Report

Bifurcations of the Higgs Potential & the Top Quark Mass

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Abstract

We have recently conjectured that the Standard Model gauge group unfolds under successive bifurcations of the Higgs potential. This brief report points out that the maximal fixed-point solution of the bifurcation process corresponds to a top-antitop quark condensate.

Keywords: Bifurcations, Feigenbaum route to chaos, gauge symmetries, Higgs potential, top quark.

It can be shown that the flow of the classical Higgs potential with the Renormalization scale takes the form [1]

$$\dot{y} = my(1-y^2) \tag{1}$$

in which y is given by

$$y = \frac{\sqrt{2}}{\mathrm{v}}\varphi \tag{2}$$

Here, φ denotes the amplitude of the complex-scalar field whose vacuum expectation value is $\mathrm{v} = 246$ GeV. Eq. (1) follows from the theory of bi-stable systems embedded in a *double-well potential* [8]. The control parameter of (1) contains the self-interaction coupling λ and a reference scale m_0 as in

$$m = \frac{2\lambda \mathrm{v}^2}{m_0^2} \tag{3}$$

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The differential equation (1) may be cast as the iterated map shown below

$$y_{n+1} = f(m, y_n) = m y_n (1 - y_n^2) \tag{4}$$

There are two trivial fixed points of (1) and (4), given by: a) $y^* = 0, m = 0, \lambda = 0$ - which resembles massless photons in an "effective" approximation, and b) a pair of maximal solutions arisen in the limit of large number of map iterations ($n \to \infty$), namely,

$$y_\infty^* = \pm 1 \tag{5}$$

whose separation along the y - axis is

$$\Delta y_\infty^* = +1 - (-1) = 2 \tag{6}$$

As suggested in [2-5], the fermionic sector of the Standard Model unfolds as the last segment of the bifurcation diagram. By (6) and (2), this conjecture leads to a separation in field space closely approximating a *top-antitop condensate*, that is,

$$\Delta \varphi_\infty^* = \sqrt{2}\,\mathrm{v} = 347.9 \text{ GeV} \tag{7a}$$

$$\Delta \varphi_\infty^* \approx 2 m_t \tag{7b}$$

where $m_t \approx 173$ GeV is the experimental value of the top quark mass [6]. As the top quark is the heaviest known fermion, relation (7) brings additional support for the self-contained flavor composition of the Standard Model near the electroweak scale [7].

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