Article

Bianchi Type-VI0 Universe with Anisotropic Dark Energy in the Presence of a Massless Scalar Field in Lyra Manifold

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Abstract

In this paper, the authors have investigated a Bianchi type-VI_0 cosmological model in the presence of an anisotropic dark energy and a massless scalar field in Lyra manifold. Exact solutions of the field equations are obtained by utilizing a special form of the average scale factor that yields a time-varying deceleration parameter. The physical and dynamical features of the model are discussed. It is shown that the anisotropic dark energy model is consistent with the recent observations on the present-day universe.

Keywords: Bianchi Type-VI_0 space-time, dark energy model, massless scalar field, Lyra manifolds.

1. Introduction

The cosmological origin of universe is based on the observations of the cosmic microwave background (CMB) anisotropy. The spatial distribution of galaxies on a large scale and their existence with isotropy was explained by the validity of the hypothesis of the spatial homogeneity and isotropy of the universe. The theoretical interest in anisotropic cosmological models has been due to the realization that the standard models, which are in good agreement with the present-day universe, do not provide a clear distribution of the early phases of evolution of the universe while physically realistic description of the early universe is best given by anisotropic models.

The spatially homogeneous and anisotropic Bianchi models present a middle way between FRW models and completely inhomogeneous and anisotropic models, thus play an important role in modern cosmology. This is due to the fact that close to the big-bang singularity, neither the assumption of the spherical symmetry nor of isotropy can be strictly valid. This stimulates

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researchers to study exact isotropic solutions of Einstein field equations, which yield cosmologically acceptable physical models of Bianchi type I-IX [1].

Bianchi type-VI_0 space-times play a fundamental role in constructing models with rather structure both geometrically and physically for describing the early stages of evolution of the universe. Bianchi type-VI_0 space-time has been extensively studied so far in different physical contexts. Here we are concerned to present Bianchi type-VI_0 cosmological model in the presence of anisotropic dark energy (DE) and a massless scalar field in Lyra manifold.

The limitations of general relativity in providing satisfactory explanation of the phases of the universe in its evolution have lead researchers to adopt various hypotheses and study their implications in this context. These hypotheses include those assigning other geometries or physical fields with the universe. Such theories are expected to bring out a number of aspects of mathematical and physical interests associated with them. As the Einstein's theory of general relativity is based on the geometrical description of gravitation, many researchers have used their efforts to generalize the idea of geometrizing the gravitation to include a geometrical description of electromagnetism.

For the purpose of unification of gravitation, electromagnetic fields and many other effects, various modifications to Riemannian geometry have been carried out. Lyra [2] has suggested one such modification to incorporate a gauge function into the structureless manifold giving rise to a displacement field. Halford [3] discussed Robertson-Walker models in Lyra's geometry with time independent gauge function. Consequently several researchers have investigated solutions of the field equations in general relativity and cosmology with time-independent and time-dependent displacement field vector in different physical contexts.

Scalar-tensor theories of gravitation is a local point of interest in many areas of gravitational physics and cosmology, as they provide the most natural generalisation of general relativity and thus provide a convenient set of representations of the observational limits on possible deviations from general relativity. To draw analogy of physics of the cosmos with experimental evidences, investigation of the nature of scalar field with or without a mass parameter interacting with matter distribution has been a topic of significant interest since long of many researchers. Bergmann and Leipnik [5] investigated a spherically symmetric space-times with zero-rest-mass scalar fields in general relativity.

Penney [6] and Gautreau [7] have extended the study to the axially symmetric fields and have shown that the scalar fields obey a flat-space Laplace Equations. Hawking and Ellis [8] have shown that the flat zero-rest-mass scalar field Robertson-Walker model can be reduced to a steady state model as $t \to \infty$. Singh [9] studied the static plane symmetric solutions of Einstein's field equations in empty space in the presence of zero-rest-mass scalar field. Chatterjee and Roy [10] have presented a plane symmetric space-time in the presence of the perfect fluid coupled with zero-rest-mass scalar field. Rahaman et al.[11] presented a Kantowski-Sachs cosmological model in the presence of zero-rest-mass scalar field. Verma et al.[12] investigated an anisotropic bulk viscous cosmological model with zero-rest-mass scalar field and time-dependent cosmological term. Venkatswarlu and Pawan Kumar [13] considered the interaction of massive string and zero-rest-mass scalar field and have obtained a plane symmetric model for the distribution. Venkateswarlu and Satish[14] obtained a bulk viscous cosmic string Kantowski-Sachs model in the presence of zero rest-mass-scalar field.

