Article

Progress in TGD: Physics as Geometry & Number Theory

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Abstract

This article is the first one in a series of 3 articles, trying to give a rough overall view about Topological Geometrodynamics (TGD) as it is towards the end of 2021. The preparation of this article series led to considerable progress in several aspects of TGD. In the first article the two views about TGD are discussed at the general level:

1. The first view generalizes Einstein's program for the geometrization of physics. Entire quantum physics is geometrized in terms of the notion of "world of classical worlds" (WCW), which by its infinite dimension has unique Kähler geometry. Physical states correspond to WCW spinor fields. WCW consists of space-time surfaces, which are minimal surfaces in $H = M^4 \times CP_2$ analogous to Bohr orbits so that they realize holography forced by general coordinate invariance. H is fixed uniquely by standard model symmetries and quantum numbers and also by the twistor lift of TGD.
2. Second vision reduces physics to number theory. Classical number fields (reals, complex numbers, quaternions, and octonions) are central as also p-adic number fields and extensions of rationals. The physics is classically coded by algebraic 4-surfaces in complexified M^8 having octonionic structure and "roots" of octonionic polynomials obtained as algebraic continuations of real polynomials with rational coefficients. M^8_c has an interpretation as an analog of momentum space.

$M^8 - H$ duality, which generalizes the momentum-position duality of wave mechanics, relates these two approaches and is discussed in the second article of the series. Third article in the series is devoted to neutrinos in the TGD framework. The motivation is that the recent results about neutrinos led to a breakthrough also in the construction of quantum theory based on the WCW approach. Several new results are obtained:

1. The realization that M^8 has an interpretation as analog of momentum space, has led to a breakthrough in the understanding of the theory.
2. One interesting result relates to the often neglected fact that the mutual entanglement of fermions (bosons) as elementary particles is always maximal so that only fermionic and bosonic degrees can entangle in QFTs. The replacement of point-like particles with 3-surfaces forces us to reconsider the notion of identical particles from the category theoretical point of view.

   The number theoretic definition of particle identity seems to be the most natural and implies that the new degrees of freedom make possible geometric entanglement. Also the notion particle generalizes: also many-particle states can be regarded as particles with the constraint that the operators creating and annihilating them satisfy commutation/anticommutation relations. This leads to a close analogy with the notion of infinite prime.

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Contents

1 Introduction

This article is the first one in the series of 3 articles devoted to the state of TGD towards the end of 2021. It must be emphasized that TGD is only a vision, not a theory able to provide precise rules for calculating scattering amplitudes. A collective theoretical and experimental effort would be needed to achieve this.

It is perhaps good to explain what TGD is not and what it is or hoped to be. The article [33] gives an overview of various aspects of TGD and is warmly recommended.

1. "Geometro-" refers to the idea about the geometrization of physics. The geometrization program of Einstein is extended to gauge fields allowing realization in terms of the geometry of surfaces so that Einsteinian space-time as abstract Riemann geometry is replaced with sub-manifold geometry. The basic motivation is the loss of classical conservation laws in General Relativity Theory (GRT)(see **Fig. 1**). Also the interpretation as a generalization of string models by replacing string with 3-D surface is natural.

   Standard model symmetries uniquely fix the choice of 8-D space in which space-time surfaces live to $H = M^4 \times CP_2$ [17]. Also the notion of twistor is geometrized in terms of surface geometry and the existence of twistor lift fixes the choice of H completely so that TGD is unique [21, 23](see **Fig. 7**).

The geometrization applies even to the quantum theory itself and the space of space-time surfaces - "world of classical worlds" (WCW) - becomes the basic object endowed with Kähler geometry (see **Fig. 8**). General Coordinate Invariance (GCI) for space-time surfaces has dramatic implications. Given 3-surface fixes the space-time surface almost completely as analog of Bohr orbit (preferred extremal).This implies holography and leads to zero energy ontology (ZEO) in which quantum states are superpositions of space-time surfaces.

2. Consider next the attribute "Topological". In condensed matter physical topological physics has become a standard topic. Typically one has fields having values in compact spaces, which are topologically non-trivial. In the TGD framework space-time topology itself is non-trivial as also the topology of $H = M^4 \times CP_2$.

   The space-time as 4-surface $X^4 \subset H$ has a non-trivial topology in all scales and this together with the notion of many-sheeted space-time brings in something completely new. Topologically trivial Einsteinian space-time emerges only at the QFT limit in which all information about topology is lost (see **Fig. 3**).

   Practically any GCI action has the same universal basic extremals: CP_2 type extremals serving basic building bricks of elementary particles, cosmic strings and their thickenings to flux tubes defining a fractal hierarchy of structure extending from CP_2 scale to cosmic scales, and massless extremals (MEs) define space-time correletes for massless particles. World as a set or particles is replaced with a network having particles as nodes and flux tubes as bonds between them serving as correlates of quantum entanglement.

   "Topological" could refer also to p-adic number fields obeying p-adic local topology differing radically from the real topology (see **Fig. 10**).

3. Adelic physics fusing real and various p-adic physics are part of the number theoretic vision, which provides a kind of dual description for the description based on space-time geometry and the geometry of "world of classical" orders. Adelic physics predicts two fractal length scale hierarchies: p-adic length scale hierarchy and the hierarchy of dark length scales labelled by $h_{eff} = nh_0$, where n is the dimension of extension of rational. The interpretation of the latter hierarchy is as phases of ordinary matter behaving like dark matter. Quantum coherence is possible in all scales.

   The concrete realization of the number theoretic vision is based on $M^8 - H$ duality (see **Fig. 12**). The physics in the complexification of M^8 is algebraic - field equations as partial differential equations are replaced with algebraic equations associating to a polynomial with rational coefficients a X^4 mapped to H by $M^8 - H$ duality. The dark matter hierarchy corresponds to a hierarchy of algebraic extensions of rationals inducing that for adeles and has interpretation as an evolutionary hierarchy (see **Fig. 9**).

   $M^8 - H$ duality provides two complementary visions about physics (see **Fig. 2**), and can be seen as a generalization of the q-p duality of wave mechanics, which fails to generalize to quantum field theories (QFTs).

4. In Zero energy ontology (ZEO), the superpositions of space-time surfaces inside causal diamond (CD) having their ends at the opposite light-like boundaries of CD, define quantum states. CDs form a scale hierarchy (see **Fig. 13** and **Fig. 14**).

   Quantum jumps occur between these and the basic problem of standard quantum measurement theory disappears. Ordinary state function reductions (SFRs) correspond to "big" SFRs (BSFRs) in which the arrow of time changes. This has profound thermodynamic implications and the question about the scale in which the transition from classical to quantum takes place becomes obsolete.

BSFRs can occur in all scales but from the point of view of an observer with an opposite arrow of time they look like smooth time evolutions.

In "small" SFRs (SSFRs) as counterparts of "weak measurements" the arrow of time does not change and the passive boundary of CD and states at it remain unchanged (Zeno effect).

In the first article the geometric and number theoretic views about TGD are discussed at the general level.

1. The geometric view generalizes Einstein's program for the geometrization of physics. Entire quantum physics is geometrized in terms of the notion of "world of classical worlds" (WCW), which by its infinite dimension has unique Kähler geometry. Physical states correspond to WCW spinor fields. WCW consists of space-time surfaces, which are minimal surfaces in $H = M^4 \times CP_2$ analogous to Bohr orbits so that they realize holography forced by general coordinate invariance. H is fixed uniquely by standard model symmetries and quantum numbers and also by the twistor lift of TGD.

2. Second vision reduces physics to number theory. Classical number fields (reals, complex numbers, quaternions, and octonions) are central as also p-adic number fields and extensions of rationals. The physics is classically coded by algebraic 4-surfaces in complexified M^8 having octonionic structure and "roots" of octonionic polynomials obtained as algebraic continuations of real polynomials with rational coefficients. M^8_c has an interpretation as an analog of momentum space.

$M^8 - H$ duality, which generalizes the momentum-position duality of wave mechanics, relates these two approaches and is discussed in the second article of the series. Third article in the series is devoted to neutrinos in the TGD framework. The motivation is that the recent results about neutrinos led to a breakthrough also in the construction of quantum theory based on the WCW approach.

In this article the two views are discussed in more detail.

1. Space-time as 4-surface in $H = M^4 \times CP_2$ generalizes Einstein's geometrization and also allows us to reduce standard model gauge fields to the geometry of the space-time surface. Quantum field theory (QFT) limit of TGD replaces many-sheeted space-time with a region of M^4 whose metric is slightly deformed. In zero energy ontology (ZEO) space-time surfaces are surfaces inside causal diamonds CD= cd$\times CP_2$ characterized by the scale of cd, which is the intersection of the future and past directed light-cones.

2. The notion of "world of classical worlds" (WCW) as the space of 3-surfaces or almost equivalently that of 4-surfaces analogous to Bohr orbits is discussed. General Coordinate Invariance forces holography and twistor lift suggests the identification of space-time surfaces as minimal surfaces which are also extremals of Kähler action analogous to Maxwell action. Twistor lift also forces M^4 to have an analog of Kähler structure and this leads to powerful predictions discussed in the third article of the series. Twistor lift requires in turn that the twistor spaces of M^4 and CP_2 have Kähler structure. These are the only 4-D spaces satisfying this condition.

   The experience with loop spaces suggests that the mere existence of the WCW Kähler metric fixes it uniquely so that the physics would be unique from its mathematical existence.

3. Physical states correspond to modes of WCW spinor structure. Gamma matrices of WCW are assumed to correspond to super generators of WCW isometries identifiable as symplectic transformations of $\delta M^4_+ \times CP_2$. Since the induction of spinor structure of H means restriction of second quantized H spinors to space-time surface and since the super-symplectic generators are obtained as Noether charges, WCW metric can be constructed in terms of anticommutators of super charges.

4. There are strong reasons to believe that TGD is completely integrable and several realizations of the complete integrability expected to be equivalent are considered.

Also the vision about physics as number theory is discussed. The number theoretic vision involves all classical number fields, all p-adic number fields and their extensions induced by extensions of rationals.

1. The motivations for the p-adic physics are discussed. p-Adic physics could be seen as describing the correlates of cognition. One can speak about p-adic counterparts of 4-surfaces. One can consider two options: either algebraic surfaces in M^8 or also p-adic analogs of space-time surfaces in H.

   A central notion is cognitive representation consisting of a subset of points of 4-surface, which belong to the intersection of real and p-adic variants of the 4-surfaces associated with a given extension of rational numbers. This requires that the coordinates of the point belong to the extension of rationals in question. This discretization is unique.

   The minimal option is that cognitive representations appear only at the level of M^8 having interpretation as analog of momentum space and are determined partially by the polynomial defining the 4-surface in M^8 as its "root". This would be natural since the coordinates of M^8_c having interpretation as complexified octonions are almost unique apart from a real translation in the extension considered. Adelic space might be seen as the M^8 counterpart of WCW at the level of H.

2. Adelic physics leads to a highly unique quantum measurement theory based on the notion of cognitive quantum measurement for the quantum states defined as wave functions in the Galois group of extension allowing hierarchical structure for extensions of extensions.... of extensions.

Several new results are obtained.

1. The realization that M^8 has an interpretation as an analog of momentum space, has led to a breakthrough in the understanding of the theory. The implications for condensed matter physics are especially significant. For instance, a universal mechanism for the formation of bound states based on Galois confinement is proposed.

2. One interesting result relates to the often neglected fact that the mutual entanglement of fermions (bosons) as elementary particles is always maximal so that only fermionic and bosonic degrees can entangle in QFTs. The replacement of point-like particles with 3-surfaces forces us to reconsider the notion of identical particles from the category theoretical point of view.

   The number theoretic definition of particle identity seems to be the most natural and implies that the new degrees of freedom make possible geometric entanglement. Also the notion particle generalizes: also many-particle states can be regarded as particles with the constraint that the operators creating and annihilating them satisfy commutation/anticommutation relations. This leads to a close analogy with the notion of infinite prime.

2 Physics as geometry

The following provides a sketchy representation of TGD based on the vision about physics as geometry which is complementary to the vision of physics as number theory. $M^8 - H$ duality relates these two visions. A longer representation can be found in [33].

2.1 Space-time as 4-surface in $H = M^4 \times CP_2$

1. The energy problem of GRT means that since space-time is curved, one cannot define Poincare charges as Noether charges (see **Fig. 1**). If space-time X^4 is a surface in $H = M^4 \times CP_2$, the situation changes. Poincare symmetries are lifted to the level of $M^4 \subset H$.

2. Generalization of the notion of particle is in question: point-like particle $\rightarrow$ 3-surface so that TGD can be seen also as a generalization of string model. String $\rightarrow$ 3-surface. String world sheet $\rightarrow X^4$. The notions of the particle and space are unified.

3. Einstein's geometrization program is extended to standard model interactions. CP_2 codes for standard model symmetries and gauge fields. Isometries $\leftrightarrow$ color SU(3). Holonomies of spinor connection $\leftrightarrow$ electroweak U(2) [17]. Genus-generation correspondence provides a topological explanation of the family replication phenomenon of fermions [5]: 3 fermion families are predicted.

4. Induction of spinors structure as projection of components of spinor connection from CP_2 to X^4 is central for the geometrization. The projections of Killing vectors of color isometries yield color gauge potentials. Parallel translation at X^4 using spinor connection of H. Also spinor structure is induced and means projection of gamma matrices.

