(\beta+\frac{\alpha}{t}\right)^2\right\}^n \\ -c_2 t^{-3\alpha(1+\omega_m-k)} e^{-3\beta t(1+\omega_m-k)} \end{array} \right\}. \tag{16}$$

In the derived hybrid law of expansion of the Universe, it is observed that the energy density of both the fluid i.e. matter and holographic dark energy always positive and decreasing function of cosmic time t. At the initial at which the Universe starts to accelerate, the Universe has infinitely large energy density but with the expansion of the Universe it declines and at very large expansion, it is null. The behavior is clearly shown in Fig. 1.

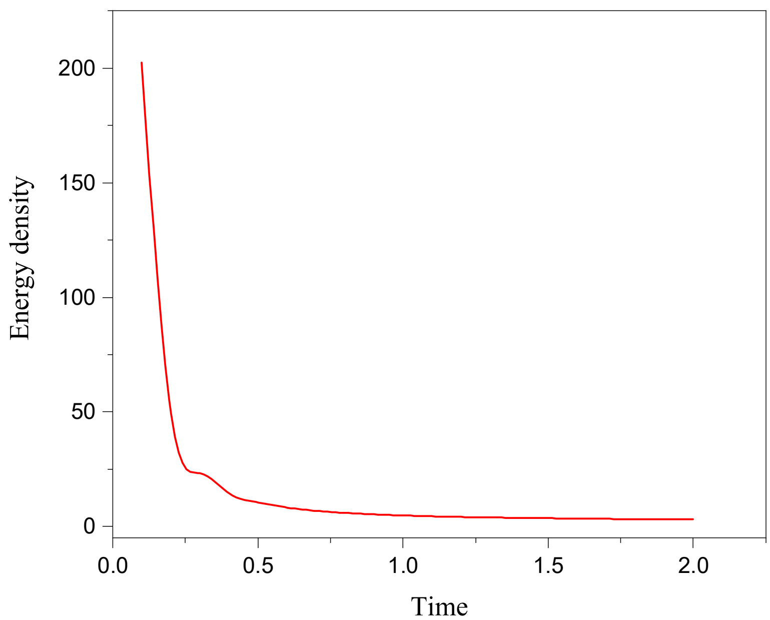

Fig. 1. Graphical representation of energy density of holographic dark energy versus cosmic time with appropriate choice of constants $m=2$, $\alpha=0.5$, $\beta=0.5$, $\gamma=0.5$, $b=0.1$, $n=1$, $k=1$, $c_2=0.1$ and $\omega_m=1/3$.

Isotropic pressure of holographic dark energy,

$$p_\Lambda = \left\{ \begin{array}{l} \dfrac{3}{(m+2)^2}\left(\beta+\dfrac{\alpha}{t}\right)^2 \left\{(m+2)\left[3m-2\gamma+\dfrac{\gamma^2(m+2)}{3}\right]t^{-\alpha\gamma}e^{-\beta t\gamma}-3\left(m^2+2m+3\right)\right\} \\ +\dfrac{\alpha}{t^2}\left\{\left[\gamma-\dfrac{3m}{(m+2)}\right]t^{-\alpha\gamma}e^{-\beta t\gamma}+3\right\}+\dfrac{b}{2}\left\{\dfrac{6\alpha}{t^2}-\dfrac{18\left(m^2+2m+3\right)}{(m+2)^2}\left(\beta+\dfrac{\alpha}{t}\right)^2\right\}^n \\ -c_2\omega_m t^{-3\alpha(1+\omega_m-k)}e^{-3\beta t(1+\omega_m-k)} \end{array} \right\}. \quad (17)$$