A number of recent astronomical data provided by type Ia supernovae [15, 16], large scale structure [17] and measurements of the cosmic microwave background (CMB) anisotropy [18] etc. have convincing evidences that the present-day universe is not only expanding but it is also accelerating. This accelerated expansion is believed to be caused by a mysterious energy present in the universe, called dark energy (DE) with strong negative pressure. The thermodynamic studies of DE reveals that the constituents of DE may be massless particles (bosons and fermions) whose collective behaviours resembles with a kind of radiation fluid having negative pressure which is a kind of repulsive force acting as antigravity responsible for gearing up the universe.

Investigation of DE models in the presence of massive as well as massless scalar fields is of vital importance role in inflationary cosmology. In particular, spatially homogeneous and anisotropic Bianchi type cosmological models in the presence of a scalar fields play important role in the study of possible effects of anisotropy in the early Universe. Reddy etal.[19] studied the dynamics of an LRS Bianchi type-II DE model in presence of massless scalar field. Katore and Hatkar [20] have investigated Kaluza-Klein universe with magnetized anisotropic DE in the context of Lyra manifolds. Shri Ram et al. [21] discussed Kantowski-Sachs universe with anisotropic DE. Aditya et al. [22] have presented a Kaluza-Klein anisotropic DE model in Lyra manifold in the presence of massive scalar field. In the existing literature a good number of exact solutions of Einstein's filed equations for anisotropic DE for spatially homogeneous Bianchi type models can be found.

In this paper, we study a Bianchi type-VI_0 anisotropic DE cosmological model in the presence of zero-rest-mass scalar field within the framework of Lyra manifold. The paper is organised as follow : In section 2,the metric and field equations are presented. In section 3, exact solutions of the field are obtained. In Section 4, the physical features of the cosmological model are discussed by using the kinematical and physical parameters. The summary and conclusions are given in section 5.

2. Metric and Basic Field Equations

We consider the spatially homogeneous and anisotropic Bianchi type-VI_0 space-time with the metric of the form

$$ds^2 = dt^2 - A^2 dx^2 - B^2 e^{-2mx} dy^2 - C^2 e^{2mx} dz^2 \tag{1}$$

where A ,B and C are functions of the cosmic time t $only$ and m is a constant parameter.

Einstein's field equations in the presence of anisotropic DE fluid and zero-rest-mass scalar field in Lyra manifold are given as

$$R_{ij} - \frac{1}{2} g_{ij} R + \frac{3}{2}\left(d_i d_j - \frac{1}{2} g_{ij} d_k d^k\right) = -T_{ij} - \frac{1}{2}\left(\phi_{,i}\phi_{,j} - \frac{1}{2} g_{ij}\phi_{,k}\phi^{,k}\right) \tag{2}$$

where ϕ is the zero-rest-mass scalar field which satisfies the Klein-Gordon equation

$$g^{ij}\phi_{;ij} = 0 \tag{3}$$

where a semicolon denotes covariant derivative and a comma denotes an ordinary derivative. In the field of equations (2) di is the time-like displacement vector given by

$$di = (\beta(t), 0, 0, 0) \tag{4}$$

here $\beta(t)$ is the time-dependent gauge function in Lyra geometry. Beesham [23] has shown that, the advantage of cosmological models based on Lyra manifold, rather than Riemannian manifolds, is that the cosmological constant arises naturally from the geometry rather than being introduced in an arbitrary adhoc fashion.

The energy-momentum tensor T_{ij} for DE fluid is defined a

$$\begin{aligned} T_i^j &= diag[\rho_\wedge, -p_x, -p_y, -p_z] \\ &= diag[1, -\omega_x, -\omega_y, -\omega_z]\rho_\wedge \end{aligned} \tag{5}$$

where $\rho_\wedge$is the dark energy density and p_x, p_y and p_z are pressure of the DE fluid along x, y and zaxes respectively. Here

$$\omega_\wedge = \frac{p_\wedge}{\rho_\wedge} \tag{6}$$

is the equation of state (EoS) parameter of the fluid and ω_x, ω_y, ω_z are the EoS parameters in the directions of x,y and z-axes respectively. For the sake of simplicity, we parametrized T_{ij} so that

$$T_i^j = diag[1, -\omega, -(\omega + 8) - (\omega + \gamma)\,]\rho \tag{7}$$

where δand γ are skewness parameters in EoS parameters in the direction of y and z-axes and we write$\omega_\wedge = \omega$ and $\rho_\wedge = \rho$. The skewness parameters need not to be constant but can be functions of cosmic time.