5. Dynamics for X^4 is determined by an action S consisting of Kähler action plus volume term (cosmological constant) following from the twistor lift of TGD [15, 23].

6. The dynamics for fermions at space-time level is determined by modified Dirac action determined by S being super-symmetrically related to it. Gamma matrices are replaced with modified gamma matrices determined by the S as contractions of canonical momentum currents with gamma matrices. Preferred extremal property follows as a condition of hermiticity for the modified Dirac operator.

   Second quantized H-spinors, whose modes satisfy free massless Dirac equation in H restricted to X^4: this induces second quantization to X^4 and one avoids the usual problems of quantization in a curved background. This picture is consistent with the modified Dirac equation satisfied by the induced spinors in X^4.

   Only quarks are needed if leptons are 3-quark composites in CP_2 scale: this is possible only if one accepts the TGD view about color symmetries. This also provides a new view about matter antimatter asymmetry [28, 36]. CP violation is forced by the M^4 part of Kähler form forced by the twistor lift.

2.1.1 Basic extremals of classical action

Practically any GCI action allows the same basic extremals (for basic questions related to classical TGD see **Fig. 3**).

1. CP_2 type extremals having light-like geodesic as M^4 projection and Euclidian signature of the induced metric serve as building bricks of elementary particles. If the volume term is absent as it might be at infinite volume limit, the geodesics become light-like curves [42]. Wormhole contacts connecting two Minkowskian space-time sheets can be regarded as a piece of a deformed CP_2 type extremal. Monopole flux through contact stabilizes the wormhole contact.

2. Massless extremals (MEs)/topological light rays are counterparts for massless modes. They allow superposition of modes with single direction of massless momentum. Ideal laser beam is a convenient analogy here.

3. Cosmic strings $X^2 \times Y^2 \subset M^4 \times CP_2$ and their thickenings to flux tubes are also a central notion.

2.1.2 QFT limit of TGD

The induced gauge fields and gravitational field are expressible in terms of only 4 H- coordinates. Locally the theory is too simple to be physical.

1. Many-sheeted space-time means that X^4 is topologically extremely complex. CP_2 coordinates are many-valued functions of M^4 coordinates or vice versa or both. In contrast to this, the space-time of EYM theory is topologically extremely simple.

2. Einsteinian space-times have 4-D projection to M^4. Small test particle experiences the sum of the classical gauge potentials associated with various space-time sheets. At QFT limit the sheets are replaced with a single region of M^4 made slightly curved and gauge potentials are defined as the sums of gauge potentials from different space-time sheets having common M^4 projection. Topological complexity and local simplicity are replaced with topological simplicity and local complexity. (see **Fig. 3**).

2.2 World of classical worlds (WCW)

The notion of WCW emerges as one gives up the idea about quantizing by path integral.

2.2.1 The failure of path integral forces WCW geometry

The extreme non-linearity implies that the path integral for surfaces space-time surfaces fails. A possible solution is to generalize Einstein's geometrization program to the level of the entire quantum theory.

1. "World of classical worlds" (WCW) can be identified as the space of 3-surfaces endowe with a metric and spinor structure (see **Fig. 8**). Hermitian conjugation must have a geometrization. This requires Kähler structure requiring also complex structure. WCW has Kähler form and metric.

2. WCW spinors are Fock states created by fermionic oscillator operators assignable to spinor modes of H basically [32]. WCW gamma matrices as linear combinations of fermionic (quark) oscillator operators defining analog of vielbein.

   WCW has also spinor connection and curvature in WCW. correspond The quantum states of world correspond formally to *classical* spinor fields in WCW. Gamma matrices of WCW expressinble in terms of fermionic oscillator operators are also purely classical objects.

2.2.2 Implications of General Coordinate Invariance

General Coordinate Invariance (GCI) in 4-D sense forces to assign to 3-surface X^3 a 4-surface $X^4(X^3)$, which is as unique as possible. This gives rise to Bohr orbitology and quantum classical correspondence (QCC), and holography. Also zero energy ontology (ZEO) emerges.

Quantum states quantum superpositions of space-time surfaces as analogs of Bohr orbits. QCC means that the classical theory is an exact part of quantum theory (QCC).

A solution to the basic paradox of quantum measurement theory emerges [27]: superposition of deterministic time evolutions is replaced with a new one in state function reduction (SFR): SFR does not force any failure of determinism for individual time evolutions.

2.2.3 WCW Kähler geometry from classical action

WCW geometry is determined by a classical action defining Kähler function $K(X^3)$ for a preferred extremal $X^4(X^3)$ defining the preferred extremal/Bohr orbit [7] (see **Fig. 8**).

1. QCC suggests that the definition of Kähler function assigns a more or less unique 4-surface $X^4(X^3)$ to 3-surface X^3. Finite non-uniqueness is however possible [42].

2. $X^4(X^3)$ is identified as a *preferred* extremal of some general coordinate invariant (GCI) action forcing the Bohr orbit property/holography/ZEO. This means a huge reduction of degrees of freedom.

   Remark:: Already the notion of induced gauge field and metric eliminates fields as primary dynamical variables and GCI leaves locally only 4 H-coordinates as dynamical variables.

3. Twistor lift [21, 23] of TGD geometrizes the twistor Grassmann approach to QFTs. The 6-D extremal X^6 of 6-D Kähler action as a 6- surface in the product $T(M^4) \times T(CP_2)$ of twistor spaces of M^4 and CP_2 represents the twistor space of X^4.

   The condition that X^6 reduces to an S^2 bundle with X^4 as base space, forces a dimensional reduction of 6-D Kähler action to 4-D Kähler action + volume term, whose value for the preferred extremal defines the Kähler function for $X^4(X^3)$.

4. The volume term corresponds to a p-adic length scale dependent cosmological constant Λ approach zero at long p-adic length scale so that a solution of the cosmological constant problem emerges. Preferred extremal/Bohr orbit property means a simultaneous extremal property for *both* Kähler action and volume term.

   This forces X^4 to have a generalized complex structure (Hamilton-Jacobi structure) so that field equations trivialize and there is no dependence on coupling parameters. Universality of dynamics follows and the TGD Universe is quantum critical. In particular, Kähler coupling strength is analogous to a critical temperature and is quantized [41].

5. Soap film analogy is extremely useful [42]: the analogs of soap film frames are singular surfaces of dimension $D < 4$. At the frame the space-time surface fails to be a simultaneous extremal of both actions separately and Kähler and volume actions couple to each other. The corresponding contributions to conserved isometry currents diverge but sum up to a finite contribution. The frames define the geometric analogs for the vertices of Feynman diagrams.

2.2.4 WCW geometry is unique

WCW geometry is fixed by the existence of Riemann connection and requires maximal symmetries.

1. Dan Freed [1] found that loop space for a given Lie group allows a unique Kähler geometry: maximal isometries needed in order to have a Riemann connection. Same expected to be true now [6, 12].

2. Twistor lift of TGD [21, 23] means that one can replace X^4 with its twistor space $X^6(X^4)$ in the product $T(M^4) \times T(CP_2)$ of the 6-D twistor spaces $T(M^4)$ and $T(CP_2)$. $X^6(X^4)$ is 6-surface with the structure of S^2 bundle.

   Dimensionally reduced 6-D Kähler action gives sum of 4-D Kähler action and volume term. Twistor space must however have a Kähler structure and only the twistor spaces of M^4,E^4, and CP_2 have Kähler structure [2]. TGD is unique both physically and mathematically!

2.2.5 Isometries of WCW

What can one say about the isometries of WCW? Certainly, they should generalize conformal symmetries of string models.

1. The crucial observation is that the 3-D light-cone boundary δM^4_+ has metric, which is effectively 2-D. Also the light-like 3-surfaces $X^3_L \subset X^4$ at which the Minkowskian signature of the induced metric changes to Euclidian are metrically 2-D. This gives an extended conformal invariance in both cases with complex coordinate z of the transversal cross section and radial light-coordinate r replacing z as coordinate of string world sheet. Dimensions $D = 4$ for X^4 and M^4 are therefore unique.

2. $\delta M^4_+ \times CP_2$ allows the group symplectic transformations of $S^2 \times CP_2$ made local with respect to the light-like radial coordinate r. The proposal is that the symplectic transformations define isometries of WCW [6].

3. To the light-like partonic orbits one can assign Kac-Moody symmetries assignable to $M^4 \times CP_2$ isometries with additional light-like coordinate. They could correspond to Kac-Moody symmetries of string models assignable to elementary particles.

   The preferred extremal property raises the question whether the symplectic and generalized Kac-Moody symmetries are actually equivalent. The reason is that isometries are the only normal subgroup of symplectic transformations so that the remaining generators would naturally annihilate the physical states and act as gauge transformations. Classically the gauge conditions would state that the Noether charges vanish: this would be one manner to express preferred extremal property.

2.2.6 A possible problem related to the twistor lift

The twistor lift strongly suggests that the Kähler form of M^4 exists. The Kähler gauge potential would be the sum of M^4 and CP_2 contributions. The definition of M^4 Kähler structure is however not straightforward [29, 30]. The naive guess would be that J represents an imaginary unit as the square root of -1 represented by the metric tensor. This would give the condition $J^2 = -g$ for the tensor square but this leads to problems.

To understand the situation, notice that the analogs of symplectic/Kähler structures in $M^4 \subset H$ have a moduli space, whose points correspond to what I have called Hamilton-Jacobi structures defined by integrable distributions of orthogonal decompositions $M^4 = M^2(x) \times E^2(x)$: $M^2(x)$ is analogous to string world sheet and Y^2 to partonic 2-surface. This means the presence of slicing by string world sheets $X^2(x)$, where x labels a point of Y^2. $X^2(x)$ is orthogonal to Y^2 at x. One can interchange the roles X^2 ad Y^2 in the slicing.

The induced Kähler form has an analogous decomposition. The decomposition is completely analogous to the decomposition of polarizations to non-physical time-like ones and physical space-like ones. This decomposition allows a natural modification of the definition of the symplectic structure so that the problem caused by $J^2 = -g$ conditions is avoided.

Consider first the problem. The $E^2(x)$ part of M^4 Kähler metric produces no problems since the signature of the metric is Euclidean. For $M^2(x)$ part, the Minkowskian signature produces problems. If one assumes that the $M^2(x)$ part of the Kähler form is non-vanishing, it should be imaginary in order to satisfy $J^2(M^2(x)) = -g(M^2(x))$. This implies that Kähler gauge potential is imaginary and this spoils the hermiticity of the modified Dirac equation [10]. Also the electric contribution to the Kähler energy is negative.

The solution of the problem turned out to be ridiculously simple and I should have noticed it a long time ago.

1. $M^2(x)$ has a hypercomplex structure, which means that the imaginary unit e satisfies $e^2 = 1$ rather than $e^2 = -1$. Hamilton-Jacobi structure allows one to decompose J locally into two parts $J = J(M^2(x)) + J(E^2(x))$ such that $J^2 = g(M^2(x)) - g(E^2(x))$. This gives $J^4 = g(M^4)$. The Kähler energy of the canonically embedded M^4 is non-vanishing and positive whereas Kähler action vanishes by self-duality. Situation is identical to that in Maxwell's electrodynamics.

2. Kähler action for the canonically embedded M^4 vanishes and it is possible to define also Lagrangian 2-surfaces as surfaces for which the induced Kähler form vanishes. These are of special interest since they would guarantee small CP violation: string world sheets could be examples of these surfaces. Note that since the magnetic part of J induces violation of CP, the violation is vanishing for CP_2 type extremals and cosmic strings and also small for flux tubes.

If the notion of symplectic/canonical transformation generated by Hamiltonian preserving J generalizes, one could generate an infinite number of slicings.

Consider first ordinary symplectic transformations.

1. For the ordinary symplectic transformations, the closedness of the symplectic for J is essential ($dJ = 0$ corresponds to topological half of Maxwell's equations).

2. Second essential element is that symplectic transformation is generated as a flow for some Hamiltonian H: $j_H = i_{dH}J$ or more explicitly: $j^l_H = J^{kl}\partial_l H$. It is essential that one has $i_{j_H}J = -dH$: having a vanishing exterior derivative. In other words, $J_{kl}j^l_H = -\partial_k H$ is a gradient vector field and has therefore a vanishing curl. Together with $dJ = 0$, this guarantees the vanishing of the Lie derivative of J: $d_{j_H}J = d(i_{j_H}J) + i_{j_H}dJ = ddH + dJ(j_H) = 0$ so that J is preserved.

Could one talk about symplectic transformations in M^4?

1. The analogs of symplectic/canonical transformations should map the Hamilton-Jacobi structure to a new one and leave $J(M^2(x))$ and $J(E^2(x))$ invariant. The induced metrics of X^2 and Y^2 need not be preserved since only the diagonal metric $g^k_l(X^2/Y^2)$ appears in the conditions $J^2 = g(X^2) - g(Y^2)$.

2. The symplectic transformation generated by the Hamiltonian H would be a flow defined by the vector field $j_H = i_{dH}J$ and one would have $i_{j_H}J = -d_1H + d_2H$, where d_1 and d_2 are gradients operators in X^2 and Y^2. Usually one would have $J_{kl}j^l = dH$ satisfying $d^2H = 0$.

   The condition $ddH = 0$ satisfied by the ordinary symplectic transformations is replaced with the condition $d(-d_1H + d_2H) = 0$. This can be written as $-d_1^2H + d_2^2H + [d_2, d_1]H = 0$, and is satisfied. Therefore this part is not a problem.

