Article

The Transversal Intersection of Special Surfaces of Mannheim Curve Pairs

Savaş Karaahmetoğlu[1], Ceremnur Tetik[2] & Abdullah Dertli[*2]

[1]Kumru Şehit Sabri Eryeler Vocational and Tech. Anatolian High School, Ordu, Turkey
[2]Math. Dept., Faculty of Arts & Sci., Ondokuz Mayıs Univ., Samsun, Turkey

Abstract

In this paper, we study the local properties of the intersection curve of the tangent, rectifying devlopable and Darboux developable surfaces of a Mannheim Curve Pair. We derive the curvature vector and curvature for the transversal intersection for intersection problem. Furthermore, we investigate some characteristic features of the intersection curve for all three cases and give some important results.

Keywords: Mannheim curve pair, transversal intersection, Darboux developable, rectifying developable.

1. Introduction

A ruled Surface is generated by a moving straight line continously in Euclidean space E^3, [1]. Ruled surfaces are one of the simplest objects in geometric modeling. One important fact about ruled surfaces is that they can be generated by straight lines. A practical application of this type surfaces is that they are used in civil engineering and physics,[2]. Izumiya and Takeuichi introduced some new special ruled surfaces such as Darboux developable and Rectifying developable surfaces and investigate their properties, [3].

The curves are a fundamental structure of differential geometry. An increasing interest of the theory of curves makes a development of special curves to be examined. Especially, Bertrand curves are well-studied classical curves,[4]. Another special curves are Mannheim curves. In recent works, Liu and Wang are curious about the Mannheim curves in both Euclidean and Minkowski 3-space and they obtained the necessary and sufficient conditions between the curvature and the torsion for a curve to be the Mannheim partner curves[5],[6]. Kasap and Orbay studied the Mannheim partner curves in Euclidean space and obtain the relationships between the curvatures and the torsions of the Mannheim partner curves with respect to each other, [7].

In this paper, we study the intersection problem for the tangent, rectifying devlopable and Darboux developable surfaces of (α, α^*) Mannheim curve pair. We investigate the charactherizations of the intersection curve for each case of surface-surface intersection. First, we express the curvature vector and the curvature of the intersection curve in terms of normal

[*]Correspondence: Abdullah Dertli, Math. Dept., Faculty of Arts & Sci., Ondokuz Mayıs University, Samsun, Turkey.
Email: abdullah.dertli@gmail.com

curvatures of both (α,α^*) Mannheim curve pair. Then for each type of ruled surfaces, we investigate the properties of the intersection curve.

2. Preliminaries

In this study, the 3-dimensional Euclidean Space E^3 is the pair ($\mathbb{R}^3$, < , >), is a three-dimensional real vector space equipped with an inner product,

$$\langle x,y\rangle = x_1y_1 + x_2y_2 + x_3y_3 \tag{1}$$

where $x=(x_1,x_2,x_3)$ and $y=(y_1,y_2,y_3)$.

For a vector $x=(x_1,x_2,x_3)\in\mathbb{R}^3$. norm of x is defined by

$$\|x\| = \langle x,x\rangle^{\frac{1}{2}} = \sqrt{x_1^2+x_2^2+x_3^2} \tag{2}$$

A vector $x=(x_1,x_2,x_3)\in\mathbb{R}^3$, which satisfies $\langle x,x\rangle=1$ is called a unit vector. Any basis $\{v_1,v_2,v_3\}$ on $\mathbb{R}^3$ is known as an orthogonal basis if the vectors are mutually orthogonal vectors.We also define the vector product of x and y (in that order)

$$x\times y = \begin{vmatrix} e_1 & e_2 & e_3 \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix} \tag{3}$$

where $\{e_1,e_2,e_3\}$ is the canonical basis of $\mathbb{R}^3$, $x=(x_1,x_2,x_3)$ and $y=(y_1,y_2,y_3)$.