In comoving coordinates, Einstein's field equation (2) for the metric (1) with the help of (4) and (7) can be explicitly written as

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} + \frac{m^2}{A^2} + \frac{3}{4}\beta^2 = -\omega\rho - \frac{\dot{\phi}^2}{2}, \tag{8}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{C}}{AC}-\frac{m^2}{A^2}+\frac{3}{4}\beta^2=-(\omega+\delta)\rho-\frac{\dot{\phi}^2}{2}, \tag{9}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}-\frac{m^2}{A^2}+\frac{3}{4}\beta^2=-(\omega+\gamma)\rho-\frac{\dot{\phi}^2}{2}, \tag{10}$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{A}\dot{C}}{AC}+\frac{\dot{B}\dot{C}}{BC}-\frac{m^2}{A^2}-\frac{3}{4}\beta^2=\rho+\frac{\dot{\phi}^2}{2}, \tag{11}$$

$$m\left(\frac{\dot{B}}{B}-\frac{\dot{C}}{C}\right)=0 \tag{12}$$

where an overdot stands for derivative with respect to cosmic time t.

Equation (3) leads to

$$\ddot{\phi}+\dot{\phi}\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)=0 \tag{13}$$

By conservation equations for DE fluid and gauge function, we obtain

$$\dot{\rho}+\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)(1+\omega)\rho=0, \tag{14}$$

$$\beta\dot{\beta}+\beta^2\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)=0. \tag{15}$$

We now define some physical and kinematical parameters which are important in cosmological observations. The spatial volume V and the average scale factor a are defined as

$$V=a^3=ABC \tag{16}$$

The anisotropy parameter A_m of the expansion is characterized by the mean and directional Hubble's parameters defined as

$$A_m=\frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i-H}{H}\right)^2 \tag{17}$$

where H is the mean Hubble parameter and H_1, H_2, H_3 are directional Hubble parameters in the direction of x,y and z-axes respectively defined as

$$H_1=\frac{\dot{A}}{A}, H_2=\frac{\dot{B}}{B}, H_3=\frac{\dot{C}}{C}$$

and

$$H=\frac{1}{3}(H_1+H_2+H_3) \tag{18}$$

The expansion scalar θ and shear scalar σ^2aregiven by

$$\theta=3H, \tag{19}$$

$$\sigma^2=\frac{1}{2}\sum_{i=1}^{3}H_i^2-3H^2 \tag{20}$$

An important observational quantity in cosmology is the declaration parameter q defined by

$$q = -\frac{a\ddot{a}}{a^2}$$

The sign of q indicates whether the model inflates or not. The positive sign of q corresponds to decelerating models and the negative sign indicates inflation.

3. Solution and DE Model

In this section, we solve the field equations (8) –(15) which are highly non-linear differential equations and present the corresponding anisotropic DEmodels with zero-rest-mass scalar field in Lyra manifold.

Integration of equation (12) provides

$$B = C \tag{22}$$

by absorbing the arbitrary constant in B or C. Using (22), equations (8)-(15) reduced to

$$\frac{2\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{m^2}{A^2} + \frac{3}{4}\beta^2 = -\omega\rho - \frac{\dot{\phi}^2}{2}, \tag{23}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{C} + \frac{\dot{A}\dot{B}}{AB} - \frac{m^2}{A^2} + \frac{3}{4}\beta^2 = -(\omega + \delta)\rho - \frac{\dot{\phi}^2}{2}, \tag{24}$$

$$\frac{2\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} - \frac{m^2}{A^2} - \frac{3}{4}\beta^2 = \rho + \frac{\dot{\phi}^2}{2}, \tag{25}$$

$$\ddot{\phi} + \dot{\phi}\left(\frac{\dot{A}}{A} + \frac{2\dot{B}}{B}\right) = 0, \tag{26}$$

$$\dot{\beta} + \beta\left(\frac{\dot{A}}{A} + \frac{2\dot{B}}{B}\right) = 0, \tag{27}$$

$$\dot{\rho} + \left(\frac{\dot{A}}{A} + \frac{2\dot{B}}{B}\right)(1 + \omega)\rho = 0 \tag{28}$$

where we take $\delta = \gamma$because of the symmetry in field equation. This is a set of five equations in seven unknowns A, B, ρ, ω, δ, ϕ and β. [Eq. (28) is the consequences of the field equations]. Hence to obtain a determinate solution of the field equations we need two conditions which are either physically or mathematically viable.