3. Also the orthogonality of $M^2(x)$ and $E^2(x)$ must be preserved. This is a highly non-trivial condition since the metrics are induced and the symplectic transformations change the slicing and the metrics.

   An arbitrary Hamiltonian flow f, which depends on the coordinates of Y^2 only, maps Y^2 to itself but takes the tangent space $E^2(x)$ to $E^2(f(x))$. Unless the slicing satisfies special conditions, $E^2(f(x))$ is not orthogonal to $M^2(x)$.

4. The orthogonality is expressed as orthogonality of the projectors $P(X^2)$ and $P(Y^2)$: $P(X^2)P(Y^2) = 0$. This condition must be respected by the Hamiltonian flow. The product involves 4 components giving 4 conditions which turn out to be partial differential equations for Hamiltonian. The naive expectation is that there are very few solutions.

   The Lie-derivative of the product must therefore vanish:

$$L_{j_H}[P(X^2)P(Y^2)] = L_{j_H}(P(X^2))P(Y^2) + P(X^2))L_{j_H}(P(Y^2)) = 0 \ . \tag{2.1}$$

   The projector $P_{mn}(X^2)$ can be expressed as

$$P^{mn} = g^{\alpha\beta}\partial_\alpha m^k \partial_\beta m^l \ . \tag{2.2}$$

   Here $g_{\alpha\beta} = m_{kl}\partial_\alpha m^k \partial_\beta m^l$ is the induced metric of X^2 or Y^2. m_{kl} is Minkowski metric and one can use linear Minkowski coordinates so that m_{kl} is constant.

   The Lie derivative of $P^{mn}(X^2) \equiv P$ can be written as

$$L_j(P^{mn}) = L_j(g^{\alpha\beta})\partial_\alpha m^k \partial_\beta m^l + g^{\alpha\beta})(\partial_r j^k \partial_\alpha m^r \partial_\beta m^l + \partial_r j^l \partial_\alpha m^r \partial_\beta m^k \ . \tag{2.3}$$

   The Lie derivative of the induced metric is in linear coordinates

$$\begin{aligned} L_j g^{\alpha\beta} &= -g^{\alpha\mu} g^{\beta\nu} L_j g_{\mu\nu} \ , \\ L_j g_{\alpha\beta} &= m_{kl}(\partial_\alpha j^k \partial_\beta m^l + \partial_\alpha m^k \partial_\beta j^l) \ . \end{aligned} \tag{2.4}$$

Although the existence of symplectic transformations in the general case seems implausible, one can construct special slicings for which symplectic transformations are possible.

1. One can start from a trivial slicing defined by $M^2 \times E^2$ decomposition and perform slicings of M^2 and E^2. The orthogonality is trivially true for all slicings of this kind since $Y^2(y)$ is orthogonal to X^2 not only at y but at every point x. Symplectic transformations of M^2 and Y^2 produce new slicings of this kind. Even symplectic flowqs defined by general Hamiltonians respect the orthogonality.

2. Second example is provided by the slicing of the light-one boundary by light-like 2-surfaces Y^2_v labelled by the value of light-like radial coordinate v with metrics differing by r^2 factor. The surfaces X^2 would be planes $X^2(y)$ orthogonal to Y^2 at y with light-like coordinates u and v. The orthogonality would be preserved by symplectic transformations.

   The open question is whether these slicings are the only possible slicings allowing symplectic transformations. Although the construction of these slicings looks trivial, they are not trivial physically.

2.3 About Dirac equation in TGD framework

2.3.1 Three Dirac equations

In TGD spinors appear at 3 levels:

1. At the level of imbedding space $H = M^4 \times CP_2$ the spinor field imbedding space $M^4 \times CP_2$spinor fields (quark field) is a superposition of the harmonics of the Dirac operator. In the complexified M^8 having interpretation as complexified octonions, spinors are octonionic spinors. In accordance with the fact that M^8 is analogous to momentum space, the Dirac equation is purely algebraic and its solutions correspond to discrete points analogous to occupied points of Fermi ball.

2. The spinors at the level of 4-surfaces $X^4 \subset H$ are restrictions of the second quantized embedding space spinor field in X^4 so that the problematic second quantization in curved background is avoided. At the level of M^8 the restriction selects the points of M^8 belonging to 4-surface and carrying quark. The simplest manner to realize Fermi statistics is to assume that there is at most a single quark at a given point.

3. The third realization is at the level of the "world of classical worlds" (WCW) assigned to H consisting of 4-surfaces as preferred extremals of the action. Gamma matrices of WCW are expressible as superpositions of quark oscillator operators so that anti-commutation relations are geometrized. The conditions stating super-symplectic symmetry are a generalization of super-Kac-Moody symmetry and of super-conformal symmetry and give rise to the WCW counterpart of the Dirac operator [12, 33] as a non-hermitian super-Virasoro generator G which however carries fermion number.

4. What the realization of WCW at the level of M^8 is, has remained unclear. The notion of WCW geometry does not generalize to his level and should be replaced with an essentially number theoretic notion.

   Adelic physics as a fusion of real and p-adic physics suggests a possible realization. Given extension of rationals induces extensions of various p-adic number fields. These can be glued to a book-like structure having as pages real numbers and the extensions of p-adic number fields.

The pages would intersect along points with coordinates in the extension of rationals. These points form a cognitive representation. The additional condition that the active points are occupied by quarks guarantees that this makes sense also for octonions, quaternions and 4-surface in M^8. The p-adic sector could consist of discrete and finite cognitive representations continued to the p-adic surface and define the counterpart of WCW at the level of M^8?

2.3.2 The relationship between Dirac operator of H and modified Dirac operator

At the level of $X^4 \subset H$, the proposal is that modified Dirac action for the induced spinor fields defines the dynamics somehow. Modified Dirac equation or operator should be also consistent with the second quantization of induced spinor fields performed at the level of H and inducing the second quantization at the level of X^4.

1. The modified gamma matrices Γ^α are defined by the contractions of H gamma matrices Γ_k and canonical momentum currents $T^{k\alpha}$ associated with the action defining space-time surface. The modified Dirac operator $D = \Gamma^\alpha D_\alpha$, where D_α is X^4 projection of the vector defined by the covariant derivative operators of H ($D_\alpha = \partial_\alpha h^k D_k$). Hermiticity requires $D_\alpha \Gamma^\alpha = 0$ implying that classical field equations are satisfied.

2. Can one assume that the modified Dirac equation is satisfied? Or is it enough to assume that this is not the case so that the modified Dirac operator defines the propagator as its inverse as the QFT picture would suggest?

   In fact, the propagators in H allow to compute N-point functions involving quarks and at the level of H the theory is free and the restriction to the space-time surface brings in the interactions. Therefore the notion of space-time propagator is not absolutely necessary. One can however ask whether some weaker condition could be satisfied and provide new insights.

   One can also ask whether the solutions of the modified Dirac equation correspond to external particles, which correspond to space-time surfaces for which the solution of the modified Dirac equation is consistent with the solution of the Dirac equation in H. Are these kinds of space-time surfaces possible?

3. The intuitive picture is that the solutions of the modified Dirac equation correspond to the external particles of a scattering diagram having an interpretation on mass shell states and are possible only for a very special kind of preferred extremals. Intuitively they should correspond to singular surfaces in M^8 and their mapping to H would involve blow-up due to the non-uniqueness of the normal space along lower than 4-D surface. String like objects and CP_2 type extremals would be basic entities of this kind. Could the modified Dirac equation or its weakened form hold true for these surfaces.

The strong form of equivalence of modified Dirac equation and ordinary Dirac equation would mean the equivalence of the actions of two Dirac operators acting on the second quantized induced spinor field.

1. The modified Dirac operator is given by $\Gamma_k T^{\alpha k} \partial_\alpha h^k D_k$ and its action should be same as H Dirac operator $\Gamma^k D_k$. This would require

$$\Gamma_k T^{\alpha k} \partial_\alpha h^k D_k \Psi = \Gamma^k D_k \Psi \ . \tag{2.5}$$

   Not surprisingly, it turns out that this condition is too strong.

2. One can express Γ_k using an overcomplete basis defined by the Killing vector fields j_A^k for H isometries. In the case of M^4 it is enough to use translations by using the identity $\sum_A j_A^k j_A^l = h^{kl}$. This allows to define gamma matrices $\Gamma_A = \Gamma_k j_A^k$ and to write the equation in the form

$$\Gamma_A T^{A\alpha}\partial_\alpha h^k D_k\Psi = \Gamma_A j_A^k D_k\Psi \ . \tag{2.6}$$

Here $T^{A\alpha}$ is the conserved isometry current associated with the Killing vector j_A^k. Is it possible to satisfy the condition

$$T^{A\alpha}\partial_\alpha h^k = j_A^k \tag{2.7}$$

or its suitably weakened form?

The strong form of the condition cannot be satisfied. The left hand side of the equation is determined by the gradients of H coordinates and parallel to X^4 whereas the right hand side also involves the component normal to X^4. Therefore the condition cannot be satisfied in the general case.

3. By projecting the condition to the tangent space, one obtains a weaker condition stating that the tangential parts of two Dirac operators are proportional to each other with a position dependent proportionality factor $\Lambda(x)$:

$$\begin{aligned} T^{A\alpha} &= \Lambda(x) j_A^\alpha \\ j_A^\alpha &= j_A^k \partial^\alpha h_k = j_A^k h_{kl} g^{\alpha\beta}\partial_\beta h^l \ . \end{aligned} \tag{2.8}$$

The conserved isometry current is proportional to the projection of the Killing vector to the tangent space of X^4. $\Lambda(x)$ is proportionality constant depending on the point of X^4. Isometry current is analogous to a Hamiltonian vector field being parallel to the Killing vector field.

4. If the action were a mere cosmological volume term, the isometry currents would be proportional to j^α so that the conditions would be automatically satisfied. The contribution to $\Lambda(x)$ is proportional to the p-adic length scale dependent cosmological constant.

   Kähler action receives contributions from both M^4 and CP_2. Both add to $T^{A\alpha}$ a term of form $T^{\alpha\beta} j_{A\beta}$ coming from the variation of the Kähler action with respect to $g_{\alpha\beta}$. $T^{\alpha\beta}$ is the energy momentum tensor with a form similar to that for Maxwell action.

   Besides this, M^4 *resp.* CP_2 contribute a term proportional to $J^{\alpha\beta} J_{kl}\partial_\beta h^k j_A^k$ coming from the variation of the Kähler action with respect to $J_{\alpha\beta}$ contributing only to M^4 *resp.* CP_2 isometries. These contributions make the conditions non-trivial. The Kähler contribution to $\Lambda(x)$ need not be constant. Note that the Kähler contributions to the energy momentum tensor vanish if X^4 is (minimal) surface of form $X^2 \times Y^2 \subset M^4 \times CP_2$ so that both X^2 and Y^2 are Lagrangian.

5. The vanishing of the divergence of $T^{A\alpha}$ using the Killing property $D_l j_{Ak} + D_k j_{Al} =$ of j_{Ak} gives

$$j^{A\alpha}\partial_\alpha \Lambda = 0 \ . \tag{2.9}$$

Λ is constant along the flow lines of $j^{A\alpha}$ and is therefore analogous to a Hamiltonian. The constant contribution from the cosmological term to Λ does not contribute to this condition.

6. An attractive hypothesis, consistent with the hydrodynamic interpretation, is that the proposed condition is true for all preferred extremals. The conserved isometry current along the X^4 projection of the flow line is proportional to the projection of Killing vector: this conservation law is analogous to the conservation of energy density $\rho v^2/2 + p$ along the flow line). One can say that isometries as flows in the embedding space are projected to flows along the space-time surface. One could speak of projected or lifted representation.

7. The projection to the normal space does not vanish in the general case. One could however ask whether a weaker condition stating that the second fundamental form $H^k_{\alpha\beta} = D_\alpha h^k$, which is normal to X^4, defines the notion of the normal space in terms of data provided by space-time surface. If X^4 is a geodesic submanifold of H, in particular a product of geodesic submanifolds of M^4 and CP_2, one has $H^k_{\alpha\beta} = 0$.

2.3.3 Gravitational and inertial representations of isometries

The lift/projection of the isometry flows to X^4 strongly suggests a new kind of representation of isometries as analog of the braid representation considered earlier.

1. Projected/lifted representation would clarify the role of the classical conserved charges and currents and generalize hydrodynamical conservation laws along the flow lines of isometries. In particular, quark lines would naturally correspond to time-like flow lines of time translations. In the case of CP_2 type extremals, quark momenta for the lifted representations would be light-like.

2. The conservation conditions along the flow lines are very strong, and one can wonder if they might provide a new formulation of the preferred extremal property. It is quite possible that the conditions apply only to a sub-algebra. Quantum classical correspondence (QCC) suggests Cartan algebra for which the quantum charges can have well-defined eigen values simultaneously. In accordance with QCC, the choice of the quantization axes would affect the space-time surfaces considered and could be interpreted as a higher level quantum measurement.

3. Projected/lifted representation provides a new insight also to the Equivalence Principle (EP) stating that gravitational and inertial masses are identical. At the level of scattering amplitudes involving isometry charges defined at the level of H, the isometries affect the entire space-time surface, and one could see EP as an almost trivial statement. QCC however forces us to consider EP more seriously.

   I have proposed that QCC could be seen as the identification of the eigenvalues of Cartan algebra isometry charges for quantum states with the classical charges associated with the preferred extremals. EP would follow from QCC: gravitational charges would correspond to the representation of the flows defined by isometries as their projections/lifts to X^4 whereas inertial charges would correspond to the representation at the level of H with isometries affecting the entire space-time surfaces.