A parametrized differentiable curve is a differentiable map $\alpha : I \to \mathbb{R}^3$ of an open interval $I=(a,b)$ of the real line $\mathbb{R}$ into $\mathbb{R}^3$. Let $\alpha(s)$ be a parametric curve in 3-dimensional Euclidean space. s is called the arc length (regular and $\|\alpha'(s)\|=1$) and $\alpha(s)$ has second derivatives. We assume that $\alpha''(s)\neq 0$, because otherwise the curve is a straight line segment or the principal normal is undefined at some point on the curve. Since $\alpha(s)$ is a regular curve with $\alpha''(s)\neq 0$ the Frenet frame $\{T(s),N(s),B(s)\}$ along $\alpha(s)$ is defined, where $T(s)=\alpha'(s)$, $N(s)=\dfrac{\alpha''(s)}{\|\alpha''(s)\|}$ and $B(s)=T(s)\times N(s)$ are the unit tangent, principal normal, and binormal vectors of the curve at the point $\alpha(s)$, respectively. Then we have the FrenetSerret formulae

$$T' = \kappa N, \quad N' = -\kappa T + \tau B, \quad B' = -\tau N \tag{4}$$

where $\kappa(s)$ and $\tau(s)$ are curvature and torsion of the curve at the point $\alpha(s)$, respectively, [4]. If there exists a corresponding relationship between the space curves α and α^* such that the principal normal lines of α coincides with the binormal lines of α^* at the corresponding points of the curves, then α is called as a Mannheim curve and α^* is called as a Mannheim partner curve of α. The pair of (α,α^*) is said to be a Mannheim pair, [6]. There exists a relationship between the position vectors as

$$\begin{aligned}\alpha(s)&=\alpha^*(s^*)+\lambda B^*(s^*)\\ \alpha^*(s^*)&=\alpha(s)-\lambda N(s)\end{aligned}\tag{5}$$

and we can write $N=B^*$ and λ is the distance between the curves $\alpha(s)$ and $\alpha^*(s^*)$ at the corresponding points. Let (α,α^*) be a Mannheim pair, $\{T(s),N(s),B(s)\}$ and $\{T^*(s^*),N^*(s^*),B^*(s^*)\}$ be their Frenet frames, respectively, we can write the following relationship between these frames:

$$\begin{aligned}T&=\cos\theta T^*+\sin\theta N^*\\ B&=-\sin\theta T^*+\cos\theta N^*\end{aligned}\tag{6}$$

and

$$\begin{aligned}T^*&=\cos\theta T-\sin\theta B\\ N^*&=\sin\theta T+\cos\theta B\end{aligned}\tag{7}$$

where θ is the angle between the tangent vectors T and T^*.

Let X be a regular surface and $\alpha: I\subset\mathbb{R}\to X$ be a unit speed curve on the surface. Then, Darboux frame $\{T,N,U=N\times T\}$ is well-defined along the curve α where T is the tangent of α and N is the unit normal of X. Darboux equations for this frame are given by

$$\begin{aligned}T'&=\kappa_g U+\kappa_n N\\ N'&=-\kappa_n T-\tau_g U\\ U'&=-\kappa_g T+\tau_g N\end{aligned}\tag{8}$$

where κ_n is the normal curvature, κ_g is the geodesic curvature and τ_g is the geodesic torsion of α. Let $\alpha(s)$ be curve on the surface X. The curve $\alpha(s)$ is an asymptotic curve, geodesic curve or line of curvature on the surface X if and only if the normal curvature $\kappa_n=0$, the geodesic curvature $\kappa_g=0$ or the geodesic torsion $\tau_g=0$, respectively. For any unit speed curve $\alpha: I\subset\mathbb{R}\to\mathbb{R}^3$, we define a vector field $\tilde{D}(s)=\left(\tau/\kappa\right)T(s)+B(s)$ along α under condition $\kappa(s)\neq 0$ and we call it the modified Darboux vector field of α. Let $\alpha(s)$ be a unit speed curve with $\kappa(s)\neq 0$, the surface $X(s,u)=\alpha(s)+u\tilde{D}(s)$ is called the rectifying developable surface of α. And also the surface $X(s,u)=T(s)+uB(s)$ is called the Darboux developable surface of α,[3].

Let $X^A(s,u)$ and $X^B(s^*,u)$ be two parametric surfaces and $c=c(s)$ the transversal intersection curve of both surfaces $X^A(s,u)$ and $X^B(s^*,u)$. This means that the tangent vector of the transversal intersection curve lies on the tangent planes of both surfaces. Therefore, it can be obtained as the cross product of the unit surface normal vectors of the surfaces at $p=c(s)$

$$t=\frac{N^A\times N^B}{\left\|N^A\times N^B\right\|} \tag{9}$$

where N^A is the unit normal vector to the surface X^A and N^B is the unit normal vector to the surface X^B.

