Article

Cosmic Acceleration of Universe in Teleparallel Gravity

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Abstract

The paper is devoted to dynamical investigations of LRS Bianchi type-I cosmological model filled with wet dark fluid in the framework of $f(T)$ gravity. The equation of state modeled on $p=\omega(\rho-\rho^*)$ in the form of wet dark fluid for the dark energy component of the universe. Solutions to the corresponding field equations are obtained for exponential law of expansion. The geometrical and physical parameters of the model are studied.

Keywords: *f*(*T*) gravity, wet dark fluid, Bianchi Type I, dark energy, expansion, universe.

1. Introduction

It is well known that the recent observational studies [1-5] have well established the accelerated expansion of the current universe. The universe consists of 76 % dark energy and 20 % dark matter. Wet dark fluid is a new candidate for dark energy in the script of generalized chaplygin gas, where a physically motivated equation of state is offered with the properties relevant for a dark energy problem. The equation of state for a wet dark fluid is

$$\frac{p_{WDF}}{\omega}+\rho^*=\rho_{WDF}. \tag{1}$$

Equation (1) is the good approximation for many fluids, including water. The parameter ω and ρ^* are taken to be positive and we restrict ourselves to $0\le\omega\le1$. Note that if c_s denotes the adiabatic sound speed in wet dark fluid, then $c_s^2=\partial p/\partial\rho\ge0$. To find the wet dark fluid energy density, we use the energy conservation equation

$$\rho^*_{WDF}+3H\left(p_{WDF}+\rho_{WDF}\right)=0. \tag{2}$$

From equation of state (1) and using $3H=\frac{\dot{V}}{V}$ in equation (2), we get

$$p_{WDF}=\left(\frac{\omega}{1+\omega}\right)\rho^*+\frac{c}{V^{(1+\omega)}} \quad , \tag{3}$$

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where c is the constant of integration and V is the volume expansion. wet dark fluid naturally includes these components, a piece that behaves as a cosmological constant as well as a standard fluid with an equation of state $p = \omega\rho$. We can show that if we take $c > 0$, this fluid will not violate the strong energy condition $p + \rho \geq 0$. Thus, we get

$$p_{WDF} + \rho_{WDF} = (1+\omega)\rho_{WDF} - \omega\rho^{*} = (1+\omega)\left(\frac{c}{V^{(1+\omega)}}\right) \geq 0. \quad (4)$$

The wet dark fluid has been used as dark energy in the homogeneous, isotropic FRW model by Holman and Naidu [6].The Bianchi type-I universe filled with dark energy from a wet dark fluid has been investigated by Singh and Chaubey [7]. The Bianchi type-V universe filled with dark energy from a wet dark fluid has been studied by Chaubey [8]. Many Relativists [9-21] studied cosmological models with WDF in General Relativity and theories of gravitations.

2. Teleparallel gravity formalism

We define the action by generalizing the TG i.e. $f(T)$ theory as

$$S = \int [T + f(T) + L_{matter}] e\, d^4x \quad . \quad (5)$$

Here, $f(T)$ denotes an algebraic function of the torsion scalar T. Making the functional variation of the action (5) with respect to the tetrads, we get the following equations of motion

$$S_{\mu}^{\nu\rho}\partial_{\rho}T f_{TT} + \left[e^{-1}e^{i}_{\mu}\partial_{\rho}\left(e e^{\alpha}_{i} S_{\alpha}^{\nu\rho}\right) + T^{\alpha}{}_{\lambda\mu} S_{\alpha}^{\nu\lambda}\right](1+f_T) + \frac{1}{4}\delta^{\nu}_{\mu}(T+f) = 4\pi(T^{\nu}_{\mu} + \overline{T}^{\nu}_{\mu}), \quad (6)$$

The field equation (6) is written in terms of the tetrad and partial derivatives and appears very different from Einstein's equation, where T^{ν}_{μ} is the energy momentum tensor, $f_T = df(T)/dT$ and by setting $f(T) = a_0 =$constant this is dynamically equivalent to the GR.

