Article

Bianchi Type-I Cosmological Model in Saez-Ballester Theory with Varying Cosmological Constant

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Abstract: The standard ΛCDM model is very successful to explain the universe, yet there are so many problems that are not still resolved. One of them are the cosmological constant problem, certain anomaly in cosmic microwave background radiation and general relativity is valid on large scale . Therefore it is necessary to describe modified theories to solve these problems.In this paper, we have analysed Bianchi type-I cosmological model in Saez-Ballester theory with varying cosmological constant.To solve field equations by using expansion anisotropy parameter ($\frac{\sigma}{\theta}$) to be a suitable function of scale factor (a) and suitable supposition of varying form of variable cosmological term. To found the physical parameter such as Hubble parameter (H), deceleration parameter (q), energy density (ρ), pressure (p), and cosmological constant (Λ) in term of redshift (z). Also we discuss cosmographic parameter such as jerk parameter (j) which foreshow that the universe in this model approaches the Lambda Cold Dark Matter model at the late time. We have seen that the our model supporting with the recent cosmological observational data.

Keywords: Saez-Ballester theory, Transit universe, Cosmological constant.

1 Introduction:

General theory of relativity is a geometric theory which represents gravitational effects and it is very useful to making mathematical models in cosmology which describe with LSS of the universe. The recent astronomical data like Supernova type Ia [1-6], Cosmic Microwave Background[7,8], Wilkinson Microwave Anisotropy Prob[9-12] have proved main features of the universe: the universe in early time anisotropic and in the present time isotropic and it is also the present universe is not only expanding but also the accelerating , its means rate of expansion is increasing. The supernova type Ia observations indicate that the universe is transition phase from early time decelerating phase to late time accelerating phase . To proving the above observations are more difficult for researchers theoretically.
Friedmann-Robertson -Walker (FRW) model represents spatially homogeneous and isotropic universe, which can be suitable for the present-time universe, but it is also does not represent a correct matter description for early universe. Therefore, these model are not capable to explain the nature of the universe early time anisotropic and late time isotropic. Bianchi spacetimes provide a correct matter to explain the behaviour of universe.
Einstein's general theory of relativity interpret many astronomical phenomena, but this theory fails to describe the expanding and late time accelerated expansion of the universe. To explain these behaviour of the universe so many modified theories has given, like as, Brans-Dicke gravity theory[13], Saez-Ballester theory of gravity [14], and modified theory of gravity such as $f(R)$ [15], $f(R,T)$[16] etc. In this paper, we have discussed Saez-Ballester theory of gravity and in S B theory, Einstein's field equations have been changed by including dimensionless scale field ϕ coupled with the metric g_{ij} in a simple manner. By changing in this theory it characterized the week fields in which an accelerated expansion regime appears. In recent years, many researchers such as Aditya and Reddy[17] had studied anisotropic new

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holograpic dark energy model in Saez -Ballester theory of gravitation. Bulk viscous string cosmological model in Saez-Ballester theory of gravity had solved by Mishra and Dua[18]. Kaluza Klein FRW dark energy model in Saez-Ballester theoryof gravitation had studied by Naidu et al.,[19]. From the above discussion we motivate to studied Bianchi type-I cosmological model in Saez-Ballester theory in the presence of cosmological constant. To solve field equation by using expansion anisotropy $(\frac{\sigma}{\theta})$ to be a suitable function of scale factor (a) and suitable supposition of varying form of variable cosmological term . The manuscript is construct as follows: In section-2 contains metric and field equations. Section-3 We have solved the field equations by taking assumption . Section-4 Discuss the parameters in terms of redshift. Section-5 we have discussed jerk parameter. In the last section-5 Conclusion Remark.