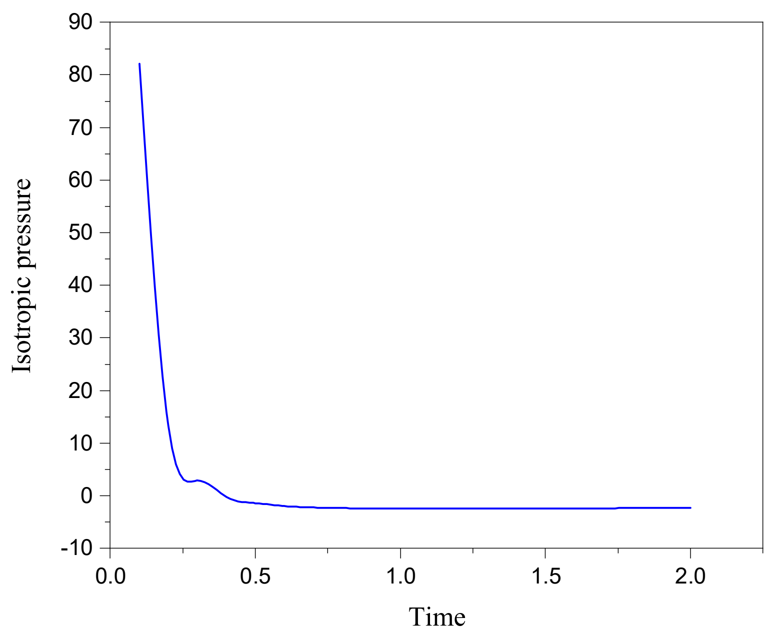

Fig. 2 Graphical representation of isotropic pressure of holographic dark energy versus cosmic time with appropriate choice of constants $m=2$, $\alpha=0.5$, $\beta=0.5$, $\gamma=0.5$, $b=0.1$, $n=1$, $k=1$, $c_2=0.1$ and $\omega_m=1/3$.

Equation of state parameter of holographic dark energy,

$$\omega_\Lambda = \frac{\left\{\begin{array}{l} \frac{3}{(m+2)^2}\left(\beta+\frac{\alpha}{t}\right)^2\left\{(m+2)\left[3m-2\gamma+\frac{\gamma^2(m+2)}{3}\right]t^{-\alpha\gamma}e^{-\beta t\gamma}-3\left(m^2+2m+3\right)\right\} \\ +\frac{\alpha}{t^2}\left\{\left[\gamma-\frac{3m}{(m+2)}\right]t^{-\alpha\gamma}e^{-\beta t\gamma}+3\right\}+\frac{b}{2}\left\{\frac{6\alpha}{t^2}-\frac{18\left(m^2+2m+3\right)}{(m+2)^2}\left(\beta+\frac{\alpha}{t}\right)^2\right\}^n \\ -c_2\omega_m t^{-3\alpha(1+\omega_m-k)}e^{-3\beta t(1+\omega_m-k)} \end{array}\right\}}{\left\{\begin{array}{l} \frac{3}{(m+2)^2}\left(\beta+\frac{\alpha}{t}\right)^2\left\{\left[\gamma(m+2)^2-3\left(m^2+2\right)\right]t^{-\alpha\gamma}e^{-\beta t\gamma}+3\left(m^2+2m+3\right)\right\} \\ +\frac{3\alpha}{t^2}\left(t^{-\alpha\gamma}e^{-\beta t\gamma}-1\right)-\frac{b}{2}\left\{\frac{6\alpha}{t^2}-\frac{18\left(m^2+2m+3\right)}{(m+2)^2}\left(\beta+\frac{\alpha}{t}\right)^2\right\}^n \\ -c_2 t^{-3\alpha(1+\omega_m-k)}e^{-3\beta t(1+\omega_m-k)} \end{array}\right\}} \tag{18}$$

