Article

Bianchi Type–V Bulk Viscous Fluid Cosmological Models with Constant Deceleration Parameter

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Abstract

A Class of Bianchi Type-V Cosmological Models with time dependent Λ is presented. Where an equation of state p = ωρ and deceleration parameter is taken to constant, are considered. The universe is decelerating throughout the evolution. The physical and geometrical behaviour of the models are also discussed.

Keywords: Bianchi Type-V, declaration parameter, bulk viscosity, cosmological model.

1. Introduction

A host of observations show that our universe is currently undergoing an accelerating expansion. This has been confirmed by astronomical observation, such as observations of type Ia supernovae (SNIa) [1, 2], observations of large scale structure [3, 4] and measurements of the Cosmic Microwave Background (CMB) anisotrophy [5-8]. Based on these observations, cosmologists have accepted the idea of dark energy, which is a fluid with negative pressure making up around 70 percent of the present energy content to be responsible for this acceleration due to repulsive gravitation. There are many theoretical models of dark energy but the Lorentz Invariant Vacuum Energy (LIVE) which can be presented by cosmological constant Λ with a constant equation of state parameter is ω = 1, is still the preferred model. The so called Λ CDM model, which in a flat universe model contain both LIVE and Cold Dark Matter (CDM), i.e. dust, is the simplest cosmological model that is in agreement with the observational data and can explain the accelerated expansion of the universe.

Vishwakarma [9] has studied axially symmetric Bianchi Type-I models with perfect fluid distribution of matter and variable. Cosmological term Λ, where cosmological term Λ measures of the energy of empty space, provides a repulsive force opposing the gravitational pull between the galaxies. A number of Workers [10, 11] have studied the cosmological models with time-dependent cosmological parameter Λ. However, in the presence of interaction with matter or radiation, a solution of Einstein's and the assumed equation of conservation of stress energy with

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time-dependent Λ can be found. Homogeneous and anisotropic Bianchi type-I, cosmological models provides, such a framework.

Kumar and Singh [12]; Pradhan and Jotania [13]; Akarsu and Kilinc [14, 15]; Agrawal *et al.* [16]; Singh and Baghel [17] have investigated the common to use a constant deceleration parameter, as it duly gives a power law for matric function or corresponding quantity. Tonry *et al.* [18] and Riess *et al.* [19, 20] have studied the Type-Ia Supernovae (S Ne) allow us to probe the expansion history of the universe leading to the conclusion that the expansion of the universe is accelerating. Thus, we can consider cosmological models with time dependent cosmological term Λ and constant deceleration parameter q.

In this paper, we consider Bianchi Type-V cosmological models for perfect fluid distribution assuming the deceleration parameter q to be not function of time which account for initial anisotropy and approaches isotropy at late times. Physical and Kinematical behaviour of the models have also discussed.

2. Model & Field Equation

We consider the Bianchi type-V space time in orthogonal form represented by the line-element

$$ds^2 = -dt^2 + A^2(t)dx^2 + e^{2\alpha x}\left\{B^2(t)dy^2 + C^2(t)dz^2\right\} \tag{1}$$

where α is a constant. We assume the cosmic matter consisting of bulk viscous fluid given by the energy momentum tensor as :

$$T_i^j = (\rho + \overline{p})v_i v^j + \overline{p} g_i^j \tag{2}$$

where $\overline{p}$ is the effective pressure given by

$$\overline{p} = p - \xi v_i^j; \tag{3}$$

satisfying linear equation of state

$$p = w\rho, \qquad 0 \le w \le 1 \tag{4}$$

ρ being energy density of matter, p is the isotropic pressure, ξ is the coefficient of bulk viscosity and v^j the flow vector of the fluid satisfying $v_i v^i = -1$. On thermodynamical grounds bulk viscous coefficient ξ is positive, assuming that the viscosity pushes the dissipative pressure $\overline{p}$ towards negative values. But correction to the thermodynamical pressure p due to bulk viscous pressure is very small.

Therefore, the dynamics of cosmic evolution does not change fundamentally by inclusion of viscous term in the energy momentum tensor. The Einstein's field equations with time-dependent cosmological term Λ are given by

$$R_i^j - \frac{1}{2} R g_i^j = -T_i^j + \Lambda(t) g_i^j \tag{5}$$

In comoving system of coordinates, the field equations (5) for the metric (1) and matter distribution (2) lead to

$$\bar{p} - \Lambda = \frac{\alpha^2}{A^2} - \frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} - \frac{\dot{B}\dot{C}}{BC} \tag{6}$$

$$\bar{p} - \Lambda = \frac{\alpha^2}{A^2} - \frac{\ddot{A}}{A} - \frac{\ddot{C}}{C} - \frac{\dot{A}\dot{C}}{AC} \tag{7}$$

$$\bar{p} - \Lambda = \frac{\alpha^2}{A^2} - \frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} - \frac{\ddot{A}\dot{B}}{AB} \tag{8}$$

$$\rho + \Lambda = -\frac{3\alpha^2}{A^2} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} \tag{9}$$

$$O = \frac{2\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C} \tag{10}$$

Covariant divergence of (5) gives

$$\dot{\rho} + (\rho + \bar{p})\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) + \frac{\dot{\Lambda}}{8\pi G} = 0$$

We observe that for a constant Λ, equation (11) reduces to the equation of continuity.

We define the average scale factor R for Bianchi type V space time as R^3 = ABC. From (6) – (8) and (10), we get

$$\frac{\dot{A}}{A} = \frac{\dot{R}}{R} \tag{12}$$

$$\frac{\dot{B}}{B} = \frac{\dot{R}}{R} - \frac{k}{R^3} \tag{13}$$

$$\frac{\dot{C}}{C} = \frac{\dot{R}}{R} + \frac{k}{R^3} \tag{14}$$

where k is constant of integration.