2 The metric and field equations

To develop cosmological model, here, we have assumed spatially homogeneous and anisotropic Bianchi type-I metric as

$$ds^2 = -dt^2 + X^2dx^2 + Y^2dy^2 + Z^2dz^2, \tag{2.1}$$

where $X = X(t), Y = Y(t), Z = Z(t)$ are metric potentials in x, y, z axes respectively.
As explained above, in Saez-Ballester theory (1986), the scalar field ϕ can not take as the part of variable G taken as in Brans-Dicke theory of gravity. Although it is taken as a dimensionless scale field. In this concept, metric g_{ij} is combing with a dimensionless scalar field ϕ in a straightforward manner and the gravitational field conditions have been gotten by characterizing the Lagrangian as

$$L = -\omega\phi^r\phi_{,\eta}\phi^{,\eta} + R, \tag{2.2}$$

The action for this theory is given by

$$S = \int_{\Omega}\sqrt{-g}(L_m + L)dx^1dx^2dx^3dx^4, \tag{2.3}$$

where Ω is an arbitrary integrating region, g is the determinant of metric tensor g_{ij}, L_m is the matter Lagrangian density and x^i are the coordinates. We have used geometrized units i.e. $8\pi G = 1$ and $c = 1$. By taking arbitrary independent variations of scalar field ϕ and metric g_{ij} vanishing at the boundary of Ω, $\delta S = 0$, gives following modified Einstein field equations:

$$R_{ij} - \frac{1}{2}Rg_{ij} - \omega\phi^r(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}) = -8\pi T_{ij} + \Lambda g_{ij}, \tag{2.4}$$

where, R_{ij} is Ricci tensor, R is Ricci Scalar, g_{ij} is a metric tensor, ω is a dimensionless coupling constant, r is an arbitrary constant, T_{ij} is the energy-momentum tensor of matter and ϕ is the scalar field satisfying the equation

$$2\phi^r\phi^{;i}_{;i} + r\phi^{r-1}\phi_{,k}\phi^{,k} = 0, \tag{2.5}$$

where, semi-colon denote covariant and comma denote partial derivative with respect to t.
The energy momentum tensor T_{ij} for the perfect fluid distribution can be written as

$$T_{ij} = (p + \rho)u_iu_j + pg_{ij}. \tag{2.6}$$

Here p and ρ are the pressure and energy density of the cosmic fluid respectively and $u_i = (0, 0, 0, 1)$ is the four velocity vector satisfying $g_{ij}u^iu^j = -1$.
Here, the field equations (2.4) and (2.5) for Bianchi type-I metric and perfect fluid distribution reduce to the following set of the differential equations:

$$\dot{H}_y + H_y^2 + \dot{H}_z + H_z^2 + H_yH_z - \frac{1}{2}\omega\phi^r\dot{\phi}^2 = -8\pi p + \Lambda, \tag{2.7}$$

$$\dot{H}_x + H_x^2 + \dot{H}_z + H_z^2 + H_xH_z - \frac{1}{2}\omega\phi^r\dot{\phi}^2 = -8\pi p + \Lambda, \tag{2.8}$$

$$\dot{H}_x + H_x^2 + \dot{H}_y + H_y^2 + H_xH_y - \frac{1}{2}\omega\phi^r\dot{\phi}^2 = -8\pi p + \Lambda, \tag{2.9}$$

$$H_xH_y + H_yH_z + H_zH_x + \frac{1}{2}\omega\phi^r\dot{\phi}^2 = 8\pi\rho + \Lambda, \tag{2.10}$$

$$\ddot{\phi} + \dot{\phi}(H_x + H_y + H_z)\frac{r}{2}\frac{\dot{\phi}^2}{\phi} = 0, \tag{2.11}$$

here $H_x = \frac{\dot{X}}{X}$, $H_y = \frac{\dot{Y}}{Y}$ and $H_z = \frac{\dot{Z}}{Z}$.
The average scale factor (a) for the Bianchi type-I model is

$$a^3 = XYZ. \tag{2.12}$$

Using equations(2.7) and (2.8), we get

$$(\dot{H}_x + H_x^2) - (\dot{H}_y + H_y^2) - H_z(H_x - H_y) = 0. \tag{2.13}$$

After integration, we get

$$H_x - H_y = \frac{k_1}{a^3}. \tag{2.14}$$

Again, by using equations (2.8) and (2.9), we obtain

$$H_y - H_z = \frac{k_2}{a^3}. \tag{2.15}$$

From equations (2.12),(2.14) and (2.15), we get

$$H_x = \frac{\dot{a}}{a} + \frac{(2k_1 + k_2)}{3a^3}, \tag{2.16}$$

$$H_y = \frac{\dot{a}}{a} + \frac{(k_2 - k_1)}{3a^3}, \tag{2.17}$$

$$H_z = \frac{\dot{a}}{a} - \frac{(k_1 + 2k_2)}{3a^3}. \tag{2.18}$$

The Hubble parameter (H), deceleration parameter (q), shear scalar (σ) and anisotropy parameter (A_m), for Bianchi type-I model is given by