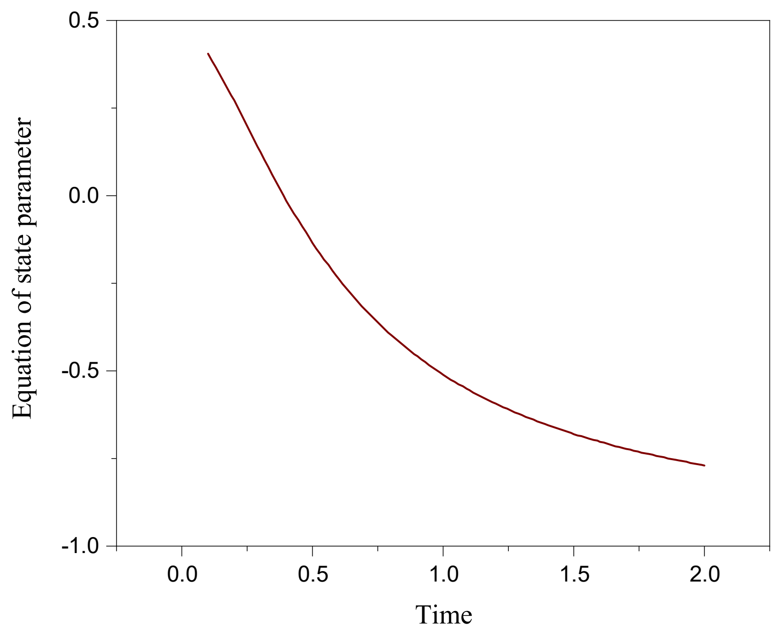

Fig. 3 Graphical representation of equation of state parameter of holographic dark energy versus cosmic time with the appropriate choice of constants $m=2$, $\alpha=0.5$, $\beta=0.5$, $\gamma=0.5$, $b=0.1$, $n=1$, $k=1$, $c_2=0.1$ and $\omega_m=1/3$.

Setare and Saridakis [30] have studied the dark energy models with the equation of state parameter across (-1), which gives a concrete justification for the Quinton paradigm. Some other limits of equation of state parameter are obtained from observational results that came from SNe-Ia data and a combination of SNe-Ia data with CMBR anisotropy and Galaxy clustering

statistics are $-1.66 < \omega_\Lambda < -0.62$ and $-1.33 < \omega_\Lambda < -0.79$ respectively. The latest result in 2009, obtained after a combination of cosmological data sets coming from CMBR anisotropy, luminosity distances of high red-shift SNe-Ia and galaxy clustering constrain the dark energy equation of state to $-1.44 < \omega_\Lambda < -0.92$. In the derived model, the equation of state parameter shows Quintessence region, whereas at large expansion it is evolving around ΛCDM model i.e. $\omega_\Lambda = -1$, which may be established from the current accelerated expansion of the Universe. From Fig. 3, we observed that at the initial time the model represents Quintessence region then it converts to a ΛCDM model and as the universe expand, the value of equation of state parameter i.e. $\omega_\Lambda = -1$ behaves like cosmological constant which resembles with the recent observations.
Stability factor of holographic dark energy,

$$\upsilon_s^2 = \frac{\left\{\begin{aligned} &\frac{-3}{(m+2)^2}\left(\beta+\frac{\alpha}{t}\right)\left\{(m+2)\left[3m-2\gamma+\frac{\gamma^2(m+2)}{3}\right]t^{-\alpha\gamma}e^{-\beta t\gamma}\left[\gamma\left(\beta+\frac{\alpha}{t}\right)^2+\frac{2\alpha}{t^2}\right]-\frac{6\alpha}{t^2}\left(m^2+2m+3\right)\right\} \\ &-\frac{\alpha}{t^2}\left\{\left[\gamma-\frac{3m}{(m+2)}\right]t^{-\alpha\gamma}e^{-\beta t\gamma}\left[\gamma\left(\beta+\frac{\alpha}{t}\right)+\frac{2}{t}\right]+\frac{6}{t}\right\} \\ &+\frac{6bn\alpha}{t^2}\left\{\frac{3\left(m^2+2m+3\right)}{(m+2)^2}\left(\beta+\frac{\alpha}{t}\right)-\frac{1}{t}\right\}\left\{\frac{6\alpha}{t^2}-\frac{18\left(m^2+2m+3\right)}{(m+2)^2}\left(\beta+\frac{\alpha}{t}\right)^2\right\}^{n-1} \\ &+3c_2\omega_m\left(1+\omega_m-k\right)\left(\beta+\frac{\alpha}{t}\right)t^{-3\alpha(1+\omega_m-k)}e^{-3\beta t(1+\omega_m-k)} \end{aligned}\right\}}{\left\{\begin{aligned} &\frac{-3}{(m+2)^2}\left(\beta+\frac{\alpha}{t}\right)\left\{\left[\gamma(m+2)^2-3\left(m^2+2\right)\right]t^{-\alpha\gamma}e^{-\beta t\gamma}\left[\gamma\left(\beta+\frac{\alpha}{t}\right)^2+\frac{2\alpha}{t^2}\right]+\frac{6\alpha}{t^2}\left(m^2+2m+3\right)\right\} \\ &-\frac{3\alpha}{t^2}\left\{t^{-\alpha\gamma}e^{-\beta t\gamma}\left[\gamma\left(\beta+\frac{\alpha}{t}\right)+\frac{2}{t}\right]-\frac{2}{t}\right\} \\ &-\frac{6bn\alpha}{t^2}\left\{\frac{3\left(m^2+2m+3\right)}{(m+2)^2}\left(\beta+\frac{\alpha}{t}\right)-\frac{1}{t}\right\}\left\{\frac{6\alpha}{t^2}-\frac{18\left(m^2+2m+3\right)}{(m+2)^2}\left(\beta+\frac{\alpha}{t}\right)^2\right\}^{n-1} \\ &+3c_2\left(1+\omega_m-k\right)\left(\beta+\frac{\alpha}{t}\right)t^{-3\alpha(1+\omega_m-k)}e^{-3\beta t(1+\omega_m-k)} \end{aligned}\right\}}. \tag{19}$$