4. The lifted/projected/gravitational representation of isometries, which seems possible in 4-D situation, is analogous to braid group representation making sense only in 2-D situation. Indeed, for the many-sheeted space-time surfaces assignable to $h_{eff} > h_0$, it can happen that rotation by 2π leads to a new space-time sheet and that the $SO(2)$ subgroup of the rotation group associated with the Cartan algebra is lifted to n-fold covering. Same can can happen in the case of color rotations. This leads to a fractionation of quantum numbers usually assigned with quantum group representations suggested to correspond to $h_{eff} > h$ [13].

   Also for the quantum groups, Cartan algebra plays a special role. In the case of the Poincare group, the 2-D nature of braid group representations would correspond to the selection $M^2 \times SO(2)$ as a Cartan subgroup implying effective 2-dimensionality in the case rotation group. Gravitational representations could therefore correspond to quantum group representations.

5. The gravitational representation provides also a new insight on M^8-H duality. The source of worries has been whether Uncertainty Principle (UP) is realized if a given 4-surface in M^8 is mapped to a single space-time surface in M^8. It seems that UP can be realized both in terms of inertial and gravitational representations.

   (a) In the case of the "inertial" representation of H-isometries at the level of H, one must regard $X^4 \subset H$ representing images of particle-like 4-surface in M^8 analog of Bohr orbit (holography) and map it to an analog of plane wave define as superposition of its translates and by the total momentum associated with the either boundary of CD associated with the particle. The same applies to the transforms to other Cartan algebra generators.

   In a cognitive representation based on extension of rationals, the shifts for Cartan algebra would be discrete: the values of the plane wave would be roots of unity belonging to the extension and satisfy periodic boundary conditions at the boundary of larger CD.

   Periodic boundary conditions pose rather strong conditions on the time evolution by scaling between two SSFRs. The scaling must respect the boundary conditions. If the momenta assignable to the plane waves of massive particles are conserved and h_{eff} is conserved, the scaling must multiply CD size by integers. The iterations of integer scalings, in particular $n = 2$ scalings (period doubling), are in a preferred position.

   (b) If one replaces the inertial representation of isometries with the gravitational representation, the quantum states can be realized at the level of a single space-time surface. One would have two representations: gravitational and inertial -subjective and objective, one might say.

   (c) Gravitational representations make also sense for the super-symplectic group acting at the boundary of light-cone as well as for the Kac-Moody type algebra associated with the isometries of H realized the light-like orbits of partonic 2-surfaces.

2.4 Different manners to understand the "complete integrability" of TGD

There are several ways to see how TGD could be a completely integrable theory.

2.4.1 Preferred extremal property

Preferred extremal property requires Bohr orbit property and holography and is an extremely powerful condition.

1. Twistor lift of TGD implies that X^4 in H is simultaneous extremal of volume action and Kähler action. Minimal surface property is counterpart for massless field equations and extremality for Kähler action gives interpretation for massless field as Kähler form as part of induced electromagnetic field.

   The simultaneous preferred extremal property strongly suggests that 2-D complex structure generalizes for 4-D space-time surfaces and so called Hamilton-Jacobi structure [25] meaning a decomposition of M^4 to orthogonal slicings by string world sheets and orthogonal partonic 2-surfaces would realize this structure.

2. Generalized Beltrami property [37] implies that 3-D Lorentz force and dissipation for Kähler form vanish. The Kähler form is analogous to the classical Maxwell field. Energy momentum tensor has vanishing divergence, which makes it plausible that QFT limit is analogous to Einstein-Maxwell theory.

   The condition also implies that the Kähler current defines an integrable flow so that there is global coordinate varying along flow lines. This is a natural classical correlate for quantum coherence. Quantum coherence would be always present but broken only by the finite size of the region of the space-time considered.

Beltrami property plus current conservation implies gradient flow and an interesting question is whether conserved currents define gradient flows: non-trivial space-time topology would allow this at the fundamental level. Beltrami condition is a very natural classical condition in the models of supraphases.

3. The condition that the isometry currents for the Cartan algebra of isometries are proportional to the projections of the corresponding Killing vectors is a strong condition and could also be at least an important aspect of the preferred extremal property.

2.4.2 Supersymplectic symmetry

The third approach is based on the super-symplectic symmetry of WCW. Isometry property would suggest that an infinite number of super-symplectic Noether charges are defined at the boundaries of CD by the action of the theory. They need not be conserved since supersymplectic symmetries cannot be symmetries of the action: if they were, the WCW metric would be trivial.

The gauge conditions for Virasoro algebra and Kac-Moody algebras suggest a generalization. Supersymplectic algebra (SSA) involves only non-negative conformal weights n suggesting extension to a Yangian algebra (this is essential!). Consider the hierarchy of subalgebras SSA_m for which the conformal weights are m-tiples of those of entire algebra. These subalgebras are isomorphic with the entire algebra and form a fractal hierarchy.

Assume that the sub-algebra SSA_m and commutator $[SSA_m, SSA]$ have vanishing classical Noether charges for $m > m_{max}$. These conditions could fix the preferred extremal. One can also assume that the fermionic realizations of these algebras annihilate physical states. The remaining symmetries would be dynamical symmetries.

The generators are Hamiltonians of $\delta M^4_+ \times CP_2$. The symplectic group contains Hamiltonians of the isometries as a normal sub-algebra. Also the Hamiltonians of and one could assume that only the isometry generators correspond to non-trivial classical and quantal Noether charges. Could the actions of SSA and Kac-Moody algebras of isometries be identical if a similar construction applies to Kac-Moody half-algebras associated with the light-like partonic orbits. Super-symplectic symmetry would reduce to a hierarchy of gauge symmetries.

3 Physics as number theory

Number theoretic physics involves the combination of real and various p-adic physics to adelic physics [19, 20], and classical number fields [14].

3.1 p-Adic physics

Motivation for p-adicization came from p-adic mass calculations [8, 5].

1. p-Adic thermodynamics for mass squared operator M^2 proportional to scaling generator L_0 of Virasoro algebra. Mass squared thermal mass from the mixing of massless states with states with mass of order CP_2 mass.

2. $exp(-E/T) \to p^{L_0/T_p}$, $T_p = 1/n$. Partition function p^{L_0/T_p}. p-Adic valued mass squared mapped to real number by canonical identification $\sum x_n p^n \to \sum x_n p^{-n}$. Eigenvalues of L_0 must be integers for the Bolztmann weights to exist. Conformal invariance guarantees this.

3. p-adic length scale $L_p \propto \sqrt{p}$ from Uncertainty Principle ($M \propto 1/\sqrt{p}$). p-Adic length scale hypothesis states that p-adic primes characterizing particles are near to power of 2: $p \simeq 2^k$. For instance, for electron one has $p = M^{127} - 1$, Mersenne prime. This is the largest not completely super-astrophysical length scale.

Also Gaussian Mersenne primes $M_{G,n} = (1+i)^n - 1$ seem to be realized (nuclear length scale, and 4 biological length scales in the biologically important range 10 nm,2.5 μm).

4. p-Adic physics [11] is interpreted as a correlate for cognition (see **Fig. 10**). Motivation comes from the observation that piecewice constant functions depending a finite number of pinary digts have vanishing derivative. Thefore they appear as integration constants in p-adic differential equations. This could provide a classical correlate for the non-determinism of imagination.

3.2 Adelic physics

Adelic physics fuses real and various p-adic physics to a single structure [20].

1. One can combine real numbers and p-adic number fields to a product: number fields would be like pages of a book intersecting along rationals acting as the back of the book.

2. Each extension of rational induces extensions of p-adic number fields and extension of the basic adele. Points in the extension of rationals are now common to the pages. The infinite hierarchy of adeles defined by the extensions forms an infinite library.

3. This leads to an evolutionary hierarchy (see **Fig. 9**) . The order n of the Galois group as a dimension of extension of rationals is identified as a measure of complexity and of evolutionary level, "IQ". Evolutionary hierarchy is predicted.

4. Also a hierarchy of effective Planck constants interpreted in terms of phases of ordinary matter is predicted. X^4 decomposes to n fundamental regions related by Galois symmetry. Action is n times the action for the fundamental region. Planck constant h is effectively replaced with $h_{eff} = nh$. Quantum coherence scales are typically proportional to h_{eff}. Quantum coherence in arbitrarily long scales is implied. Dark matter at the magnetic body of the system would serve as controller of ordinary matter in the TGD inspired quantum biology [43].

   $h_{eff} = nh_0$ is a more general hypothesis. Reasons to believe that h/h_0 could be the ratio R^2/L_p^2 for CP_2 length scale R deduced from p-adic mass calculations and Planck length L_P [41]. The CP_2 radius R could actually correspond to L_P and the value of R deduced from the p-adic mass calculations would correspond to a dark CP_2 radius $\sqrt{h/h_0}l_P$.

3.3 Adelic physics and quantum measurement theory

Adelic physics [20] forces us to reconsider the notion of entanglement and what happens in state function reductions (SFRs). Let us leaven the question whether the SFR can correspond to SSFR or BSFR or both open for a moment.

1. The natural assumption is that entanglement is a number-theoretically universal concept and therefore makes sense in both real and various p-adic senses. This is guaranteed if the entanglement coefficients are in an extension E of rationals associated with the polynomial Q defining the space-time surface in M^8 and having rational coefficients.

   In the general case, the diagonalized density matrix ρ produced in a state function reduction (SFR) has eigenvalues in an extension E_1 of E. E_1 is defined by the characteristic polynomial P of ρ.

2. Is the selection of one of the eigenstates in SFR possible if E_1 is non-trivial? If not, then one would have a number-theoretic entanglement protection.

3. On the other hand, if the SFR can occur, does it require a phase transition replacing E with its extension by E_1 required by the diagonalization?

Let us consider the option in which E is replaced by an extension coding for the measured entanglement matrix so that something also happens to the space-time surface.

1. Suppose that the observer and measured system correspond to 4-surfaces defined by the polynomials O and S somehow composed to define the composite system and reflecting the asymmetric relationship between O and S. The simplest option is $Q = O \circ S$ but one can also consider as representations of the measurement action deformations of the polynomial $O \times P$ making it irreducible. Composition conforms with the properties of tensor product since the dimension of extension of rationals for the composite is a product of dimensions for factors.

2. The loss of correlations would suggest that a classical correlate for the outcome is a union of uncorrelated surfaces defined by O and S or equivalently by the reducible polynomial defined by the $O \times S$ [39]. Information would be lost and the dimension for the resulting extension is the sum of dimensions for the composites. O however gains information and quantum classical correspondence (QCC) suggests that the polynomial O is replaced with a new one to realize this.

3. QCC suggests the replacement of the polynomial O the polynomial $P \circ O$, where P is the characteristic polynomial associated with the diagonalization of the density matrix ρ. The final state would be a union of surfaces represented by $P \circ O$ and S: the information about the measured observable would correspond to the increase of complexity of the space-time surface associated with the observer. Information would be transferred from entangled Galois degrees of freedom including also fermionic ones to the geometric degrees of freedom $P \circ O$. The information about the outcome of the measurement would in turn be coded by the Galois groups and fermionic state.

4. This would give a direct quantum classical correspondence between entanglement matrices and polynomials defining space-time surfaces in M^8. The space-time surface of O would store the measurement history as kinds of Akashic records. If the density matrix corresponds to a polynomial P which is a composite of polynomials, the measurement can add several new layers to the Galois hierarchy and gradually increase its height.

   The sequence of SFRs could correspond to a sequence of extensions of extensions of..... This would lead to the space-time analog of chaos as the outcome of iteration if the density matrices associated with entanglement coefficients correspond to a hierarchy of powers P^k [31, 38].

Does this information transfer take place for both BSFRs and SSFRs? Concerning BSFRs the situation is not quite clear. For SSFRs it would occur naturally and there would be a connection with SSFRs to which I have associated cognitive measurement cascades [34, 35].

1. Consider an extension, which is a sequence of extensions $E_1 \rightarrow ..E_k \rightarrow E_{k+1}.. \rightarrow E_n$ defined by the composite polynomial $P_n \circ .... \circ P_1$. The lowest level corresponds to a simple Galois group having no non-trivial normal subgroups.

2. The state in the group algebra of Galois group $G = G_n$ having G_{n-1} as a normal subgroup can be expressed as an entangled state associated with the factor groups G_n/G_{n-1} and subgroup G_{n-1} and the first cognitive measurement in the cascade would reduce this entanglement. After that the process could but need not to continue down to G_1. Cognitive measurements considerably generalize the usual view about the pair formed by the observer and measured system and it is not clear whether $O - S$ pair can be always represented in this manner as assumed above: also small deformations of the polynomial $O \times S$ can be considered.

   These considerations inspire the proposal the space-time surface assigned to the outcome of cognitive measurement G_k, G_{k-1} corresponds to polynomial the $Q_{k,k-1} \circ P_n$, where $Q_{k,k-1}$ is the characteristic polynomial of the entanglement matrix in question.

3.4 Entanglement paradox and new view about particle identity

A brain teaser that the theoretician sooner or later is bound to encounter, relates to the fermionic and bosonic statistics. This problem was also mentioned in the article of Keimer and Moore [4] discussing quantum materials https://cutt.ly/bWdTRj0. The unavoidable conclusion is that both the fermions and bosons of the entire Universe are maximally entangled. Only the reduction of entanglement between bosonic and fermionic states of freedom would be possible in SFRs. In the QFT framework, gauge boson fields are primary fields and the problem in principle disappears if entanglement is between states formed by elementary bosons and fermions.