$$H = \frac{\dot{a}}{a}, \tag{2.19}$$

$$q = -\frac{\dot{H}}{H^2} - 1, \tag{2.20}$$

$$\sigma_{ij} = \frac{1}{2}\sigma_{ij}\sigma^{ij}, \tag{2.21}$$

where

$$\sigma_{11} = H_x - H = \frac{(2k_1 + k_2)}{3a^3}, \tag{2.22}$$

$$\sigma_{22} = H_y - H = \frac{(k_2 - k_1)}{3a^3}, \tag{2.23}$$

$$\sigma_{33} = H_z - H = -\frac{(k_1 + 2k_2)}{3a^3}, \tag{2.24}$$

$$\sigma_{44} = 0, \tag{2.25}$$

$$\sigma = \frac{k}{a^3}, \tag{2.26}$$

where $3k^2 = k_1^2 + k_2^2 + k_1 k_2$, and

$$A_m = \frac{1}{3} \sum_{i=x,y,z} \left(\frac{H_i - H}{H} \right)^2. \tag{2.27}$$

3 Solution of field equations

The field equations given by (2.7)-(2.11), involve seven know variables $X, Y, Z, \rho, p, \Lambda$ and ϕ . We must have two more conditions for solving the field equations.
At first, we assume that $\frac{\sigma}{\theta}$ is a suitable function for scale factor (a) given by J. P. Singh et al.[20]

$$\frac{\sigma}{\theta} = \frac{1}{(n + a^3)}, \tag{3.1}$$

where n is positive integer.
We find previously $\frac{\sigma}{\theta} \neq 0$ and later $\frac{\sigma}{\theta} = 0$ for this supposition.
We find on integration

$$a(t) = [n(e^{kt} - 1)]^{\frac{1}{3}}. \tag{3.2}$$

The expansion scalar (θ), shear scalar (σ) and deceleration parameter (q) are given by

$$\theta = \frac{k}{1 - e^{-kt}}, \tag{3.3}$$

$$\sigma = \frac{k}{n(e^{kt} - 1)}, \tag{3.4}$$

$$q = -1 + 3e^{-kt}. \tag{3.5}$$

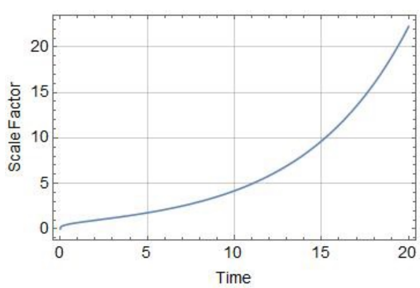

Fig.-1. Scale factor vs time.

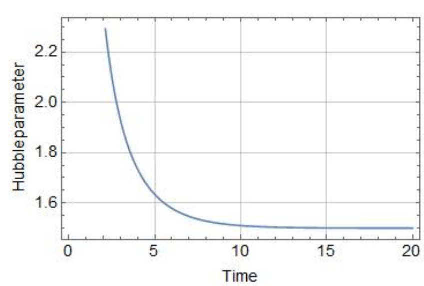

Fig.-2. Hubble parameter vs time.

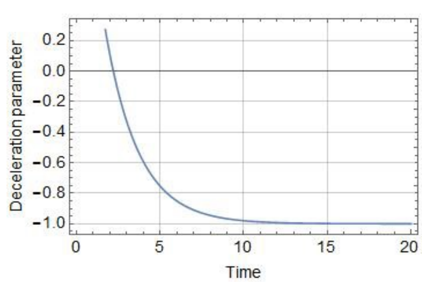

Fig.-3 Deceleration parameter vs time.