For the stability of corresponding solutions of the derived Universe, we should check that our Universe is physically acceptable. For this, firstly it is required that the velocity of sound should be less than velocity of light i.e. within the range $0 < \upsilon_s = \frac{\partial p}{\partial \rho}$. From the Fig. 4, it is observed that from initial to infinite expansion of the Universe, the stability factor having range $\upsilon_s > 0$. Hence the Universe is stable.

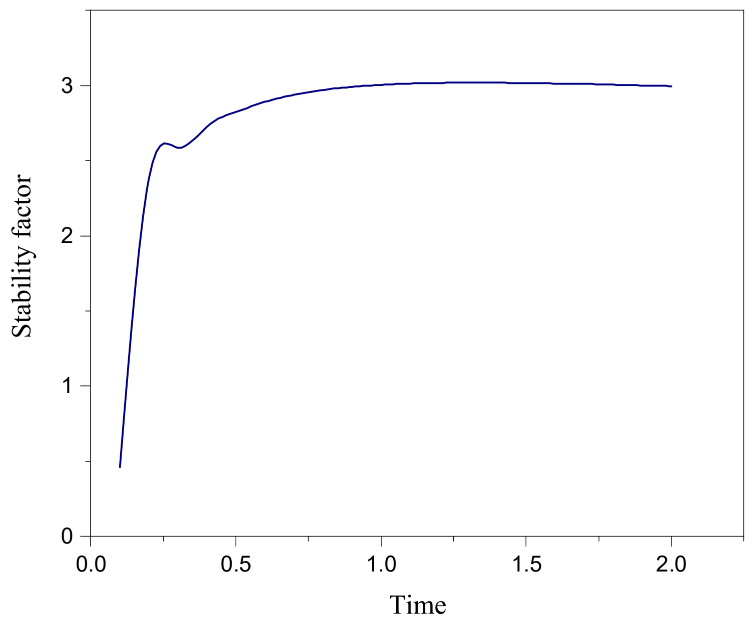

Fig. 4 Graphical representation of stability factor of holographic dark energy versus cosmic time with the appropriate choice of constants $m = 2$, $\alpha = 0.5$, $\beta = 0.5$, $\gamma = 0.5$, $b = 0.1$, $n = 1$, $k = 1$, $c_2 = 0.1$ and $\omega_m = 1/3$.

(2) Geometrical parameters

The geometrical parameters which describes the kinematics of the Universe that are spatial volume, mean scale factor, mean Hubble parameter, expansion scalar, deceleration parameter, anisotropy parameter and shear scalar are found as $V = t^{3\alpha} e^{3\beta t}$,mean scale factor $a = t^{\alpha} e^{\beta t}$ .We observe that mean scale factor and spatial volume both vanish at $t = 0$. Therefore, at $t = 0$, point-type singularity exists in the resultant model. The mean Hubble parameter $H = \left(\beta + \frac{\alpha}{t}\right)$.

The expansion scalar $\theta = 3\left(\beta + \frac{\alpha}{t}\right)$.