Subtracting equation (23) from equation (24), we obtain

$$\frac{\ddot{A}}{A} - \frac{\ddot{B}}{C} + \frac{\dot{A}\dot{B}}{AB} - \frac{\dot{B}^2}{B^2} = \left(\frac{2m}{A^2} - \rho\delta\right) \tag{29}$$

Integrating equation (29), we get

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = \frac{X}{V}\, exp \int \left[\left(\frac{2m}{A^2} - \rho\delta\right)\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)^{-1}\right] dt \tag{30}$$

where X is the constant of integration.The integral term in (30)vanishes for

$$\delta = \frac{2m}{\rho A^2}. \tag{31}$$

Using equation (31) inequation (30),we obtain

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = \frac{X}{V}. \tag{32}$$

Differentiation of equation (16) provides

$$\frac{\dot{A}}{A} + \frac{2\dot{B}}{B} = \frac{\dot{V}}{V} \tag{33}$$

Solving equations(32) and (33), we obtain

$$\frac{\dot{A}}{A} = \frac{2X}{3V} + \frac{\dot{V}}{V}\,, \tag{34}$$

$$\frac{\dot{B}}{B} = -\frac{X}{3V} + \frac{\dot{V}}{3V}. \tag{35}$$

Equations (34) and (35) determine the scale functions A and B if the spatial volume V is explicitly known as a function of cosmic time t.

In order to obtain a model of the universe consistent with recent observations, we need a Hubble parameters H such that the model starts with a decelerating expansion followed by an accelerated expansion at late time. Following Ellis and Madsen [23], Singh [24] has chosen a functional form for H as

$$H = \frac{\dot{a}}{a} = k(1 - a^{-n}) \tag{36}$$

where k>0, n>1 are constants. From equation (36), we can easily obtain the deceleration parameter of the form

$$q = -1 + \frac{n}{1+a^n}\,. \tag{37}$$

Solving equations (21) and (37), we obtain

$$a = (e^{knt} - 1)^{k/n}. \tag{38}$$

Adhav et al. [25] utilized this form of average scale factor to study spatially homogeneous and anisotropic Bianchi type I, III, V, VI_0 and Kantowski-Sachs early decelerating and late times accelerating cosmological models of the universe.

On putting k=1 and n=3 in equation (38), we obtain

$$V = a^3 = e^{3t} - 1\,. \tag{39}$$

Substitution of equation (39) into equations (34) and (35) and integration of resulting equations provides

$$A = k_1 e^{\frac{-2Xt}{3}} (e^{3t} - 1)^{\frac{2X+3}{9}}, \tag{40}$$

$$B=k_2 e^{\frac{Xt}{3}}(e^{3t}-1)^{\frac{-X+3}{9}} \tag{41}$$

where $k_1$and $k_2$are arbitrary constants.Without loss of any generality, we can take $k_1 = k_2 = 1$.

The general solution of equation (26) can be written as

$$\emptyset = (1-e^{-3t})^{\frac{\emptyset_0}{3}} \tag{42}$$

where $\emptyset_0$is an arbitrary constant of integration.Equation (27) has the solution

$$\beta = \frac{\beta_0}{e^{3t}-1} \tag{43}$$

β_0 being integration constant.

Hence, the metric of our solution can be written in the form

$$ds^2 = dt^2 - e^{\frac{-4X}{3}}(e^{3t}-1)^{\frac{4X+6}{9}}dx^2 - e^{2\left(\frac{Xt}{3}-mx\right)}(e^{3t}-1)^{\frac{-2X+6}{9}}dy^2$$
$$-e^{2\left(\frac{Xt}{3}+mx\right)}(e^{3t}-1)^{\frac{-2X+6}{9}}dz^2. \tag{44}$$

For the cosmological model (44), the mean Hubble parameter H is obtained as

$$H = e^{3t}(e^{3t}-1)^{-1} \quad . \tag{45}$$

The expansion scalar θ, shear scalar σand anisotropy parameter A_m have the values obtained as

$$\theta = \frac{3e^{3t}}{e^{3t}-1}, \tag{46}$$

$$\sigma = \sqrt{\left(\frac{2}{3}\right)}\left(\frac{X}{e^{3t}-1}\right), \tag{47}$$

$$A_m = \frac{2X^2}{9e^{6t}}. \tag{48}$$

The deceleration parameter q has the value

$$q = -1 + 3e^{-3t} \tag{49}$$

From equation (18),we obtained the energy density of DE as

$$\rho = \frac{27e^{6t}-X^2}{9(e^{3t}-1)^2} - \frac{m^2 e^{\frac{4Xt}{3}}}{9(e^{3t}-1)^{\frac{4X+6}{9}}} - \frac{3}{4}\frac{(\beta_0^2+2\emptyset_0^2)}{(e^{3t}-1)^2}. \tag{50}$$