From the above we see that at $t = 0$, the scale factor (a) is zero and expansion scalar (θ) is infinity and when $t \to \infty$, the scale factor (a) is infinite and expansion scalar (θ) is constant. At initial stage, the model is singular with supporting the big-bang theory. In our model the deceleration parameter (q) in transition phase early time decelerating phase and late time accelerating phase. In this model deceleration parameter lies between 2 to -1. Due to supernova type Ia observation at present the universe is accelerating and the value of deceleration parameter lies between $-1 < q < 0$. This feature of the model supporting very well with the observational data.
The anisotropy parameter

$$\frac{\sigma}{\theta} = \frac{(1 - e^{-kt})}{n(e^{kt} - 1)}. \tag{3.6}$$

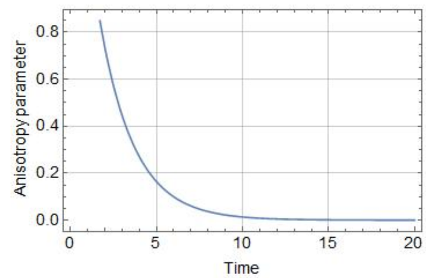

Fig.-4 Anisotropy parameter vs time.

We see that $\frac{\sigma}{\theta}$ is non-zero finite, at $t = 0$ and $t \to \infty$ as $\frac{\sigma}{\theta} \to 0$, then the model approaches to late time isotropy.
From equation (11), we get scale field ϕ

$$\phi = \left[\frac{r+2}{2}\left(\phi_0 \int \frac{dt}{a^3}\right)\right]^{\frac{2}{r+2}} = \left[\frac{r+2}{2}\left(\frac{\phi_0}{n}\left\{-t + \frac{log(1 - e^{kt})}{k}\right\}\right)\right]^{\frac{2}{r+2}}, \tag{3.7}$$

where ϕ_0 is constant of integration.
Secondly, we have considered the variable cosmological constant Λ are obtained by

$$\Lambda = 3\beta H^2, \tag{3.8}$$

where β is constant.
The physical parameters such as energy density, pressure and cosmological constant are obtained as

$$\rho = \frac{1}{8\pi}\left[\frac{(1-\beta)k^2}{3(e^{kt} - 1)^2} + \frac{1}{n^2(e^{kt} - 1)^2}\left(\frac{1}{2}\omega\phi_0^2 - k^2\right)\right], \tag{3.9}$$

$$p = -\frac{1}{8\pi}\left[\frac{(2e^{-kt} - (1+\beta))k^2}{3(e^{kt} - 1)^2} - \frac{1}{n^2(e^{kt} - 1)^2}\left(\frac{1}{2}\omega\phi_0^2 + k^2\right)\right], \tag{3.10}$$

and

$$\Lambda = \frac{\beta k^2}{3(1 - e^{-kt})^2}. \tag{3.11}$$

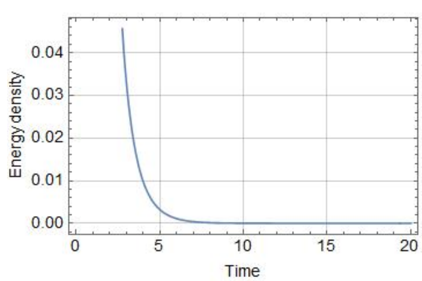

Fig.-5. Energy density vs time.

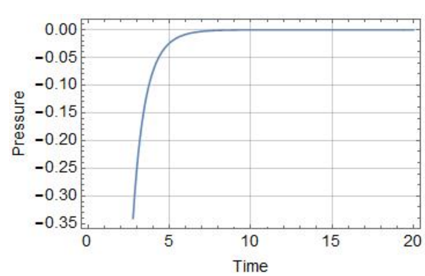

Fig.-6. pressure vs time.

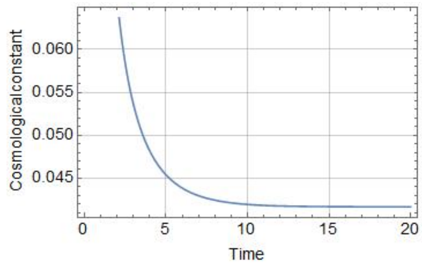

Fig.-7 Cosmological constant vs time.