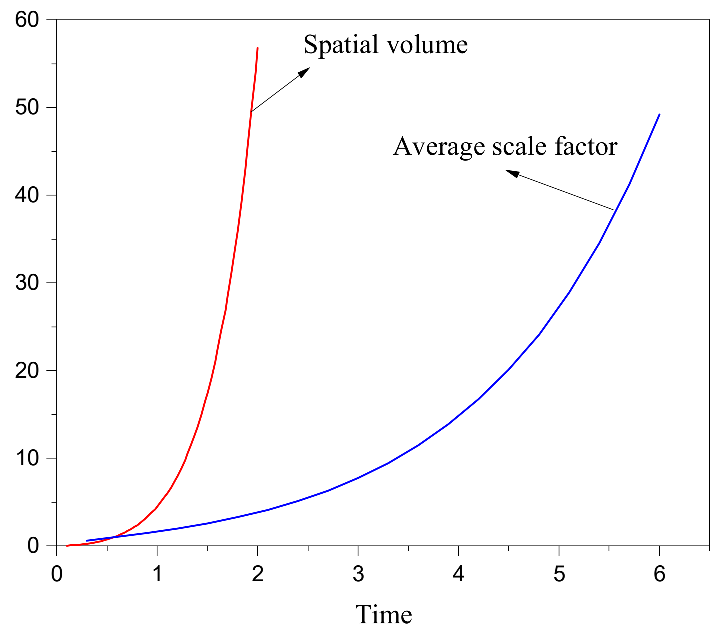

Fig. 5. Graphical representation of average scale factor and spatial volume versus cosmic time with the appropriate choice of constants $\alpha = 0.5$ and $\beta = 0.5$.

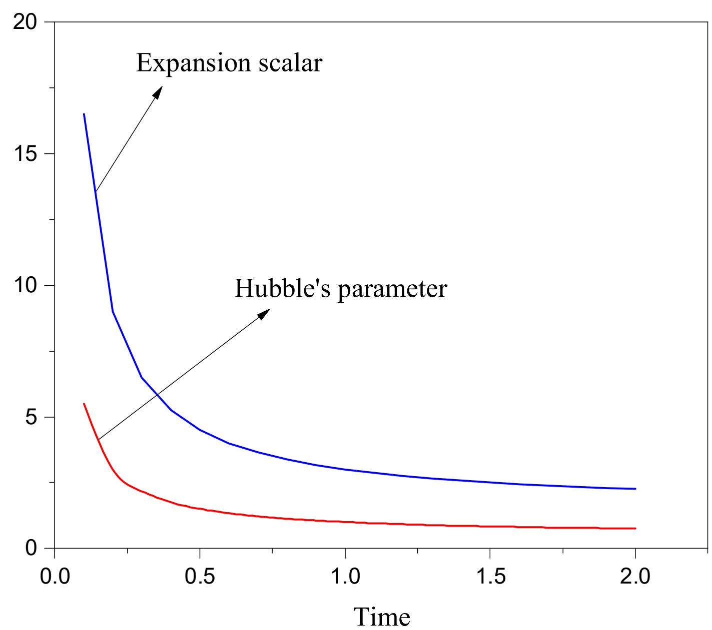

Fig. 6. Graphical representation of Hubble's parameter and expansion scalar versus cosmic time with the appropriate choice of constants $\alpha = 0.5$ and $\beta = 0.5$.

From Fig. 5, we observe that the spatial volume and average scale factor both disappear at $t \to 0$. Thus, the point-type singularity exists at $t \to 0$ in the subsequent model. The model starts evolving with a big-bang at $t \to 0$. From Fig. 6, the expansion scalar decreases as time increases and the mean Hubble parameter is initially large at $t \to 0$ and constant at $t \to \infty$. The expansion scalar $\theta \to 3\beta$ as $t \to \infty$ which indicates that the Universe is expanding with increase of time and the rate of expansion decreases with the increase of time. This suggests that at initial stage of the Universe the expansion of the model is much faster and then slows down for later time i.e. the evolution of the Universe starts with infinite rate, and with expansion, it declines. The deceleration parameter,