In the TGD Universe, all elementary particles are composites of fundamental fermions (quarks in the simplest scenario) so that if Fock space the Fock states of fermions and bosons express everything worth expressing, SFRs would not be possible at all!

Remark: In the TGD Universe all elementary particles are composites of fundamental fermions (quarks in the simplest scenario) localized at the points of space-time surface defining a number theoretic discretization that I call cognitive representation. Besides this there are also degrees of freedom associated with the geometry of 3-surfaces representing particles. These degrees of freedom represent new physics. The quantization of quarks takes place at the level of H so that anticommutations hold true over the entire H.

Obviously, something is entangled and this entanglement is reduced. What these entangled degrees of freedom actually are if Fock space cannot provide them?

1. Mathematically entanglement makes sense also in a purely classical sense. Consider functions $\Psi_i(x)$ and $\Psi_j(y)$) and form the superposition $\Psi(x) = \sum_{ij} c_{ij}\Psi_i(x)\Psi_j x)$. This function is completely analogous to an entangled state.

2. Number theoretical physics implies that the Galois group becomes the symmetry group of physics and quantum states are representations of the Galois group [34, 35]. For an extension of extension of ...., the Galois group has decomposition by normal subgroups to a hierarchy of coset groups.

   The representation of a Galois group can be decomposed to a tensor product of representations of these coset groups. The states in irreps of the Galois group are entangled and the SFR cascade produces a product of the states as a product of representations of the coset groups. Galois entanglement allows us to express the asymmetric relation between observer and observed very naturally. This cognitive SSFR cascade - as I have called it - could correspond to what happens in at least cognitive SFRs.

   If so, then SFR would in TGD have nothing to do with fermions and bosons (consisting of quarks too) since the maximal fermionic entanglement remains. For instance, when one for instance talks about long range entanglement the entanglement that matters would correspond to entanglement between degrees of freedom, which do not allow Fock space description.

In the TGD framework, the replacement of particles with 3-surfaces brings in an infinite number of non-Fock degrees of freedom. Could it make sense to speak about the reduction of entanglement in WCW degrees of freedom? There is no second quantization at WCW level so that one cannot talk about Fock spaces WCW level but purely classical entanglement is possible as observed.

1. In WCW unions of disjoint 3-surfaces correspond to classical many-particle states. One can form single particle wave functions for 3-surfaces with a single component, products of these single particle wave functions, and also analogs of entangled states as their superposition realized as building bricks of WCW spinor fields.

   If one requires that these wave functions are completely symmetric under the exchange of 3-surfaces, maximal entanglement in this sense would be realized also now and SFR would not be possible. But can one require the symmetry? Under what conditions one can regard two 3-surfaces as identical? For point-like particles one has always identical particles but in TGD the situation changes.

Table 1: Differences and similarities between GRT and TGD

	GRT	**TGD**
Scope of geometrization	classical gravitation	all interactions and quantum theory
Spacetime		
Geometry	abstract 4-geometry	sub-manifold geometry
Topology	trivial in long length scales	many-sheeted space-time
Signature	Minkowskian everywhere	also Euclidian
Fields		
classical	primary dynamical variables	induced from the geometry of H
Quantum fields	primary dynamical variables	modes of WCW spinor fields
Particles	point-like	3-surfaces
Symmetries		
Poincare symmetry	lost	Exact
GCI	true	true - leads to SH and ZEO
	Problem in the identication of coordinates	$H = M^4 \times CP_2$ provides preferred coordinates
Super-symmetry	super-gravitation	super variant of H: super-surfaces
Dynamics		
Equivalence Principle	true	true
Newton's laws and		
notion of force	lost	generalized
Einstein's equations	from GCI and EP	remnant of Poincare invariance at QFT limit of TGD
Bosonic action	EYM action	Kähler action + volume term
Cosmological constant	suggested by dark energy	length scale dependent coefficient of volume term
Fermionic action	Dirac action	Modified Dirac action for induced spinors
Newton's constant	given	predicted
Quantization	fails	Quantum states as modes of WCW spinor field

2. Here theoretical physics and category theory meet since the question when two mathematical objects can be said to be identical is the basic question of category theory. The mathematical answer is they are isomorphic in some sense. The physical answer is that the two systems are identical if they cannot be distinguished in the measurement resolution used.

4 Appendix

4.1 Comparison of TGD with other theories

Table 1 compares GRT and TGD and **Table 2** compares standard model and TGD.

4.2 Brief glossary of the basic concepts of TGD

The following glossary explains some basic concepts of TGD and TGD inspired biology.

Table 2: Differences and similarities between standard model and TGD

	SM	**TGD**
Symmetries		
Origin	from empiria	reduction to CP_2 geometry
Color symmetry	gauge symmetry	isometries of CP_2
Color	analogous to spin	analogous to angular momentum
Ew symmetry	gauge symmery	holonomies of CP_2
Symmetry breaking	Higgs mechanism	CP_2 geometry
Spectrum		
Elementary particles	fundamental	consist of fundamental fermions
Bosons	gauge bosons, Higgs	gauge bosons, Higgs, pseudo-scalar
Fundamental fermions	quarks and leptons	quarks: leptons as local 3-quark composites
Dynamics		
Degrees of freedom	gauge fields, Higgs, and fermions	3-D surface geometry and spinors
Classical fields	gauge fields, Higgs	induced spinor connection
	SU(3) Killing vectors of CP_2	
Quantal degrees of freedom	gauge bosons,Higgs,	quantized induced spinor fields
Massivation	Higgs mechanism	p-adic thermodynamics with superconformal symmetry

- ***Space-time as surface***. Space-times can be regarded as 4-D surfaces in an 8-D space $M^4 \times CP_2$ obtained from empty Minkowski space (M^4) by adding four small dimensions (CP_2). The study of field equations characterizing space-time surfaces as "orbits" of 3-surfaces (3-D generalization of strings) forces the conclusion that the topology of space-time is non-trivial in all length scales.

- ***Geometrization of classical fields***. Both weak, electromagnetic, gluonic, and gravitational fields are known once the space-time surface in H as a solution of field equations is known.

  Many-sheeted space-time (see **Fig. 4**) consists of space-time sheets with various length scales with smaller sheets being glued to larger ones by ***wormhole contacts*** (see **Fig. 5**) identified as the building bricks of elementary particles. The sizes of wormhole contacts vary but are at least of CP_2 size (about 10^4 Planck lengths) and thus extremely small.

  Many-sheeted space-time replaces reductionism with ***fractality*** . The existence of scaled variants of physics of strong and weak interactions in various length scales is implied, and biology is especially interesting in this respect.

- ***Topological field quantization (TFQ)*** . TFQ replaces classical fields with space-time quanta. For instance, magnetic fields decompose into space-time surfaces of finite size representing flux tubes or -sheets. Field configurations are like Bohr orbits carrying "archetypal" classical field patterns. Radiation fields correspond to topological light rays or massless extremals (MEs), magnetic fields to magnetic flux quanta (flux tubes and sheets) having as primordial representatives "cosmic strings", electric fields correspond to electric flux quanta (e.g. cell membrane), and fundamental particles to CP_2 type vacuum extremals. Flux tubes come in two varieties depending on whether they carry monopole flux or not. The flux tubes carrying monopole flux (see **Fig. 6**) do not need any currents to generate the magnetic field.

- ***Field body*** (FB) and ***magnetic body*** (MB). Any physical system has field identity - FB or MB

- in the sense that a given topological field quantum corresponds to a particular source (or several of them - e.g. in the case of the flux tube connecting two systems).

Maxwellian electrodynamics cannot have this kind of identification since the fields created by different sources superpose. Superposition is replaced with a set theoretic union: only the *effects* of the fields assignable to different sources on test particle superpose. This makes it possible to define the QFT limit of TGD.

- ***p-Adic physics*** [11] as a physics of cognition and intention and the fusion of p-adic physics with real number based physics are new elements.
- ***Adelic physics*** [19, 22] is a fusion of real physics of sensory experience and various p-adic physics of cognition.
- ***p-Adic length scale hypothesis*** states that preferred p-adic length scales correspond to primes p near powers of two: $p \simeq 2^k$, k positive integer.
- A ***Dark matter hierarchy*** realized in terms of a hierarchy of values of effective Planck constant $h_{eff} = nh_0$ as integers using $h_0 = h/6$ as a unit. Large value of h_{eff} makes possible macroscopic quantum coherence which is crucial in living matter.
- ***MB as an intentional agent using biological body (BB) as a sensory receptor and motor instrument*** . The personal MB associated with the living body - as opposed to larger MBs assignable with collective levels of consciousness - has a hierarchical onion-like layered structure and several MBs can use the same BB making possible remote mental interactions such as hypnosis [18].
- ***Magnetic flux tubes and sheets*** serve as "body parts" of MB (analogous to body parts of BB), and one can speak about magnetic motor actions. Besides concrete motion of flux quanta analogous to ordinary motor activity, basic motor motor actions include the contraction of magnetic flux tubes by a phase transition reducing Planck constant, and the change in thickness of the magnetic flux tube, thus changing the value of the magnetic field, and in turn the cyclotron frequency. Reconnections of the flux tubes allow two MBs to get in contact and temporal variations of magnetic fields inducing motor actions of MBs favor the formation of reconnections. Flux tube connections at the molecular level bring a new element to biochemistry making it possible to understand bio-catalysis. Flux tube connections serve as a space-time correlates for attention in the TGD inspired theory of consciousness.
- ***Cyclotron Bose-Einstein condensates (BECs)*** of various charged particles can accompany MBs. Cyclotron energy $E_c = hZeB/m$ is much below thermal energy at physiological temperatures for magnetic fields possible in living matter. In the transition $h \rightarrow h_{eff}$ E_c is scaled up by a fractor $h_{eff}/h = n$. For sufficiently high value of h_{eff} cyclotron energy is above thermal energy $E = h_{eff}ZeB/m$. Cyclotron Bose-Einstein condensates at MBs of basic biomolecules and of cell membrane proteins - play a key role in TGD based biology.
- ***Josephson junctions*** exist between two superconductors. In TGD framework, ***generalized Josephson junctions*** accompany membrane proteins such as ion channels and pumps. A voltage between the two super-conductors implies a ***Josephson current***. For a constant voltage the current is oscillating with the ***Josephson frequency*** . The Josephson current emits ***Josephson radiation***. The energies come as multiples of ***Josephson energy***.

  In TGD generalized Josephson radiation consisting of dark photons makes communication of sensory input to MB possible. The signal is coded to the modulation of Josephson frequency depending on the membrane voltage. The cyclotron BEC at MB receives the radiation producing a sequence of resonance peaks.

- ***Negentropy Maximization Principle*** (NMP). NMP [9] [40] is the variational principle of consciousness and generalizes SL. NMP states that the negentropy gain in SFR is non-negative and maximal. NMP implies SL for ordinary matter.

- ***Negentropic entanglement*** (NE). NE is possible in adelic physics and NMP does not allow its reduction. NMP implies a connection between NE, the dark matter hierarchy, p-adic physics, and quantum criticality. NE is a prerequisite for an experience defining abstraction as a rule having as instances the state pairs appearing in the entangled state.

- ***Zero energy ontology (ZEO)*** In ZEO physical states are pairs of positive and negative energy parts having opposite net quantum numbers and identifiable as counterparts of initial and final states of a physical event in the ordinary ontology. Positive and negative energy parts of the zero energy state are at the opposite boundaries of a ***causal diamond*** defined as a double-pyramid-like intersection of future and past directed light-cones of Minkowski space (see **Fig. 13**).

  CD defines the "spot-light of consciousness": the contents of conscious experience associated with a given CD is determined by the space-time sheets in the imbedding space region spanned by CD. CDs are assumed to form a hierarchy analogous to an atlas of charts used to define manifold (see **Fig. 14**).

- ***SFR*** is an acronym for state function reduction. The measurement interaction is universal and defined by the entanglement of the subsystem considered with the external world [27] [16]. What is measured is the density matrix characterizing entanglement and the outcome is an eigenstate of the density matrix with eigenvalue giving the probability of this particular outcome. SFR can in principle occur for any pair of systems.

  SFR in ZEO solves the basic problem of quantum measurement theory since the zero energy state as a superposition of classical deterministic time evolutions (preferred extremals) is replaced with a new one. Individual time evolutions are not made non-deterministic.

  One must however notice that the reduction of entanglement between fermions (quarks in TGD) is not possible since Fermi- and als Bose statistics predicts a maximal entanglement. Entanglement reduction must occur in WCW degrees of freedom and they are present because point-like particles are replaced with 3-surfaces. They can correspond to the number theoretical degrees of freedom assignable to the Galois group - actually its decomposition in terms of its normal subgroups - and to topological degrees of freedom.

- ***SSFR*** is an acronym for "small" SFR as the TGD counterpart of weak measurement of quantum optics and resembles classical measurement since the change of the state is small [27] [16]. SSFR is preceded by the TGD counterpart of unitary time evolution replacing the state associated with CD with a quantum superposition of CDs and zero energy states associated with them. SSFR performs a localization of CD and corresponds to time measurement with time identifiable as the temporal distance between the tips of CD. CD is scaled up in size - at least in statistical sense and this gives rise to the arrow of time.