We show that the energy density ρ has huge positive qualities at early stage and it approaches zero with late time and pressure p is negative. The negative-pressure matter can be considered as the justification of the late time accelerating expansion of our universe. The cosmological constant is a positive and decreasing function of cosmic time t which supports accelerating universe.

4 Solution of model in terms of Redshift

It is observe that when the galaxies moves relative to us, wavelength of light be change emitted by galaxies. In physics increase in the wavelength and corresponding decrease in the frequency and photon energy of electromagnetic radiation is called red shift.
Redshift z is related to scale factor $a(t)$ through

$$\frac{a}{a_0} = \frac{1}{1+z}. \tag{4.1}$$

As redshift z is related to scale factor $a(t)$, it is important to evince cosmological parameters in terms of z to study the development of the universe in more wisely. A expression for cosmic time t in terms of redshift z is obtained as

$$t(z) = \frac{1}{k} log\left[\frac{1}{n}(1+z)^{-3} + 1\right]. \tag{4.2}$$

The Hubble parameter and deceleration parameter are obtained in terms of redshift as

$$H = \frac{k}{3}\left[1 + \frac{1}{n}(1+z)^3\right], \tag{4.3}$$

$$q = -1 + \frac{3(1+z)^3}{\frac{1}{n} + (1+z)^3}. \tag{4.4}$$

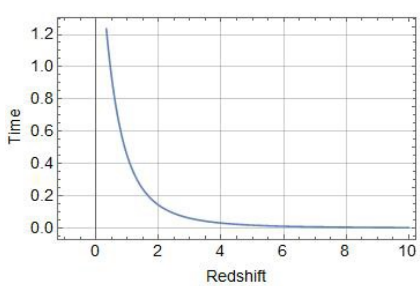

Fig.-8. Time vs Redshift.

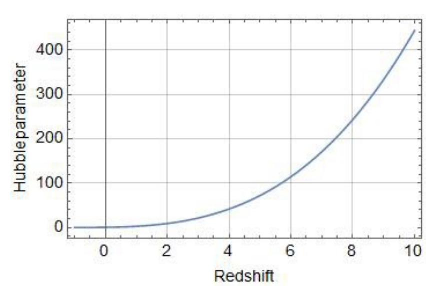

Fig.-9. Hubble parameter vs Redshift.

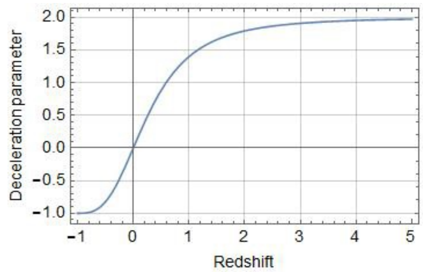

Fig.-10 Deceleration parameter vs Redshift.

We see that the change of time t with redshift z in our model. In this model $t \to \infty$ at $z \to -1$. We have seen transition of deceleration parameter with decreasing redshift, the deceleration parameter from decelerating phase to accelerating phase and $q \to -1$ as $z \to -1$. Recently S. Capozziello et al[21] have been studied the cosmographic bounds on the cosmological deceleration-acceleration transition reshift in $f(R)$ gravity.

The energy density, pressure and cosmological constant are obtained in term of redshift

$$\rho = \frac{1}{8\pi}\left[\frac{(1-\beta)k^2}{3[1+\frac{1}{n}(1+z)^3]^2} + (1+z)^6\left(\frac{1}{2}\omega\phi_0^2 - k^2\right)\right], \tag{4.5}$$

$$p = -\frac{1}{8\pi}\left[\frac{k^2}{3[1+\frac{1}{n}(1+z)^3]^2}\left\{\frac{2}{\frac{1}{n}(1+z)^3+1} - (1+\beta)\right\} - (1+z)^6\left(\frac{1}{2}\omega\phi_0^2 + k^2\right)\right], \tag{4.6}$$

$$\Lambda = \frac{\beta k^2}{3}\left[1+\frac{1}{n}(1+z)^3\right]^2. \tag{4.7}$$

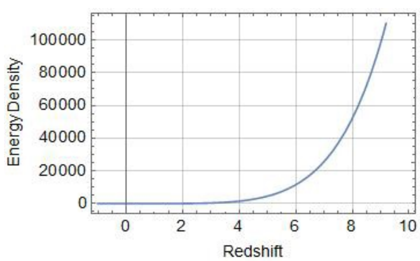

Fig.-11. Energy density vs Redshift.