3. Field equations and some physical quantities

We consider spatially homogeneous and anisotropic Bianchi type-I (LRS) space-time of the form

$$ds^2 = dt^2 - A^2(t)dx^2 - B^2(t)\left[dy^2 + dz^2\right], \quad (7)$$

where A and B be the metric potential which is the functions of cosmic time t only.The corresponding Torsion scalar is given by

$$T = -2\left(2\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{B}^2}{B^2}\right). \quad (8)$$

Let us consider that the matter content is dark energy in the form of wet dark fluid such that the energy momentum tensor T^{ν}_{μ} given by $T^{\nu}_{\mu} = (\rho_{wdf} + p_{wdf})u^{\nu}u_{\mu} - p_{wdf}\, g^{\nu}_{\mu}$, together with commoving coordinates $u^{\nu}=(0,0,0,1)$ and $u^{\nu}u_{\nu}=1$, where u^{ν} is the four-velocity vector of the fluid, p and ρ be the pressure and energy density of the fluid respectively. From the equation of motion (6), Bianchi type-I space-time (7) for the fluid of stress energy tensor can be written as

$$(T+f)+4(1+f_T)\left\{\frac{\ddot{B}}{B}+\frac{\dot{B}^2}{B^2}+\frac{\dot{A}}{A}\frac{\dot{B}}{B}\right\}+4\frac{\dot{B}}{B}\dot{T}\ f_{TT}=k^2(-p_{wdf}), \tag{9}$$

$$(T+f)+2(1+f_T)\left\{\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{B}^2}{B^2}+3\frac{\dot{A}}{A}\frac{\dot{B}}{B}\right\}+2\left\{\frac{\dot{A}}{A}+\frac{\dot{B}}{B}\right\}\dot{T}\ f_{TT}=k^2(-p_{wdf}), \tag{10}$$

$$(T+f)+4(1+f_T)\left\{\frac{\dot{B}^2}{B^2}+2\frac{\dot{A}}{A}\frac{\dot{B}}{B}\right\}=k^2(\rho_{wdf}). \tag{11}$$

where the dot $(\cdot)$ denotes the derivative with respect to time *t*.

We have three differential equations with five unknowns namely $A, B, f, p_{wdf}, \rho_{wdf}$. Some kinematical quantities of the space-time are average scale factor and volume respectively as $V=a^3=\sqrt{AB^2}$. The mean deceleration parameter which tells whether the universe exhibits accelerating volumetric expansion or not is $q=-1+\frac{d}{dt}\left(\frac{1}{H}\right)$, for $-1\le q<0$, $q>0$ and $q=0$ the universe exhibit accelerating volumetric expansion, decelerating volumetric expansion and expansion with constant-rate respectively. The mean Hubble parameter, which expresses the volumetric expansion rate of the universe, given as $H=\frac{1}{3}(H_1+H_2+H_3)$, where H_1, H_2, H_3 are the directional Hubble parameter in the direction of *x, y* and *z*-axis respectively . We have

$$H=\frac{1}{3}\frac{\dot{V}}{V}=\frac{1}{3}(H_1+H_2+H_3)=\frac{\dot{a}}{a}. \tag{12}$$

To discuss whether the universe either approach isotropy or not, we define an anisotropy parameter as

$$A_m=\frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i-H}{H}\right)^2. \tag{13}$$

The expansion scalar and shear scalar are defined as follows

$$\theta=u^{\mu}_{;\mu}=\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}, \tag{14}$$

$$\sigma^2=\frac{3}{2}H^2A_m. \tag{15}$$

4. Law of Variation for Hubble's parameter

Akarsu and Dereliwe [22] proposed a linearly varying deceleration parameter of the form (which can be used in obtaining accelerating cosmological solutions)

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -kt + m - 1, \tag{16}$$

where $k \geq 0$ and $m \geq 0$ are constant. Solving (16) one obtains different form of solutions for the scale factor:

$$a = a_1 (mt + m_1)^{\frac{1}{m}}, \quad \text{for} \quad k = 0 \text{ and } m > 0, \tag{17}$$

$$a = a_2 e^{m_2 t}, \quad \text{for} \quad k = 0 \text{ and } m = 0, \tag{18}$$

where a_1, a_2, m_1, m_2 are the constants of integration.