3. Results

3.1. Transversal Intersection Curve of Tangent Surfaces of Mannheim Curve Pairs

In this section, we compute the curvature of the transversal intersection curve of tangent surfaces of Mannheim curve pairs. Let α and α^* are Mannheim curve pair, $X^A(s,u)$ and $X^B(s^*,u)$ are their tangent surfaces, respectively. Let N^A be the unit surface normal of the tangent surface $X^A(s,u)$ and N^B be the unit surface normal of the tangent surface $X^B(s^*,u)$, we can compute N^A and N^B as;

$$N^A=\frac{X_s^A\times X_u^A}{\left\|X_s^A\times X_u^A\right\|}=\pm B \tag{10}$$

and

$$N^B=\frac{X_{s^*}^B\times X_u^B}{\left\|X_{s^*}^B\times X_u^B\right\|}=\pm B^* \tag{11}$$

where B is the binormal frenet vector of α and B^* is the binormal frenet vector of α^*. Let $c=c(s)$ be the transversal intersection curve of both tangent surfaces of X^A and X^B. This means that the tangent vector of the transversal intersection curve $c=c(s)$ lies on the tangent planes of both surfaces. Therefore, it can be obtained as the cross product of the unit surface normal vectors of the surfaces at $p=c(s)$

$$t=\frac{N^A\times N^B}{\left\|N^A\times N^B\right\|}=\pm\left\{\cos\theta T^*+\sin\theta N^*\right\}=\pm T \tag{12}$$

where N^A be the unit surface normal of the tangent surface $X^A(s,u)$ and N^B be the unit surface normal of the tangent surface $X^B(s^*,u)$.

Result 1. Let α and α^* are Mannheim curve pair, $X^A(s,u)$ and $X^B(s^*,u)$ are their tangent surfaces, respectively then, the curve $c=c(s)$ is parallel to the curve α.

Let investigate the angle between surfaces $X^A(s,u)$ and $X^B(s^*,u)$. The angle between the surfaces $X^A(s,u)$ and $X^B(s^*,u)$ is the angle between the unit surface normal vectors N^A and N^B. If η denote the angle between N^A and N^B, then we can write

$$\cos\eta = \langle N^A, N^B\rangle = \langle \pm B, \pm B^*\rangle = \langle B, N\rangle = 0 \tag{13}$$

Result 2. Let α and α^* are Mannheim curve pair, $X^A(s,u)$ and $X^B(s^*,u)$ are their tangent surfaces, respectively. The angle between the surfaces $X^A(s,u)$ and $X^B(s^*,u)$ is equal to the angle $\frac{\theta}{2}+2k\pi$, $k \in \mathbb{Z}$, that is the surfaces $X^A(s,u)$ and $X^B(s^*,u)$ intersect orthogonally.

Since, the curvature vector c'' of the transversal intersection curve at p is perpendicular to t, it must lie in the normal plane spanned by N^A and N^B. Hence, we can write it as

$$c'' = \lambda_1 N^A + \lambda_2 N^B \tag{14}$$

where λ_1 and λ_2 are the coefficients that we need to compute. Due to the inner product of the curvature vector c'' with the unit surface normals N^A and N^B, we obtain the following linear equations system;

$$\begin{aligned} \kappa_n^A &= \langle c'', N^A\rangle = \lambda_1 \\ \kappa_n^B &= \langle c'', N^B\rangle = \lambda_2 \end{aligned} \tag{15}$$

when this linear system is solved and put the coefficients in the Eq (14), we can express the curvature vector of the intersection curve as

$$c'' = \kappa_n^A N^A + \kappa_n^B N^B \tag{16}$$

Theorem 1. Let α and α^* are Mannheim curve pair, $X^A(s,u)$ and $X^B(s^*,u)$ are their tangent surfaces, respectively. Then, the curvature κ of the curve $c=c(s)$ is given by

$$\kappa = \left\{\left(\kappa_n^A\right)^2 + \left(\kappa_n^B\right)^2\right\}^{\frac{1}{2}}. \tag{17}$$

Lemma 1. Let α and α^* are Mannheim curve pair, $X^A(s,u)$ and $X^B(s^*,u)$ are their tangent surfaces, respectively. If κ_g^A and κ_g^B are the geodesic curvatures of $X^A(s,u)$ and $X^B(s^*,u)$ respectively, then we have

$$\begin{aligned} \kappa_g^A &= \kappa_n^B \\ \kappa_g^B &= \kappa_n^A . \end{aligned} \tag{18}$$

Theorem 2. Let α and α^* are Mannheim curve pair, $X^A(s,u)$ and $X^B(s^*,u)$ are their tangent surfaces, respectively. Then $c=c(s)$ is a geodesic curve of the surface $X^A(s,u)$ if and only if $c=c(s)$ is a asymptotic curve of the surface $X^B(s^*,u)$.