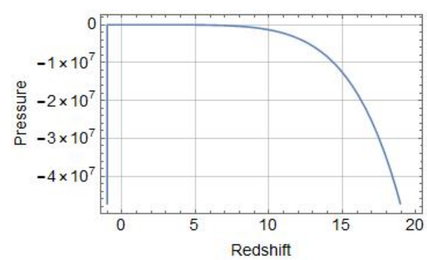

Fig.-12. pressure vs Redshift.

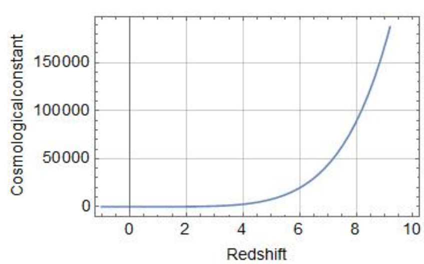

Fig.-13 Cosmological constant vs Redshift.

It is shown that energy density , pressure and cosmological constant tends to zero as $z \to -1$. The energy density and cosmological constant both are positive and increasing function of redshift and pressure is negative , the negative pressure is responsible for cosmic acceleration.

5 The Jerk parameter

The dimensionless jerk parameter (j) is third derivative of scale factor with respect to cosmic time (t) and provides a perfect diagnosis to ΛCDM dynamics . A deceleration to acceleration transition occurs for models with a positive value of j_0 and negative value of q_0. Flat ΛCDM models have a constant jerk $j = 1$. The jerk parameter (j) is defined as

$$j(t) = \frac{\dddot{a}}{aH^3}, \tag{5.1}$$

over dot denote derivative with respect to cosmic time (t).
Equation (5.1) can be written as

$$j(t) = q + 2q^2 - \frac{\dot{q}}{H}. \tag{5.2}$$

Using equations (3.3) and (3.5), we get

$$j(t) = 1 + 9e^{-2kt}. \tag{5.3}$$

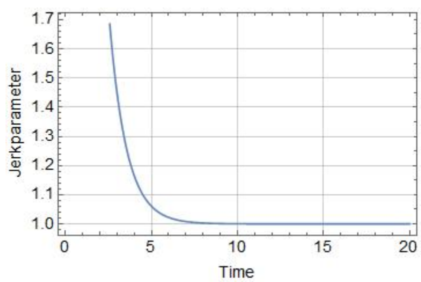

Fig.-14 jerk parameter vs time.

For a flat ΛCDM model, jerk parameter has the value $j = 1$.

Conclusion

In this manuscript, we have analysed Bianchi type-I cosmological model in Saez-Ballester theory with varying cosmological constant.To solve field equations by using expansion anisotropy parameter ($\frac{\sigma}{\theta}$) to be a suitable function of scale factor (a)and suitable supposition of varying form of variable cosmological term. In this supposition ,which yields deceleration parameter is a transition phase from early time decelerating and late time accelerating. The main feature of the model as follows:

• The supposition from an expansion anisotropy parameter yields model of universe which notice that early time anisotropic and late time isotropic. Such behaviour of the model supports with recent observations.

• The deceleration parameter of the model in early time is positive and late time is negative it means universe is early time decelerating and late time accelerating phase of expansion.

• The energy density and cosmological constant both are positive and decreasing function of cosmic time t. In our model the value of pressure is negative . The negative pressure contribute to the accelerated expansion of the universe. Also ρ, p and $\Lambda \to 0$ as $z \to -1$.

• Jerk parameter is positive over all evaluation of the universe and late time $j = 1$ i.e., the model predicts ΛCDM model at present.

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