  The unitary process and SSFR represent also the counterpart for Zeno effect in the sense that the passive boundary of CD as also CD is only scaled up but is not shifted. The states remain unchanged apart from the addition of new fermions contained by the added part of the passive boundary. One can say that the size of the CD as analogous to the perceptive field means that more and more of the zero energy state at the passive boundary becomes visible. The active boundary is however both scaled and shifted in SSFR and states at it change. This gives rise to the experience of time flow and SSFRs as moments of subjective time correspond to geometric time as a distance between the tips of CD. The analog of unitary time evolution corresponds to "time" evolution induced by the exponential of the scaling generator L_0. Time translation is thus replaced by scaling. This is the case also in p-adic thermodynamics. The idea of time evolution by scalings has emerged also in condensed matter physics.

- ***BSFR*** is an acronym for "big" SFR, which is the TGD counterpart of ordinary state function reduction with the standard probabilistic rules [27] [16]. What is new is that the arrow of time changes since the roles of passive and active boundaries change and CD starts to increase in an opposite time direction.

  This has profound thermodynamic implications. Second law must be generalized and the time corresponds to dissipation with a reversed arrow of time looking like self-organization for an observed with opposite arrow of time [26] (see **Fig. 15**) . The interpretation of BSFR is as analog of biological death and the time reversed period is analogous to re-incarnation but with non-standard arrow of time. The findings of Minev et al [3] give support for BSFR at atomic level [24]. Together with h_{eff} hierarchy BSFR predicts that the world looks classical in all scales for an observer with the opposite arrow of time .

4.3 Figures

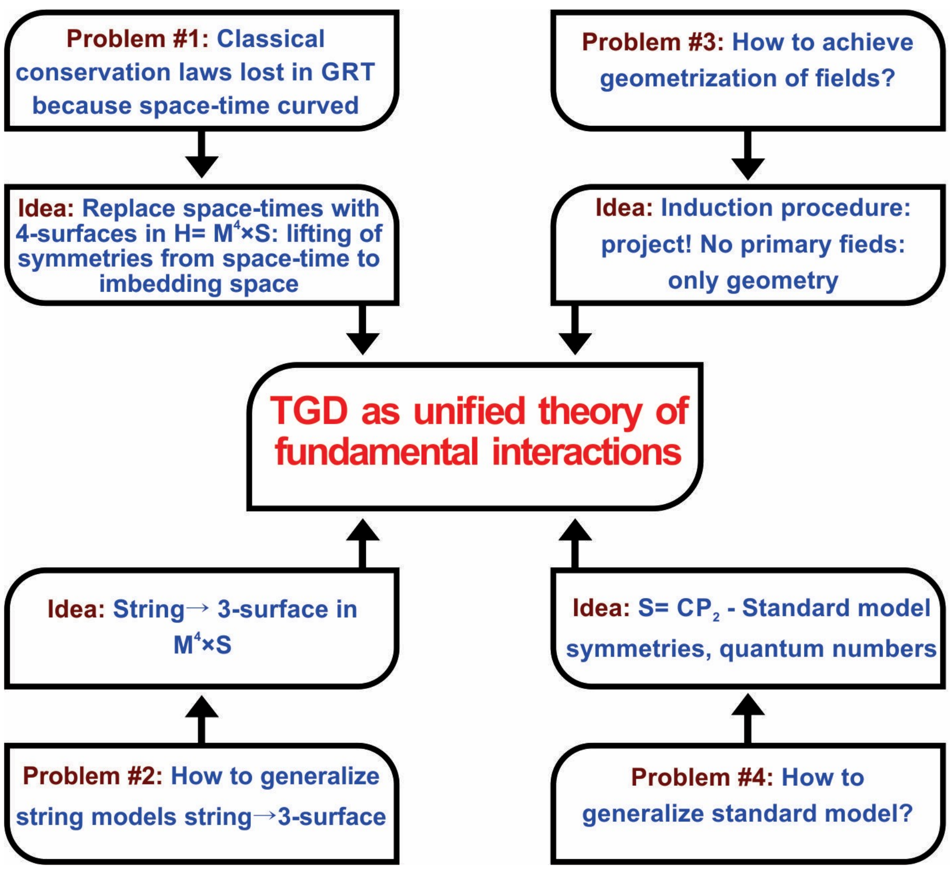

Figure 1: The problems leading to TGD as their solution.

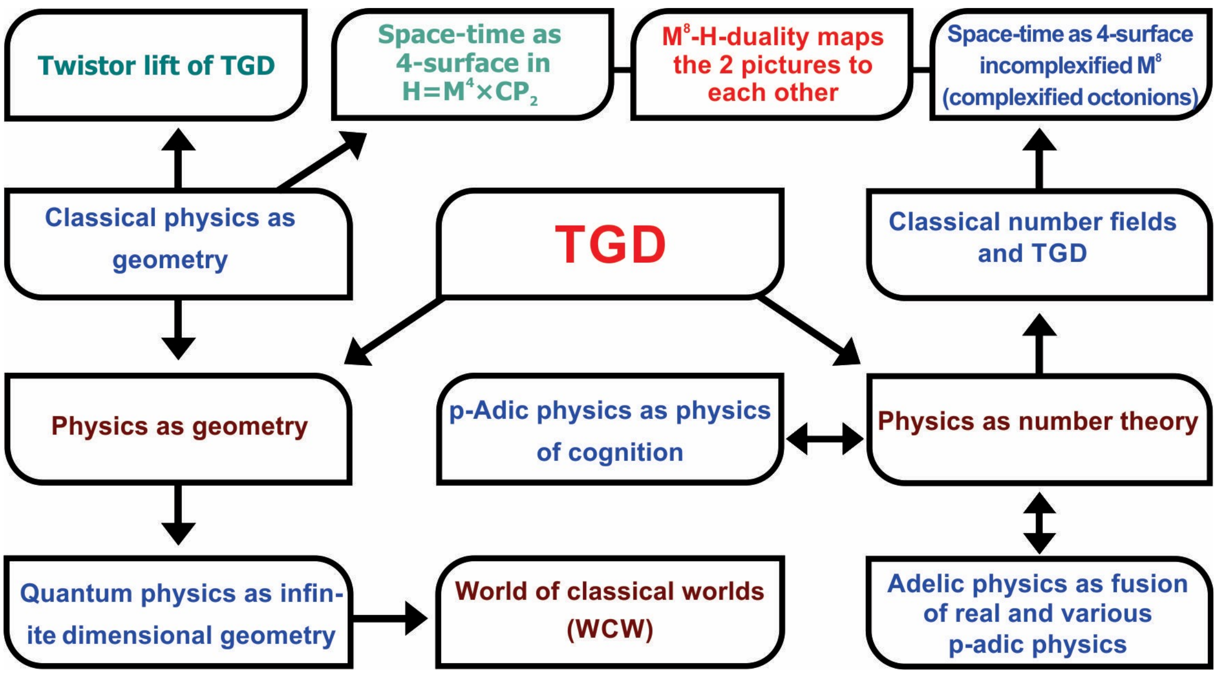

Figure 2: TGD is based on two complementary visions: physics as geometry and physics as number theory.

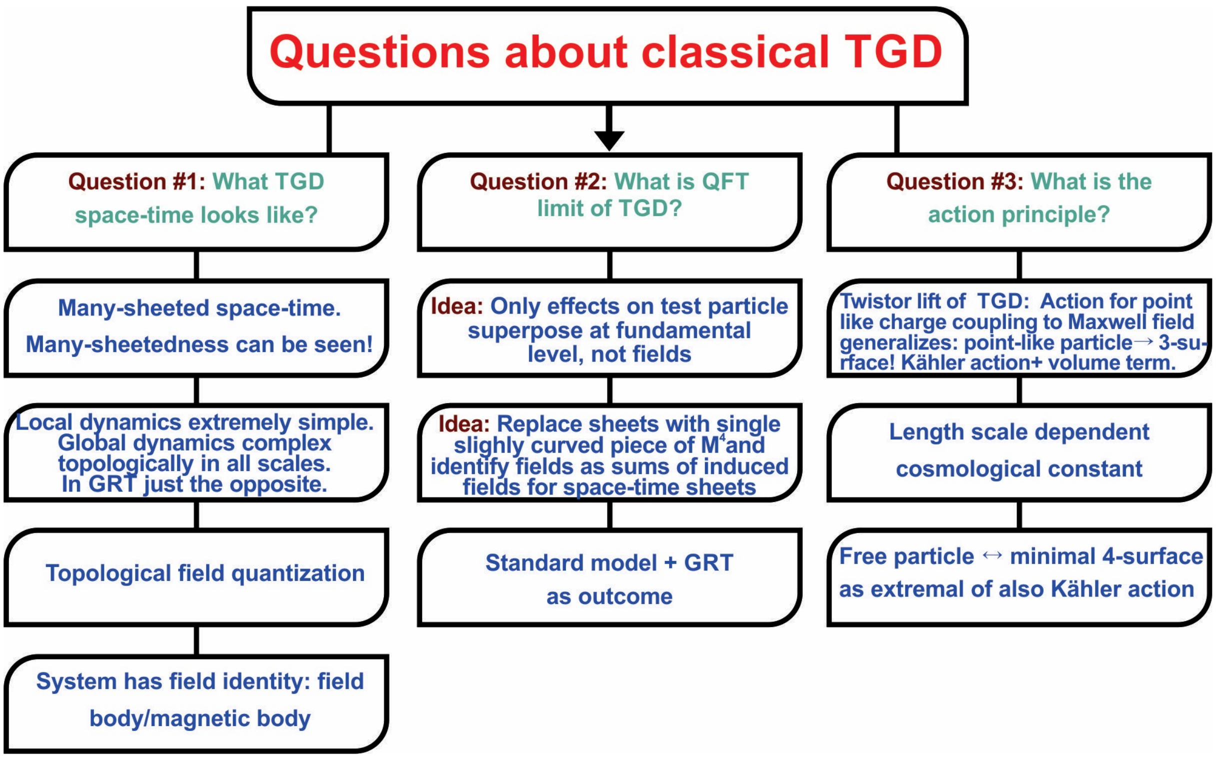

Figure 3: Questions about classical TGD.

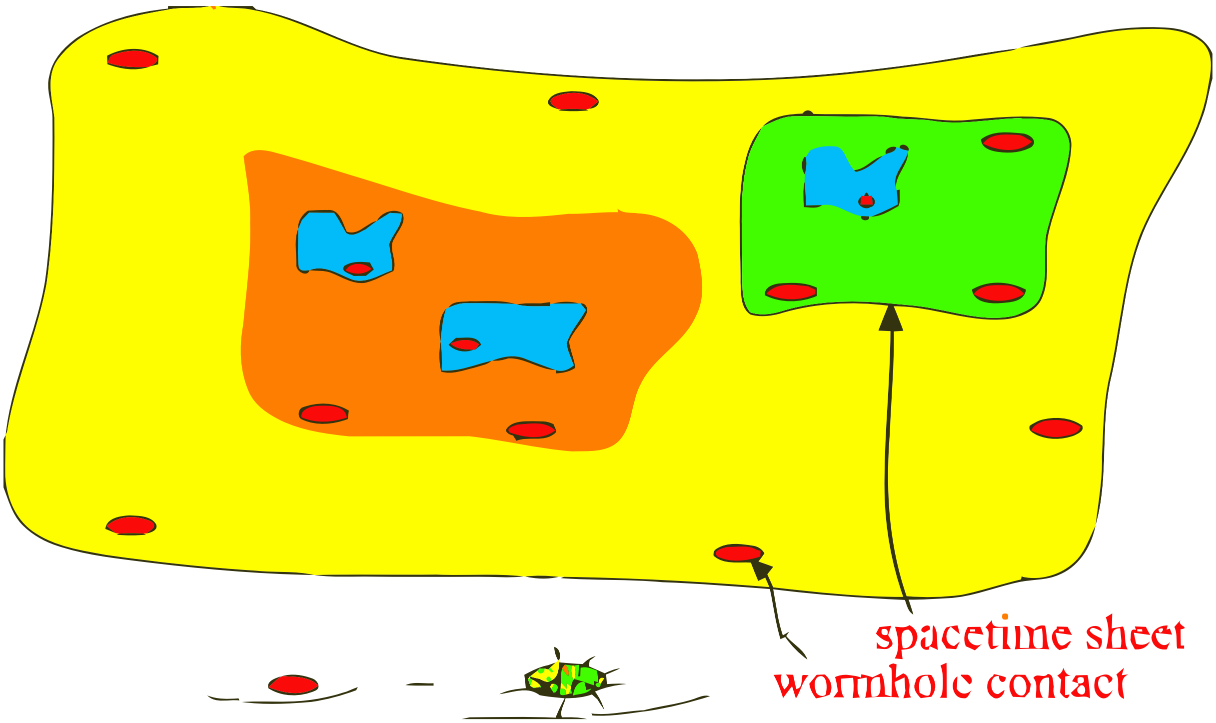

Figure 4: Many-sheeted space-time.