Theorem 3. Let α and α^* are Mannheim curve pair, $X^A(s,u)$ and $X^B(s^*,u)$ are their tangent surfaces, respectively. Then $c=c(s)$ is a geodesic curve of the surface $X^B(s^*,u)$ if and only if $c=c(s)$ is a asymptotic curve of the surface $X^A(s,u)$.

Theorem 4. Let α and α^* are Mannheim curve pair, $X^A(s,u)$ and $X^B(s^*,u)$ are their tangent surfaces, respectively. Then we have

$$\tau_g^A = -\tau_g^B. \tag{19}$$

where τ_g^A and τ_g^B are the geodesic torsions of the surfaces $X^A(s,u)$ and $X^B(s^*,u)$, respectively.

Result 3. Let α and α^* are Mannheim curve pair, $X^A(s,u)$ and $X^B(s^*,u)$ are their tangent surfaces, respectively. $c(s)$ is a line of curvature of the surface $X^A(s,u)$ if and only if $c(s)$ is a line of curvature of the surface $X^B(s^*,u)$.

3.2. Transversal Intersection Curve of Rectifying Developable Surfaces of Mannheim Curve Pairs

In this section, we compute the curvature of the transversal intersection curve of rectifying developable surfaces of Mannheim curve pairs. Let α and α^* are Mannheim curve pair, $X^A(s,u)$ and $X^B(s^*,u)$ are their rectifying developable surfaces, respectively. Let N^A be the unit surface normal of the rectifying developable surface $X^A(s,u)$ and N^B be the unit surface normal of the rectifying developable surface $X^B(s^*,u)$, we can compute N^A and N^B as;

$$N^A = \frac{X_s^A \times X_u^A}{\left\| X_s^A \times X_u^A \right\|} = \pm N \tag{20}$$

and

$$N^B = \frac{X_{s^*}^B \times X_u^B}{\left\| X_{s^*}^B \times X_u^B \right\|} = \pm N^* \tag{21}$$

where N is the binormal frenet vector of α and N^* is the binormal frenet vector of α^*. Let $c=c(s)$ be the transversal intersection curve of both rectifying developable surfaces of X^A and X^B. This means that the tangent vector of the transversal intersection curve $c=c(s)$ lies on the tangent planes of both surfaces. Therefore, it can be obtained as the cross product of the unit surface normal vectors of the surfaces at $p=c(s)$

$$t = \frac{N^A \times N^B}{\left\| N^A \times N^B \right\|} = \pm\left\{\cos\theta T - \sin\theta B\right\} = \pm T^* \tag{22}$$

where N^A be the unit surface normal of the rectifying developable surface $X^A(s,u)$ and N^B be the unit surface normal of the rectifying developable surface $X^B(s^*,u)$.

Result 4. Let α and α^* are Mannheim curve pair, $X^A(s,u)$ and $X^B(s^*,u)$ are their rectifying developable surfaces, respectively then, the curve $c=c(s)$ is parallel to the curve α^*.

Let investigate the angle between surfaces $X^A(s,u)$ and $X^B(s^*,u)$. The angle between the surfaces $X^A(s,u)$ and $X^B(s^*,u)$ is the angle between the unit surface normal vectors N^A and N^B. If η denote the angle between N^A and N^B, then we can write

$$\cos\eta = \langle N^A, N^B \rangle = \langle \pm N, \pm N^* \rangle = \langle B^*, N^* \rangle = 0 \tag{23}$$

Result 5. Let α and α^* are Mannheim curve pair, $X^A(s,u)$ and $X^B(s^*,u)$ are their rectifying developable surfaces, respectively. The angle between the surfaces $X^A(s,u)$ and $X^B(s^*,u)$ is equal to the angle $\frac{\theta}{2}+2k\pi$, $k \in \mathbb{Z}$, that is the surfaces $X^A(s,u)$ and $X^B(s^*,u)$ intersect orthogonally.