We introduce volume expansion θ and shear scalar σ as usual

$$\theta = v_i^j \quad \text{and} \quad \sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} \tag{15}$$

where σ_{ij} is the shear tensor. For Bianchi type V metric, expansion scalar θ and shear scalar σ come out to be

$$\theta = \frac{3\dot{R}}{R} \tag{16}$$

$$\sigma = \frac{k}{R^3} \tag{17}$$

In analogy with FRW universe, we define the generalized Hubble parameter H and generalized deceleration parameter q as

$$H = \frac{\dot{R}}{R} \tag{18}$$

$$q = -1 - \frac{\dot{H}}{H^2} \tag{19}$$

We can write equation (6) – (9) and (11) in terms of H, σ and ρ as

$$\overline{p} - \Lambda = \frac{\alpha^2}{R^2} + H^2(2q-1) - \sigma^2 \tag{20}$$

$$\rho + \Lambda = -\frac{3\alpha^2}{R^2} + 3H^2 - \sigma^2 \tag{21}$$

$$\dot{\rho} + 3(\rho + \overline{p})H + \dot{\Lambda} = 0 \tag{22}$$

From equations (20) and (21), we get

$$\frac{\ddot{R}}{R} = -\frac{(\rho + 3p)}{6} - \frac{2}{3}\sigma^2 + \frac{\xi}{2}\theta + \frac{\Lambda}{3} \tag{23}$$

We observe that the positive cosmological term and bulk viscosity contribute positively in driving the acceleration of the universe. Also, from equation (21), we get

$$\frac{3\dot{R}^2}{R^2} = \frac{3\alpha^2}{R^2} + \sigma^2 + \rho + \Lambda \tag{24}$$

when $\Lambda \geq 0$, each term on the right hand side of (24) is non-negative. Thus $\dot{R}$ does not change sign and we get ever – expanding models. For $\Lambda < 0$, however, we can get universe that expand and then recontract.

From equation (11), we get

$$R^{-3(w+1)}\frac{d}{dt}\left\{\rho R^{3(w+1)}\right\}=3\xi H^{2}-\dot{\Lambda} \tag{25}$$

Thus, decaying vacuum energy and viscosity of the fluid lead to matter creation.

3. Solution & Discussion

We assume the form of deceleration parameter q given by :

$$q=\frac{-R\ddot{R}}{\dot{R}^{2}}=\text{constant.} \tag{26}$$

For this choice, the average scale R come out to be

$$R=[R_{0}(t+t_{0})]^{\frac{1}{1+n}}=(R_{0}T)^{\frac{1}{1+n}} \tag{27}$$

Where S_0 and t_0 are constant of integration and $T = t_0 + t$.

Coefficient of bulk viscosity ξ in the form

$$\xi=\frac{1}{\xi_{0}+t}.$$

The expression for expansion scalar θ, special volume V, shear scalar σ of the model are given by

$$\theta=\frac{3}{(1+n)T}$$

$$V=R^{3}=(S_{0}T)^{\frac{1}{1+n}}$$

$$\sigma=\frac{k}{(S_{0}T)^{\frac{3}{1+n}}}$$

The matter density ρ and cosmological constant Λ for the model are given by

$$(1+w)\rho=\frac{2}{T^{2}}-\frac{2k}{(R_{0}T)^{\frac{6}{1+n}}}-\frac{2\alpha^{2}}{(R_{0}T)^{\frac{2}{1+n}}}+\frac{3}{(\xi_{0}+t)(1+n)T}$$

$$\Lambda=\frac{(3w-2n-1)}{(w+1)(1+n)^{2}T^{2}}+\frac{(1-w)k^{2}}{(1+w)(R_{0}T)^{\frac{6}{1+n}}}$$

$$-\frac{(3w+1)}{(w+1)}\frac{\alpha^2}{(R_0T)^{\frac{2}{1+n}}}-\frac{3}{(\xi_0+t)(1+n)T}$$

We observe that model has singularity at T = 0. The model begins with a big-bang start at T = 0 and spatial volume of model increases as time increases. At T = 0, ρ, Λ, θ, σ are all infinite whereas $\xi \to \frac{1}{\xi_0}$. For the large values of T (i.e. T → ∞), ρ, Λ, θ, σ become zero. Since q = n (constant) > 0, the universe is decelerating throughout the evaluation. For the model

$$\frac{\sigma}{\theta}=\frac{k(1+n)}{3R_0^{\frac{3}{1+n}}T^{\frac{2-n}{1+n}}}$$

We observe that $\frac{\sigma}{\theta}$ becomes infinite at T = 0. For T → ∞, $\frac{\sigma}{\theta} \to 0$. Therefore, the model approaches isotropy at late times.

We also observe that the presence of bulk viscosity increase the value of ρ and decrease the value of Λ. In absence of bulk viscosity i.e. ξ → 0, we can obtain the solution of Einstein's field equation for perfect fluid distribution.

4. Conclusion

In this paper we have studied Bianchi Type – V Bulk viscous fluid cosmological models with constant deceleration parameter q in the context of general relativity. The Einstein's field equation have been solved exactly by considering a deceleration parameter q = constant with yields time dependent scale factor

$$R=(R_0T)^{\frac{1}{1+n}}$$

We have found that cosmological term Λ being very large at initial times relaxes to genuine cosmological constant at late times. The models are found to be compatible with the results of recent observations.

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