Using equation (50) in (23),we find the EoS parameter of DE as

$$\omega = -\left[\frac{27e^{6t}+(48x-54)e^{3t}+3x^2}{9(e^{3t}-1)^2} - \frac{m^2 e^{\frac{4xt}{3}}}{9(e^{3t}-1)^{\frac{4x+6}{9}}} + \frac{3}{4}\frac{(\beta_0^2+2\emptyset_0^2)}{(e^{3t}-1)^2}\right] \Big/ \left[\frac{27e^{6t}-X^2}{9(e^{3t}-1)^2} - \frac{m^2 e^{\frac{4Xt}{3}}}{9(e^{3t}-1)^{\frac{4X+6}{9}}} - \frac{3}{4}\frac{(\beta_0^2+2\emptyset_0^2)}{(e^{3t}-1)^2}\right] \tag{51}$$

From equation (31), we obtain skewness parameter as

$$\delta = -2m^2 \left[\frac{e^{\frac{4Xt}{3}}}{(e^{3t}-1)^{\frac{4X+6}{9}}} \right] \Big/ \left[\frac{27e^{6t}-X^2}{9(e^{3t}-1)^2} - \frac{m^2 e^{\frac{4Xt}{3}}}{9(e^{3t}-1)^{\frac{4X+6}{9}}} + \frac{3}{4}\frac{(\beta_0^2+2\phi_0^2)}{(e^{3t}-1)^2} \right] \tag{52}$$

4. Discussion

We now use the physical and kinematical parameters to discuss the physical features of the derived model of the universe.

- We observe that the spatial volume of the model (44) is zero at t=0 and increases continuously and tends to infinity as $t \to \infty$. The physical parameters H, θ, and σ^2 are all infinite at t=0 and tend to zero as t approaches infinity. Therefore, the model has a big-bang type singularity at t=0.

- From equation (48), we see that the anisotropy parameter of the expansion $A_m$is constant at t=0 which decreases continuously as t increases and tends to zero for large time.Thus, this anisotropic universe isotropizes for large time. From equations(46) and (47), we also have

$$\lim_{t\to\infty} \frac{\sigma}{\theta} = 0 \tag{53}$$

  which shows that the shear scalar tends to zero faster than the expansion scalar.

- The deceleration parameter q plays an important role in the description of the nature of the cosmological models. From Eq. (49), we observe that the deceleration parameter q=2 is positive for $0 < t < \frac{1}{3}\log 3$and is negative for $t > \frac{1}{3}\log 3$. This means that the universe starts evolving from t=0 with decelerated expansion and the expansion changes from deceleration to acceleration at time t and q has a signature flip at the epoch $t = \frac{1}{3}log3$. The signature flip in q is essential for the conclusion that the universe is accelerating. For large time, *q=-1* which indicates that the universe is exponentially expanding due to the dominance of DE.

- The jerk parameter j(t) in the universe is given by

$$j(t) = q + 2q^2 - \frac{\dot{q}}{H} = 1 + 9e^{-6t} \tag{54}$$

  which is positive throughout the evolution of the universe. The parameter j(t) attains a constant value 1 at late time. Hence the expansion in the model from decelerating phase to accelerating phase is smooth one.

- From equation (43), it is clear that the gauge function βis infinite at the initial singularity and is a decreasing function of time which ultimately tend to zero as $t \to \infty$. Thus, β plays the roleas cosmological constant, preserving the same character as Λ term. Hence the concept of Lyra manifold does not remain for a very long time. Similar conclusion has also been drawn by Rahaman[11], Mohanty et al.[26], Nagpure[27].

- From equation (50), we observe that the energy density of DE is infinite at t=0 which decreases as time increases and ultimately tend to zero for large time. We also observe from equation (52) that the skewness parameter δis positive in the beginning of the evolution of the which tends to zero as $t \to \infty$. The behavior of skewness parameter shows that the dark energy always remains anisotropic throughout the evolution of the universe and ultimately vanishes for very large time.
- From equation (42), we may observe that the scalar ϕin the model decreases as V increases and ultimately tends to a constant at late time of evolution of the universe. The scalar field ϕ contributes significantly to the dark energy density.

5. Concluding Remarks

In this paper we have studied the dynamics of Bianchi type-VI_0 cosmological model in the presence an anisotropic dark energy within the framework of Lyra manifold. In order to obtain a determinate solutions of the field equations we have utilised two physically valid conditions.We have computed all the cosmological parameters and discussed their physical significance in the light of present cosmological scenario and observations. We have observed that the dark energy in our model is anisotropic throughout the evolution of the Universe. It helps to study the anisotropic at small angular scales which play a key role in the formation of largescale structures of the universe. We believe that our model puts ample light on the understanding of the physical universe.

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