Since, the curvature vector c'' of the transversal intersection curve at p is perpendicular to t, it must lie in the normal plane spanned by N^A and N^B. Hence, we can write it as

$$c'' = \lambda_1 N^A + \lambda_2 N^B \tag{24}$$

where λ_1 and λ_2 are the coefficients that we need to compute. Due to the inner product of the curvature vector c'' with the unit surface normals N^A and N^B, we obtain the following linear equations system;

$$\begin{aligned} \kappa_n^A &= \langle c'', N^A \rangle = \lambda_1 \\ \kappa_n^B &= \langle c'', N^B \rangle = \lambda_2 \end{aligned} \tag{25}$$

when this linear system is solved and put the coefficients in the Eq (24), we can express the curvature vector of the intersection curve as

$$c'' = \kappa_n^A N^A + \kappa_n^B N^B \tag{26}$$

Theorem 5. Let α and α^* are Mannheim curve pair, $X^A(s,u)$ and $X^B(s^*,u)$ are their rectifying developable surfaces, respectively. Then, the curvature κ of the curve $c=c(s)$ is given by

$$\kappa = \left\{ \left(\kappa_n^A\right)^2 + \left(\kappa_n^B\right)^2 \right\}^{\frac{1}{2}}. \tag{27}$$

Lemma 2. Let α and α^* are Mannheim curve pair, $X^A(s,u)$ and $X^B(s^*,u)$ are their rectifying developable surfaces, respectively. If κ_g^A and κ_g^B are the geodesic curvatures of $X^A(s,u)$ and $X^B(s^*,u)$ respectively, then we have

$$\begin{aligned}\kappa_g^A &= \kappa_n^B \\ \kappa_g^B &= \kappa_n^A .\end{aligned} \tag{28}$$

Theorem 6. Let α and α^* are Mannheim curve pair, $X^A(s,u)$ and $X^B(s^*,u)$ are their rectifying developable surfaces, respectively. Then $c=c(s)$ is a geodesic curve of the surface $X^A(s,u)$ if and only if $c=c(s)$ is a asymptotic curve of the surface $X^B(s^*,u)$.

Theorem 7. Let α and α^* are Mannheim curve pair, $X^A(s,u)$ and $X^B(s^*,u)$ are their rectifying developable surfaces, respectively. Then $c=c(s)$ is a geodesic curve of the surface $X^B(s^*,u)$ if and only if $c=c(s)$ is a asymptotic curve of the surface $X^A(s,u)$.

Theorem 8. Let α and α^* are Mannheim curve pair, $X^A(s,u)$ and $X^B(s^*,u)$ are their rectifying developable surfaces, respectively. Then we have

$$\tau_g^A = -\tau_g^B . \tag{29}$$

where τ_g^A and τ_g^B are the geodesic torsions of the surfaces $X^A(s,u)$ and $X^B(s^*,u)$, respectively.

Result 6. Let α and α^* are Mannheim curve pair, $X^A(s,u)$ and $X^B(s^*,u)$ are their rectifying developable surfaces, respectively. $c(s)$ is a line of curvature of the surface $X^A(s,u)$ if and only if $c(s)$ is a line of curvature of the surface $X^B(s^*,u)$.

3.3. **Transversal Intersection Curve of Darboux Developable Surfaces of Mannheim Curve Pairs**

In this section, we compute the curvature of the transversal intersection curve of Darboux developable surfaces of Mannheim curve pairs. Let α and α^* are Mannheim curve pair, $X^A(s,u)$ and $X^B(s^*,u)$ are their Darboux developable surfaces, respectively. Let N^A be the unit surface normal of the Darboux developable surface $X^A(s,u)$ and N^B be the unit surface normal of the Darboux developable surface $X^B(s^*,u)$, we can compute N^A and N^B as;

$$N^A = \frac{X_s^A \times X_u^A}{\left\| X_s^A \times X_u^A \right\|} = \pm B \tag{30}$$

and

$$N^B = \frac{X_{s^*}^B \times X_u^B}{\left\| X_{s^*}^B \times X_u^B \right\|} = \pm B^* \tag{31}$$

where N is the binormal frenet vector of α and N^* is the binormal frenet vector of α^*. Let $c=c(s)$ be the transversal intersection curve of both Darboux developable surfaces of X^A and

X^B. This means that the tangent vector of the transversal intersection curve $c=c(s)$ lies on the tangent planes of both surfaces. Therefore, it can be obtained as the cross product of the unit surface normal vectors of the surfaces at $p=c(s)$

$$t=\frac{N^A\times N^B}{\left\|N^A\times N^B\right\|}=\pm\left\{\cos\theta T^*+\sin\theta N^*\right\}=\pm T \tag{32}$$

where N^A be the unit surface normal of the Darboux developable surface $X^A(s,u)$ and N^B be the unit surface normal of the Darboux developable surface $X^B(s^*,u)$.