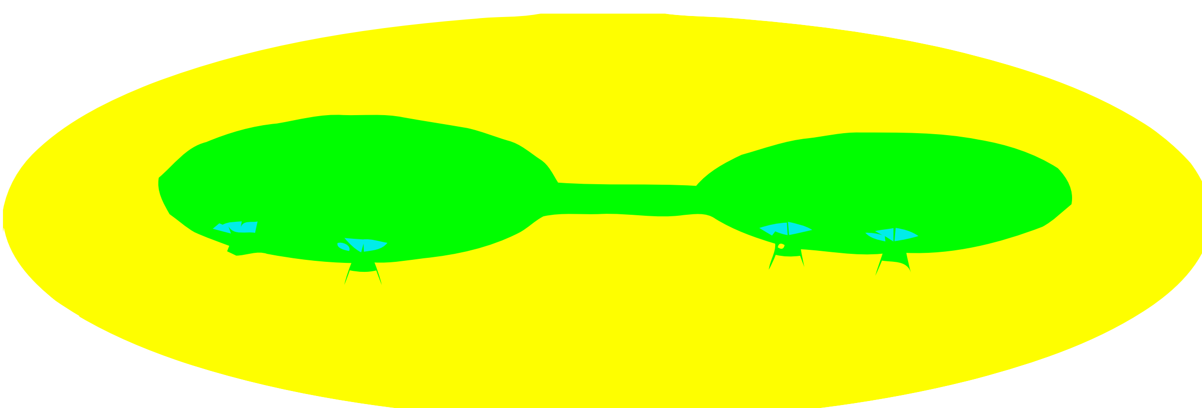

Figure 5: Wormhole contacts.

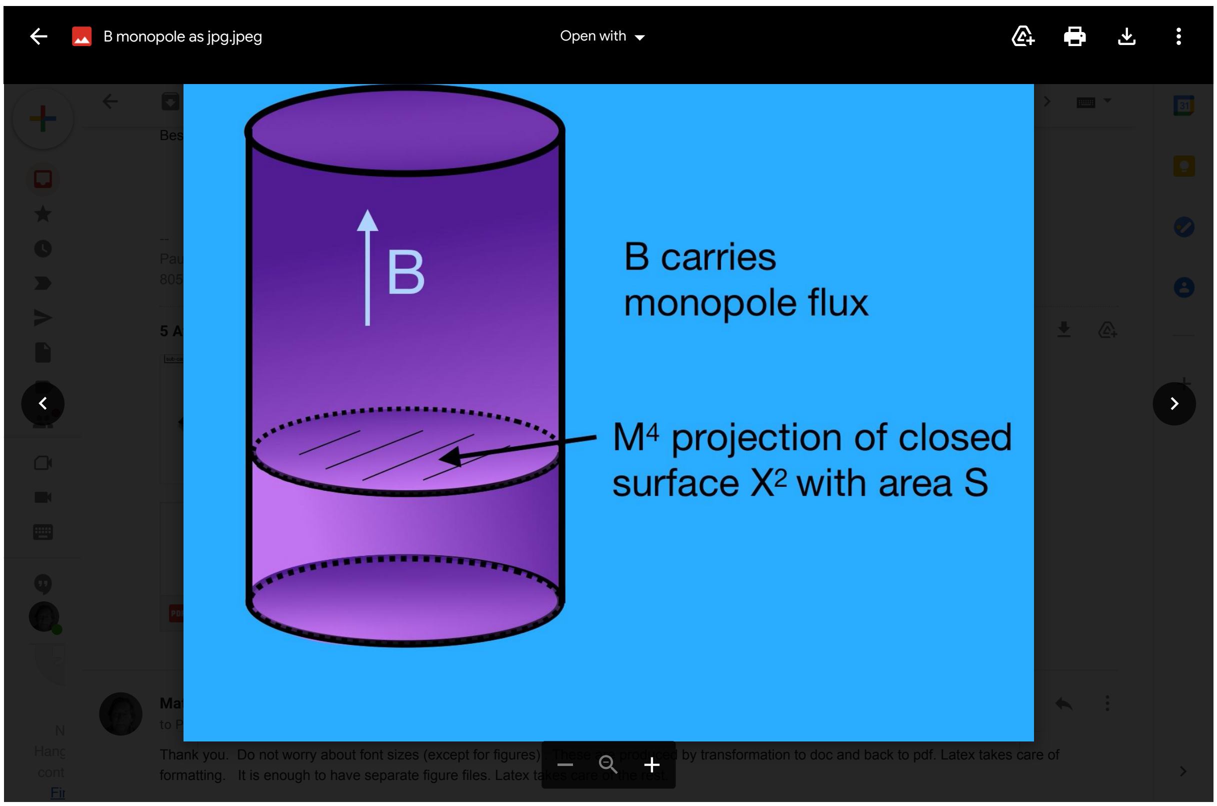

Figure 6: The M^4 projection of a closed surface X^2 with area S defining the cross section for monopole flux tube. Flux quantization $e\oint B\cdot dS = eBS = kh$ at single sheet of n-sheeted flux tube gives for cyclotron frequency $f_c = ZeB/2\pi m = khZ/2\pi mS$. The variation of S implies frequency modulation.

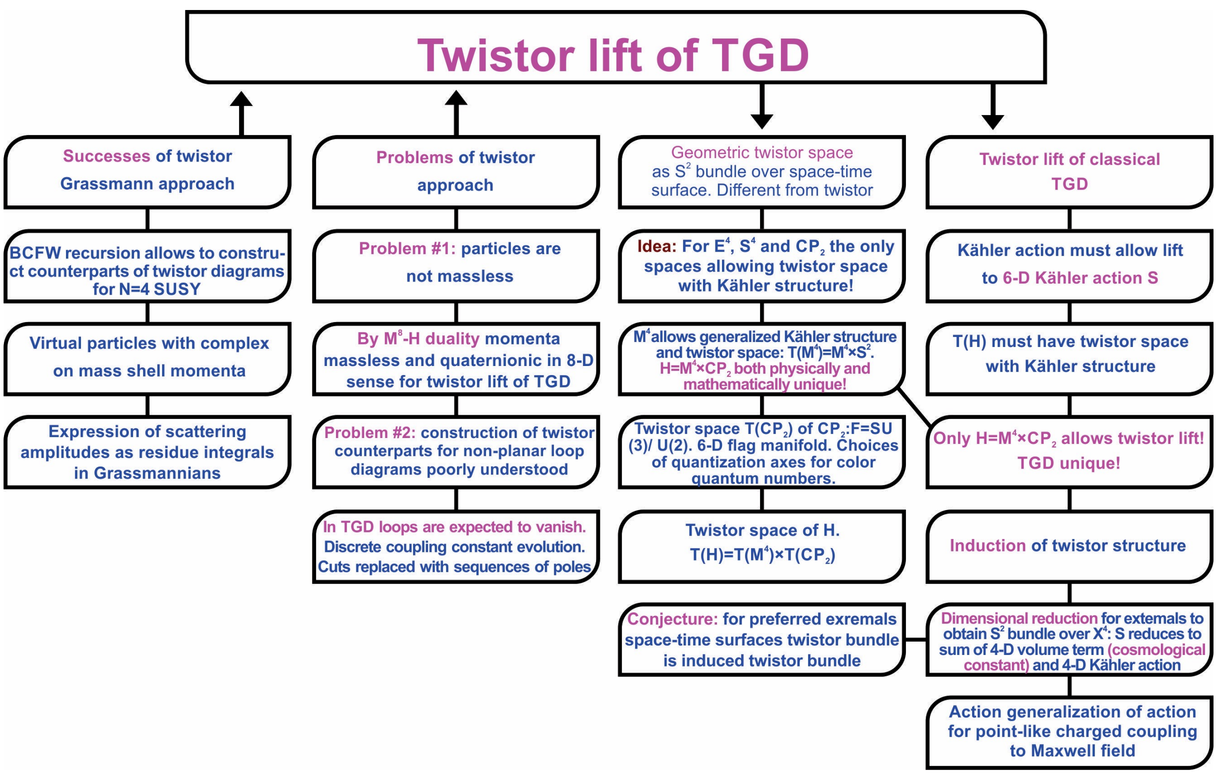

Figure 7: Twistor lift

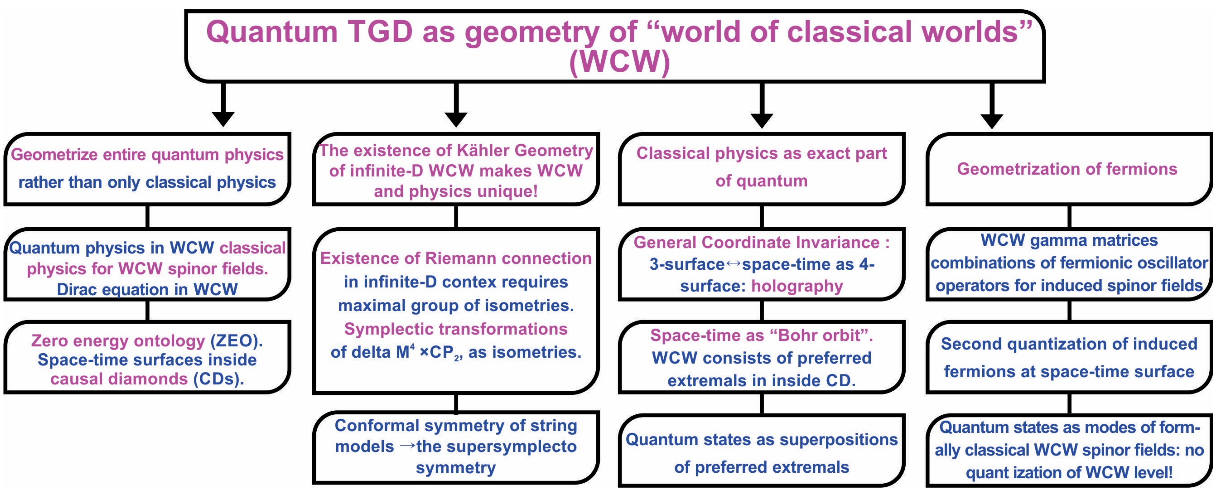

Figure 8: Geometrization of quantum physics in terms of WCW

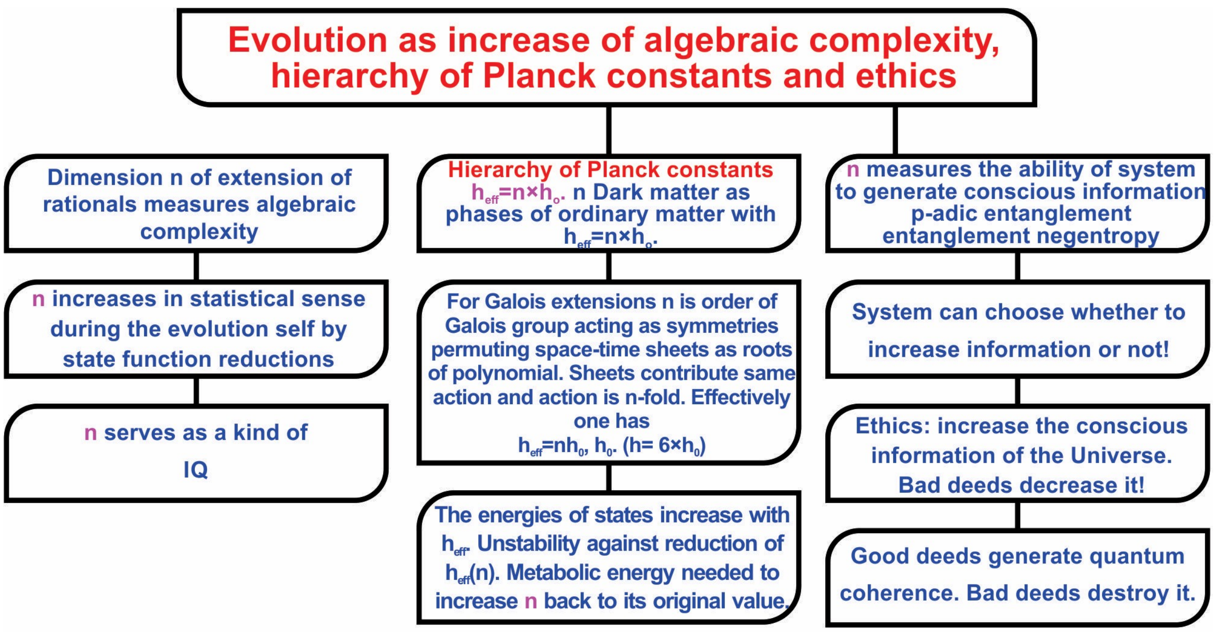

Figure 9: Number theoretic view of evolution

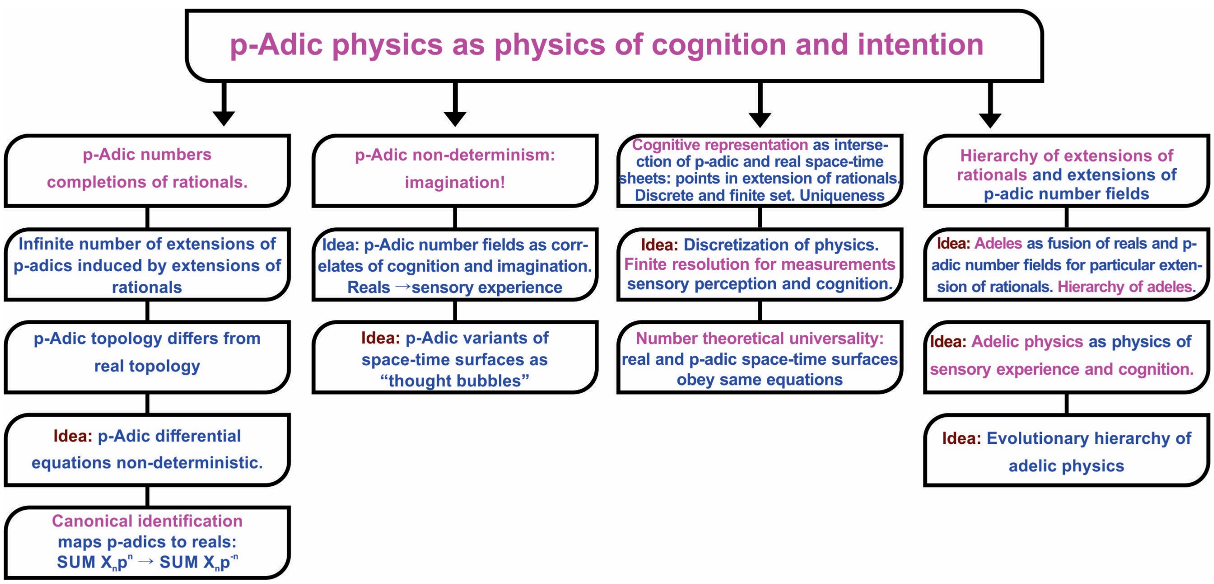

Figure 10: p-Adic physics as physics of cognition and imagination.