$$q = -1 + \frac{\alpha}{t^2\left(\beta + \frac{\alpha}{t}\right)^2}.$$

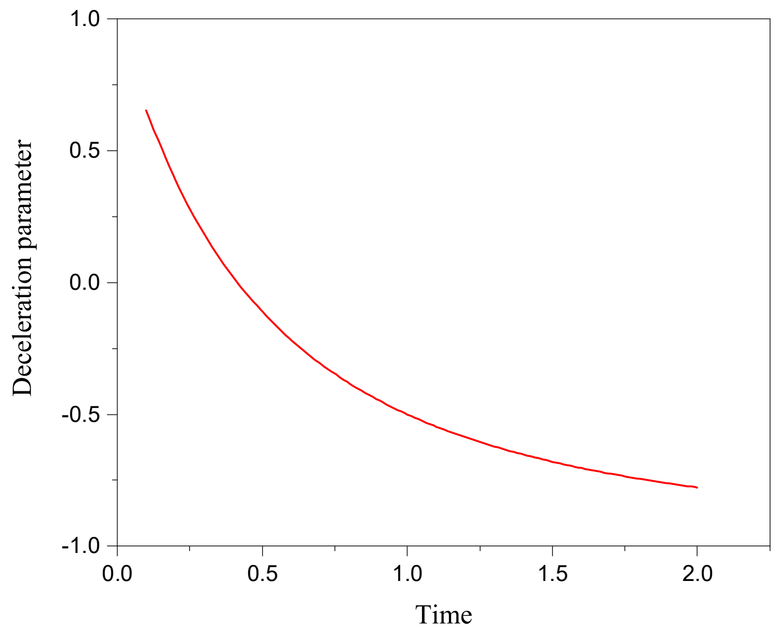

Fig. 7. Graphical representation of deceleration parameter versus cosmic time with the appropriate choice of constants $\alpha = 0.5$ and $\beta = 0.5$. The deceleration parameter comes out to be a time dependent. $q = \alpha - 1(>0)$ for $\alpha > 1$, the sign of q becomes positive which correspond to the standard decelerating expansion, with the expansion of the Universe the sign of q become negative which correspond to the standard accelerating expansion.

From Fig. 7, we observe that initially the deceleration parameter is positive but with the passage of time it is going to be negative and as $t \to \infty$, $q \to -1$. This scenario is consistent with recent theoretical observations [31-33].

Anisotropy parameter $\Delta = \frac{2(m-1)^2}{(m+2)^2}$. The shear scalar $\sigma^2 = \frac{3(m-1)^2}{(m+2)^2}\left(\beta + \frac{\alpha}{t}\right)^2$.

Since the evolution of the Universe starts with zero volume and the rate of expansion is infinite. As t increases, the spatial volume increases but the expansion scalar decreases. The mean anisotropy parameter is independent on time t and constant throughout the evolution of the Universe from early to infinite expansion but shear scalar is time dependent and decreases with time. Hence, the rate of expansion of the Universe decreases as the time increases. When $t \to \infty$, the spatial volume becomes infinitely large but the expansion stops and shear becomes zero.

6. Conclusion

In the analysis of interacting two fluid dark energy LRS Bianchi type-I cosmological model in $f(R)$ gravity with hybrid expansion law, it is observed that the energy density of matter and holographic dark energy are always positive and decreasing function of time. At the initial at which the Universe starts to accelerate, the Universe has infinitely large energy density but with the expansion it declines and at very large it is null. The equation of state parameter shows Quintessence region, whereas at large expansion it is evolving around ΛCDM model i.e. $\omega_\Lambda = -1$, which may be established from the current accelerated expansion of the Universe. Also, it is observed that from initial to infinite expansion of the Universe, the stability factor having range $\upsilon_s > 0$.

Hence the Universe is stable. The model has singularity at $t \to 0$ due to which the spatial volume and average scale factor both disappear at $t \to 0$. Thus, in the subsequent model, the point-type singularity exists. The model starts evolving with a big-bang at $t \to 0$. The expansion scalar decreases as time increases and the mean Hubble parameter is initially large at $t \to 0$ and constant at $t \to \infty$. The expansion scalar $\theta \to 3\beta$ as $t \to \infty$ which indicates that the Universe is expanding with increase of time and the rate of expansion decreases with the increase of time. This suggests that at initial stage of the Universe the expansion of the model is much faster and then slows down for later time i.e. the evolution of the Universe starts with infinite rate and with expansion, it declines, with the expansion of the Universe the sign of q become negative which correspond to the standard accelerating expansion. This scenario is consistent with recent theoretical observations.

The mean anisotropy parameter is independent on time t and constant throughout the evolution of the Universe from early to infinite expansion but shear scalar is time dependent and decreases with time. Hence, at infinitely large expansion at which the spatial volume becomes infinity the expansion stops and shear becomes zero.

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