Result 7. Let α and α^* are Mannheim curve pair, $X^A(s,u)$ and $X^B(s^*,u)$ are their Darboux developable surfaces, respectively then, the curve $c=c(s)$ is parallel to the curve α.

Let investigate the angle between surfaces $X^A(s,u)$ and $X^B(s^*,u)$. The angle between the surfaces $X^A(s,u)$ and $X^B(s^*,u)$ is the angle between the unit surface normal vectors N^A and N^B. If η denote the angle between N^A and N^B, then we can write

$$\cos\eta=\left\langle N^A,N^B\right\rangle=\left\langle \pm B,\pm B^*\right\rangle=\left\langle B,N\right\rangle=0 \tag{33}$$

Result 8. Let α and α^* are Mannheim curve pair, $X^A(s,u)$ and $X^B(s^*,u)$ are their Darboux developable surfaces, respectively. The angle between the surfaces $X^A(s,u)$ and $X^B(s^*,u)$ is equal to the angle $\frac{\theta}{2}+2k\pi$, $k\in\mathbb{Z}$, that is the surfaces $X^A(s,u)$ and $X^B(s^*,u)$ intersect orthogonally.

Since, the curvature vector c'' of the transversal intersection curve at p is perpendicular to t, it must lie in the normal plane spanned by N^A and N^B. Hence, we can write it as

$$c''=\lambda_1 N^A+\lambda_2 N^B \tag{34}$$

where λ_1 and λ_2 are the coefficients that we need to compute. Due to the inner product of the curvature vector c'' with the unit surface normals N^A and N^B, we obtain the following linear equations system;

$$\begin{aligned}\kappa_n^A&=\left\langle c'',N^A\right\rangle=\lambda_1\\ \kappa_n^B&=\left\langle c'',N^B\right\rangle=\lambda_2\end{aligned} \tag{35}$$

when this linear system is solved and put the coefficients in the Eq (34), we can express the curvature vector of the intersection curve as

$$c''=\kappa_n^A N^A+\kappa_n^B N^B \tag{36}$$

Theorem 9. Let α and α^* are Mannheim curve pair, $X^A(s,u)$ and $X^B(s^*,u)$ are their Darboux developable surfaces, respectively. Then, the curvature κ of the curve $c=c(s)$ is given by

$$\kappa=\left\{\left(\kappa_n^A\right)^2+\left(\kappa_n^B\right)^2\right\}^{\frac{1}{2}}. \tag{37}$$

Lemma 3. Let α and α^* are Mannheim curve pair, $X^A(s,u)$ and $X^B(s^*,u)$ are their Darboux developable surfaces, respectively. If κ_g^A and κ_g^B are the geodesic curvatures of $X^A(s,u)$ and $X^B(s^*,u)$ respectively, then we have

$$\begin{aligned}\kappa_g^A&=\kappa_n^B\\ \kappa_g^B&=\kappa_n^A.\end{aligned} \tag{38}$$

Theorem 10. Let α and α^* are Mannheim curve pair, $X^A(s,u)$ and $X^B(s^*,u)$ are their Darboux developable surfaces, respectively. Then $c=c(s)$ is a geodesic curve of the surface $X^A(s,u)$ if and only if $c=c(s)$ is a asymptotic curve of the surface $X^B(s^*,u)$.

Theorem 11. Let α and α^* are Mannheim curve pair, $X^A(s,u)$ and $X^B(s^*,u)$ are their Darboux developable surfaces, respectively. Then $c=c(s)$ is a geodesic curve of the surface $X^B(s^*,u)$ if and only if $c=c(s)$ is a asymptotic curve of the surface $X^A(s,u)$.

Theorem 12. Let α and α^* are Mannheim curve pair, $X^A(s,u)$ and $X^B(s^*,u)$ are their Darboux developable surfaces, respectively. Then we have

$$\tau_g^A=-\tau_g^B. \tag{39}$$

where τ_g^A and τ_g^B are the geodesic torsions of the surfaces $X^A(s,u)$ and $X^B(s^*,u)$, respectively.

Result 9. Let α and α^* are Mannheim curve pair, $X^A(s,u)$ and $X^B(s^*,u)$ are their Darboux developable surfaces, respectively. $c(s)$ is a line of curvature of the surface $X^A(s,u)$ if and only if $c(s)$ is a line of curvature of the surface $X^B(s^*,u)$.

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