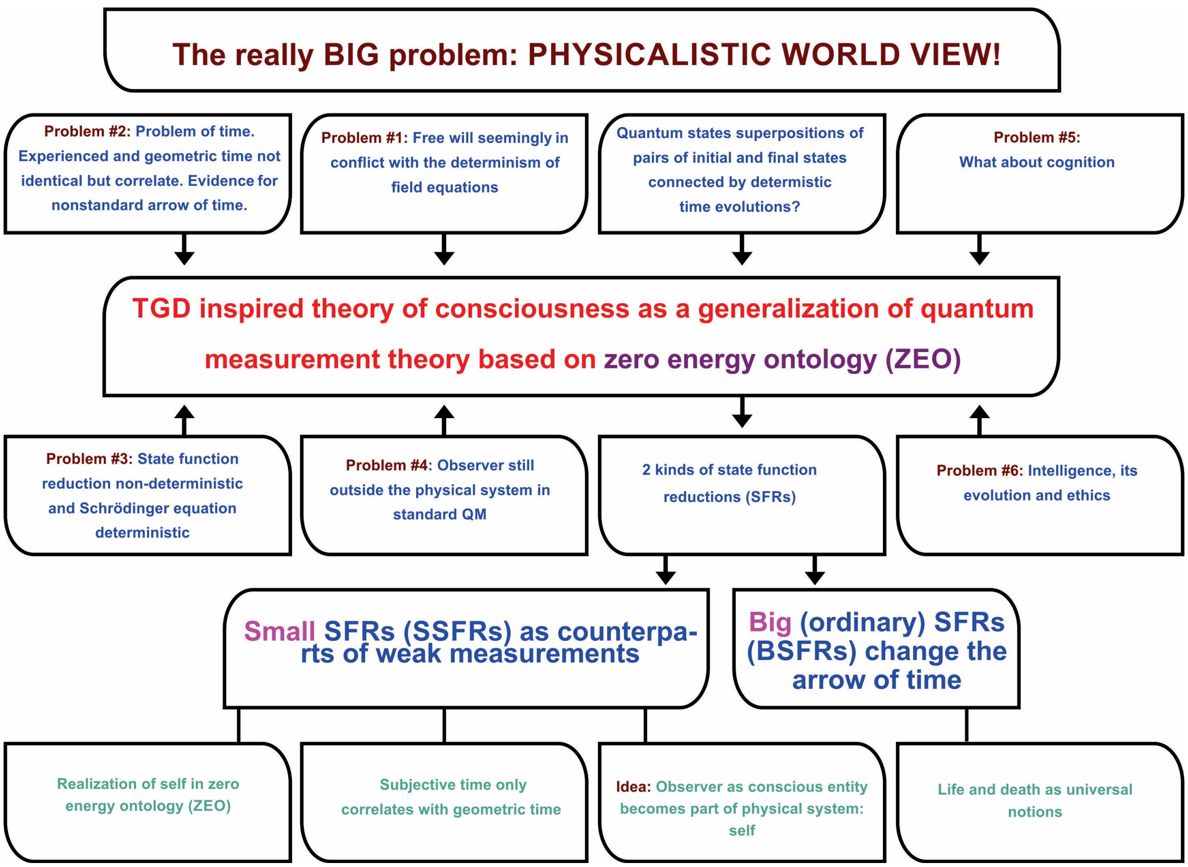

Figure 11: Consciousness theory from quantum measurement theory

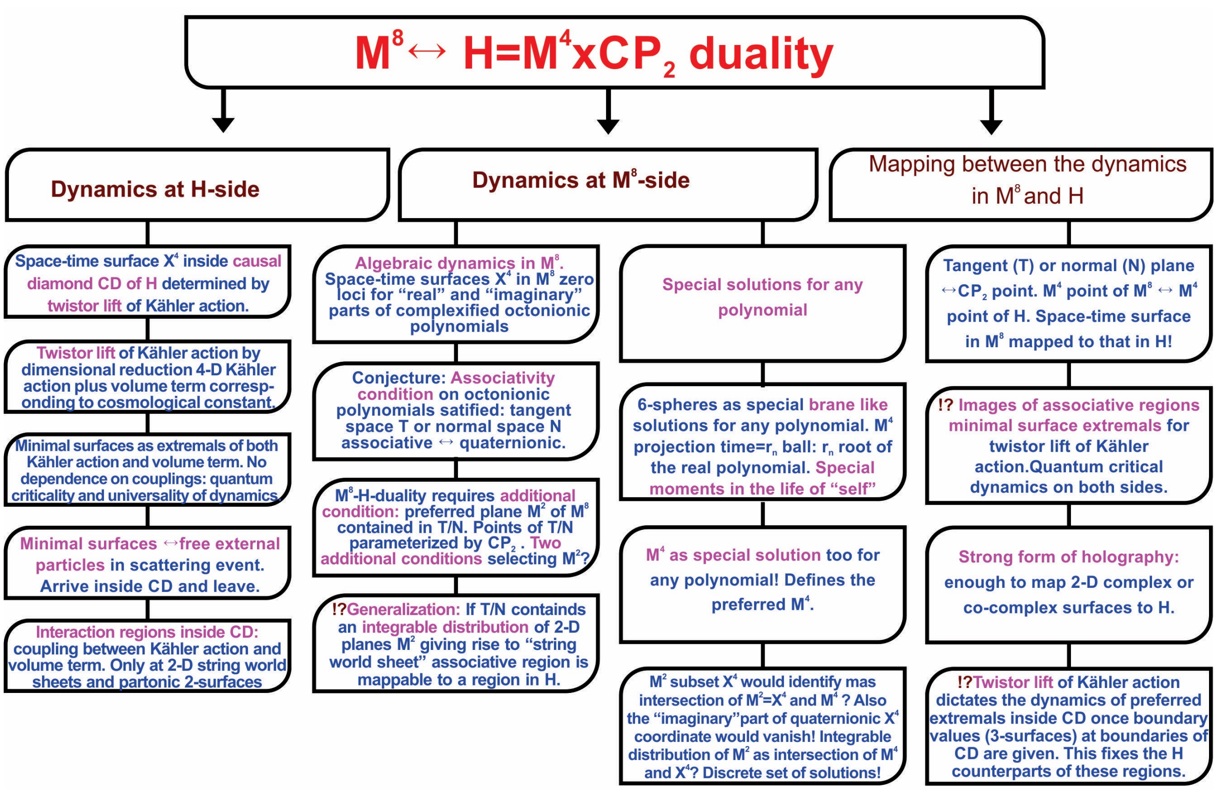

Figure 12: $M^8 - H$ duality

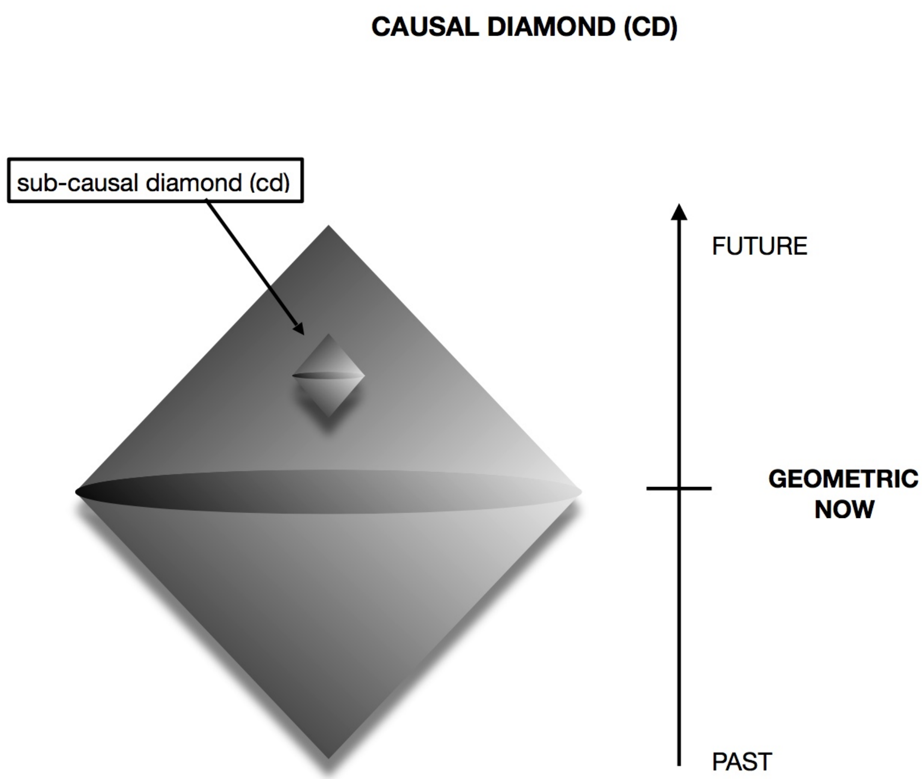

Figure 13: Causal diamond

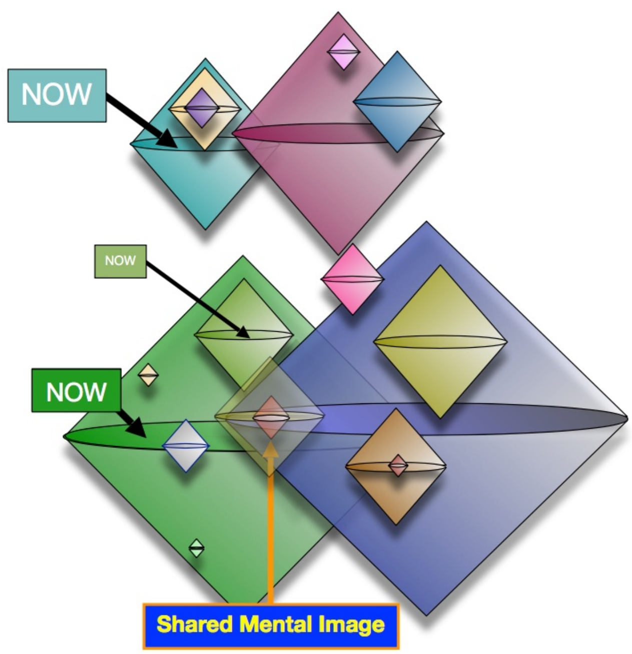

Figure 14: CDs define a fractal "conscious atlas"

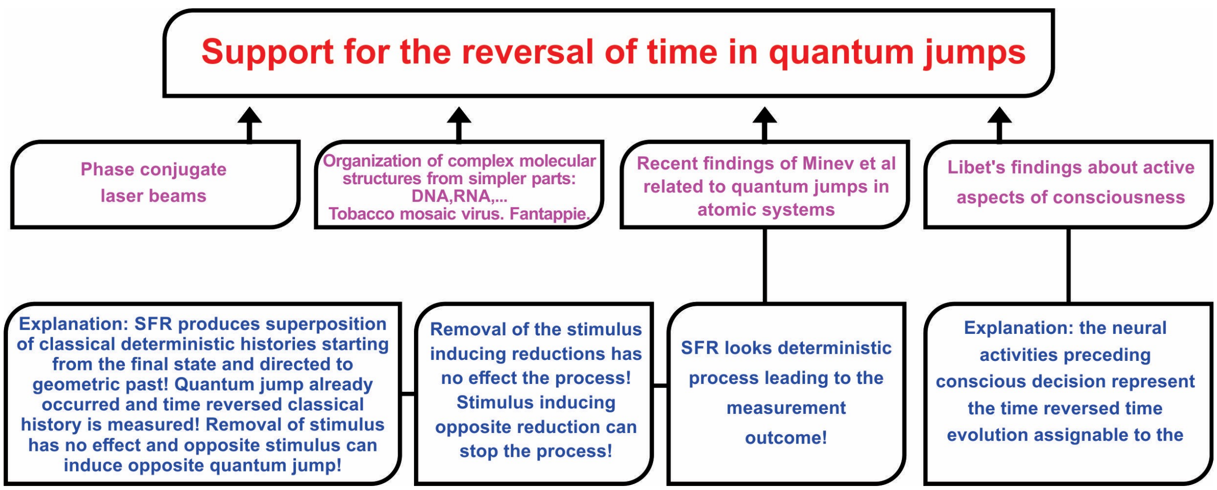

Figure 15: Time reversal occurs in BSFR

Received November 13, 2021; Last Modified December 1, 2021; Accepted April 30, 2022

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