Article

Entropy & Viscosity of Spatial Flow

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Abstract

This letter illustrates how viscosity of spatial flow in general relativity is proportional to the entropy of that spacetime.

Keywords: Hydrodynamics, flow of space, entropy, viscosity.

General relativity is often cited as the geodesic flow of particles in curved spacetime. However, more generally it can be seen as the dynamics of spatial surfaces, selected from a moduli of diffeomorphic freedoms by a coordinate condition, that foliate spacetime. The points on the spatial surface may separate or converge depending on the dynamics. This is then analogous to a fluid flow. Here, the fluid flow of space is considered from the case of the Schwarzschild black hole. The viscosity of this fluid flow, such as proposed in [1], is found to have a linear dependence on the entropy of the black hole. This then connects the entropy of spacetime, such as spacetime composed of entanglements[2], entropy as a form of quantum entropy $S = -k\,Tr[\rho log(\rho)]$ with the viscosity that is determines the geometrodynamics of space.

We consider the dynamics of space as a type of fluid flow. This is similar to our understanding of how spatial surfaces are embedded in spacetime, say within some coordinate condition, so there is a flow of space. The frame dragging of galaxies in the expansion of the universe by the flow of space is one example. The flow of space into a black hole is another. We may consider this within the construct of both Newtonian logic of $F = \frac{dp}{dt}$ and where this force, or acceleration, is given by the geodesic deviation equation.

The $\frac{dp}{dt}$ or ma portion is considered to act on a fluid with density ρ and velocity v. The term ρv is a momentum density, where for the flow of the vacuum or space there is a mass-energy density. With $\frac{d}{dt} = \frac{\partial}{\partial t} + \frac{d\mathbf{r}}{dt}\cdot\nabla$ the Navier-Stokes equation is easily derived

$$\frac{d\rho\mathbf{v}}{dt} = \frac{\partial\rho\mathbf{v}}{\partial t} + \frac{d\mathbf{r}}{dt}\cdot\nabla\rho\mathbf{v}.$$

The general form of the Navier Stokes equation is expressed according to a force term

$$\frac{d\rho\mathbf{v}}{dt}\ \nabla\cdot(\mathbf{v}\otimes\mathbf{v}) = F$$

The force term on the right is the main subject of concern. This is both the fluid force terms with viscosity and for the case with spacetime we will see the geodesic deviation equation.

The case with the geodesic deviation equation is derived in a manner like the entropy force of gravity by Verlinde[3]. The force ma uses the Unruh acceleration with $a = 2\pi ckT/\hbar$. The equipartition theorem tells us that temperature is determined by the energy so $kT = 2E/N$, for $E = Mc^2$ the mass-energy of the gravitating source or vacuum energy and N the number of harmonic oscillator modes. This force ma through a displacement δx is an increment of work or energy. This increment of energy is

$$\delta W = \frac{4\pi Mmc^3}{N\hbar}\delta x.$$

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For the case of a blackhole the number of oscillators is determined by a volume given by the horizon area times the separation distance $d\ N\ =\ Adc^3/G\hbar$ for A the area of the black hole. We then have the following equation for an increment of work

$$\delta W\ =\ \frac{GMm}{r^2}\delta x.$$

This equation is much the same as the entropy force of gravity. However, it is more desirable to work with a curvature that is defined by some area that does not enclose the gravitating mass.

The displacement is considered in a thickened area around the horizon. This thickened area, or volume collar, is a region of the quantum vacuum near the horizon with quantum field modes entangled with quantum fields just inside the black hole[4]. This is an appeal to the entropy as due to entanglement of quantum states across a holographic screen. This is similar to the concept proposed by Susskind for stitching spacetime, also with his octopus wormhole, and the argument for nonlocality of fields in spacetime quantum physics[5]. This collar region is considered to be a volume thickening of the horizon area by δr. The physics we wish to consider then involves the displacement δx be within this collar region so $\delta r\ >>\ \delta x$. For a large enough black hole then $\frac{\delta x}{\delta r}\ \simeq\ \frac{d}{r}$, for d the radial extent of the fluid, or equivalently the vacuum region of interest. This then leads to the increment in work

$$\delta W\ =\ \frac{GMmd}{r^3}\delta r\ =\ 4\pi G\rho m d\delta r.$$

The acceleration defined in this increment of work $\frac{GMd}{r^3}\ =\ R^t_{rtr}U^t dU^t$, for the 4-vectors $U^t\ \simeq\ 1$. The 4-vectors are approximately unit on the in falling frame. This means the equation is a form of the geodesic deviation equation.

This geodesic deviation equation can be shown to be

$$a\ =\ \frac{A\rho c^3 d}{3N\hbar},$$

where N is the number of Planck areas on the black hole horizon. The density is the number of Planck masses in a volume vol and so

$$a\ =\ \frac{Am_p c^3 d}{3\hbar}vol^{-1}\ =\ \frac{Ac^2 d}{3\ell_p}vol^{-1}.$$

The acceleration according to black hole entropy

$$a\ =\ \frac{4\ell_p dc^2}{3k}vol^{-1}$$

where the acceleration has linear dependence on entropy.

The Navier-Stokes equation is used to model the flow of space in the geometrodynamics of space as given by general relativity. The $\frac{d\rho\mathbf{v}}{dt}$, the ma part of the second law is equal to a force term with viscosity. These terms are[6]

$$\frac{d\rho\mathbf{v}}{dt}\ =\ \eta\nabla\cdot\left(\nabla v\ +\ \frac{2}{3}\nabla\cdot v\right),$$

for η the viscosity. We first restrict the velocity to the radial direction v_r. The second term is due to the contraction of the volume with $\nabla\cdot\nabla v\neq\ 0$. These have contribution from the curvatures $R^t_{\theta t\theta}$ which for the Schwarzschild solution is negative half the $R^t_{rtr}U^t dU^t$ This means the compressible fluid leaves us with the simplified form

$$\frac{d\rho\mathbf{v}}{dt}\ =\ \eta\nabla^2 v_r.$$

The surface bounding a volume is an invariant, which means $\int d\sigma\nabla\cdot\mathbf{r}$ may be evaluated as some charge. called Qv. We may then write $\nabla^2\mathbf{v} = 4\pi Q\hat{\mathbf{v}}vol^{-1}$. By dimensional reasoning $Q = cn\ell_p$, where the length $n\ell_p = d$. We may now put this together with the righthand side of the geodesic deviation equation to find the viscosity per unit mass is

$$\eta = \frac{s\ell_p c}{\pi k},$$

For s the entropy density of spacetime.

This illustrates a connection between an effective viscosity for the flow of space and entropy or equivalently quantum information composing space. This may apply as well to cosmology with the cosmological horizon at $r = \sqrt{3/\Lambda}$[7]. The horizon area is $A = 12\pi/\Lambda$ and defines the entropy $\simeq 10^{145}j/K$ of this region., This viscosity per mass is then a constant associated with the accelerated expansion of the universe.

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