5. Solution of Field equations

Using equations (9) and (10), we get

$$\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} - \frac{\dot{B}^2}{B^2} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} = 0, \tag{19}$$

After mathematical manipulation, we have

$$\frac{d}{dt}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) + \left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)\frac{\dot{V}}{V} = 0, \tag{20}$$

which on integration gives

$$\frac{A}{B} = k_2 \exp\left[k_1 \int \frac{dt}{V}\right], \tag{21}$$

where k_1 and k_2 are constants of integration.

In view of $V = AB^2 = a^3$, we write A and B in the explicit form as

$$A = D_1 V^{\frac{1}{3}} \exp\left(\chi_1 \int \frac{1}{V} dt\right), \tag{22}$$

$$B = D_2 V^{\frac{1}{3}} \exp\left(\chi_2 \int \frac{1}{V} dt\right), \tag{23}$$

where $D_i (i = 1, 2)$ and $\chi_i (i = 1, 2)$ satisfy the relation $D_1 D_2^2 = 1$ and $\chi_1 + 2\chi_2 = 0$. Thus, the metric functions are represented explicitly in terms of average scale factor a. Once we obtain the value of a, we can find the metric functions.

6. Model for $k=0$ and $m=0$

Using equations (18),(22) and (23), we get the following expressions for metric potentials:

$$A = D_1 a_2 e^{m_2 t} \exp\left(\chi_1 \int \frac{1}{a_2^3 e^{3m_2 t}} dt \right), \tag{24}$$

$$B = D_2 a_2 e^{m_2 t} \exp\left(\chi_2 \int \frac{1}{a_2^3 e^{3m_2 t}} dt \right), \tag{25}$$

We obtain the expressions for pressure and energy density for the model as

$$\rho_{WDF} = \left(\frac{\omega}{1+\omega} \right) \rho^* + \frac{c}{a_2^3 e^{\left(\frac{3m_2 t(1+\omega)}{m} \right)}}, \tag{26}$$

and

$$p_{WDF} = \frac{c\omega}{a_2^3 e^{\left(\frac{3m_2 t(1+\omega)}{m} \right)}} - \left(\frac{\omega}{1+\omega} \right) \rho^* . \tag{27}$$

The expressions for the scalar of expansion, magnitude of shear, the average anisotropy parameter and deceleration parameter for the model are given by

$$\theta = 3m_2, \; \sigma^2 = \frac{2\left(\chi_1^2 + 2\chi_2^2\right)}{9a_2^6} e^{-6m_2 t}, A_m = \frac{\left(\chi_1^2 + 2\chi_2^2\right)}{3m_2^2 a_2^6} e^{-6m_2 t}.$$

The average generalized Hubble's parameter is given by $H = m_2$.Deceleration parameter is given by $q = -1$

7. Discussion and Concluding Remarks

In this paper we have obtained some exact Bianchi type-I space-time in $f(T)$ theory of gravitation with wet dark fluid. To find the solution of the field equation, the proposal of a law of variation for Hubble's parameter that yields a constant value of deceleration parameter is discussed. Hubble parameter tends to a constant value and the universe asymptotically approaches to de sitter space [23-25].The value of the anisotropy parameter shows that as time tends to infinity the anisotropy parameter tends to zero that is the universe tends to isotropy [26-29].At large time shear becomes insignificant. As t increases, the anisotropy of the expansion decreases exponentially to null. Thus, the space approaches isotropy in this model [30-36]. The expansion scalar is constant throughout the evolution of the universe. The shear scalar is also constant at $t=0$ and becomes zero as $t \to \infty$.The spatial volume is constant at $t=0$. It expands exponentially as t increases and becomes infinitely large as $t \to \infty$. The deceleration parameter shows that the universe is accelerating. The value of expansion scalar is constant i.e. the rate of expansion of the universe is constant [37-42]. The ratio of shear scalar to expansion scalar is non-zero i.e. the universes is anisotropic and as time increases it tends to zero i.e. at the late time the universe tending to isotropy [43-45]. We have obtained the deceleration parameter $q=-1$ and $dH/dt=0$ for this model. Hence, it gives the largest value of the Hubble parameter and the fastest rate of expansion of the universe. The model represents an accelerated universe. Therefore the model is consistent with the cosmological observations [46-47].

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