Essay

Can Leptons Be Considered Effectively as Local 3-Quark Composites?

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Abstract

The idea about leptons as local composites of 3 quarks is strongly suggested by the mathematical structure of TGD. Later it was realized that it is enough that leptons look like local composites in scales longer than CP_2 scale defining the scale of the partonic 2-surface assignable to the particle. The proposal has profound consequences. One might say that SUSY in the TGD sense has been below our nose for more than a century. The proposal could also solve matter-antimatter asymmetry since the twistor-lift of TGD predicts the analog of Kähler structure for Minkowski space and a small CP breaking, which could make possible a cosmological evolution in which quarks prefer to form baryons and antiquarks to form leptons. The basic objection is that the leptonic analog of Δ might emerge. One must explain why this state is at least experimentally absent and also develop a detailed model. In this article the construction of leptons as effectively local 3 quark states allowing effective description in terms of the modes of leptonic spinor field in $H = M^4 \times CP_2$ having H-chirality opposite to quark spinors is discussed in detail.

1 Introduction

The idea about leptons as local composites of 3 quarks [3] is strongly suggested by the mathematical structure of TGD. Later it was realized [6, 4, 5] that it is enough that leptons look like local composites in scales longer than CP_2 scale defining the scale of the partonic 2-surface assignable to the particle.

A strong mathematical motivation for the proposal is that quark oscillator operators are enough to construct the gamma matrices of the "world of classical worlds" (WCW) and leptonic oscillator operators corresponding to opposite chirality for $H = M^4 \times CP_2$ spinors are somehow superfluous.

The proposal has profound consequences. One might say that SUSY in the TGD sense has been below our nose for more than a century. The proposal could also solve matter-antimatter asymmetry since the twistor-lift of TGD predicts the analog of Kähler structure for Minkowski space and a small CP breaking, which could make possible a cosmological evolution in which quarks prefer to form baryons and antiquarks to form leptons.

The objection against the proposal is that the leptonic analog of Δ might emerge. One must explain why this state is at least experimentally absent. In [3] I did not develop a detailed argument for the intuition that one indeed avoids the leptonic analog of Δ. In this article the construction of leptons as effectively local 3 quark states allowing effective description in terms of the modes of leptonic spinor field in $H = M^4 \times CP_2$ having H-chirality opposite to quark spinors is discussed in detail.

2 Leptons as 3-quark composites

2.1 Some background

Some background is necessary before proceeding.

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1. In TGD color is not spin-like quantum number but corresponds to color partial waves in CP_2 for H-spinors describing fundamental fermions distinguished from fermions as elementary particles.

   Different chiralities of H-spinors were identified in the original model as leptons and quarks. If quarks couple to $n = 1$ Kähler gauge potential of CP_2 and leptons to its $n = 3$ multiple, ew quantum numbers of quarks and leptons come out correctly and lepton and quark numbers are separately conserved.

2. Few years ago emerged the idea that fundamental leptons to be distinguished from physical leptons are bound states of 3-quarks. They could be either local composites or look like local composites in scales larger than CP_2 size scale assignable to partonic 2-surface associated with the lepton.

3. The spin, ew quantum numbers associated with $SU(2)_L \times U(1)_R$ are additive and these quantum numbers should come out correctly for states with leptonic spin and ew numbers.

   Fundamental leptons/quarks are not color singlets/triplets although have vanishing triality. The color quantum numbers also correlate with ew quantum numbers and M^4 helicity/handedness. Only the right-handed neutrino ν_R is a color singlet. The mass squared values of the resulting states deducible from the massless Dirac equation in H are non-vanishing since CP_2 partial waves carry mass of order CP_2 mass.

   The application of color octet generators of super-symplectic algebra (SSA) of super-Kac-Moody algebra (SKMA) with non-vanishing conformal weight contributing to mass squared can guarantee that color quantum numbers are those of physical leptons and quarks. In p-adic mass calculations one must assume negative half-integer valued ground state conformal weight $h_{vac} < 0$.

There are two challenges.

1. One must construct leptons as local of the effectively local 3-quark composites. The challenge is to prove that the resulting states with spin and ew quantum numbers possess the color quantum numbers of fundamental leptons.

2. A priori one cannot exclude leptonic analog of Δ resonance obtained in the quark model of baryons as states for which the wave functions in spin and ew spin degrees of freedom are completely symmetric. The color wave function would be indeed completely antisymmetric also for the leptonic Δ. The challenge is to explain why they do not exist or are not observed.

2.2 Color representations and masses for quarks and leptons as modes of $M^4 \times CP_2$ spinor field

It would be also highly desirable to obtain for the masses of 3-quark states the same expressions as imbedding space Dirac operator predicts for leptonic masses. The masses depend on ew spin but are same for right and left-handed modes except in the case of right-handed neutrino. This could fixes the value of h_{vac} for leptons if it is assumed to be representable as 3-quark state. Empirical data are consistent with its absence from the spectrum.

The color representations associated with quark and lepton modes of $M^4 \times CP_2$ spinor fields were originally discussed by Hawking and Pope [1] and are considered from TGD point of view in [2].

Consider first quarks. For U_R the representations $(p+1,p)$ with triality 1 are obtained and $p = 0$ corresponds to color triplet 3. For D_R the representations $(p,p+2)$ are obtained and color triplet is missing from the spectrum ($p = 0$corresponds to $\bar{6}$). The representations and masses are the same for the left handed representations in both cases since the left handed modes are obtained by applying CP_2 Dirac operator to the right-handed modes.

The CP_2 contributions to the quark masses are given by the formula

$$\begin{aligned} m^2(U,p) &= \frac{m_1^2}{3}\left[p^2+3p+2\right] \ , \ p\geq 0 \ , \\ m^2(D,p) &= \frac{m_1^2}{3}\left[p^2+4p+4\right] = \frac{m_1^2}{3}(p+2)^2 \ , \ p\geq 0 \ , \\ m_1^2 &\equiv 2\Lambda \ . \end{aligned} \tag{2.1}$$

Here Λ is cosmoloigal constant characterizing the CP_2 metric. The mass squared splitting between U and D type states is given by

$$\Delta m^2(D,U) = m^2(D,p) - m^2(U,p) = \frac{m_1^2}{3}(p+2) \ . \tag{2.2}$$

Consider next leptons. Right handed neutrino ν_R corresponds to (p,p) states with $p\geq 0$ with mass spectrum

$$m^2(\nu) = \frac{m_1^2}{3}\left[p^2+2p\right] \ , \ p\geq 0 \ . \tag{2.3}$$

Charged handed charged leptons L correspond to $(p,p+3)$ states with mass spectrum

$$m^2(L) = \frac{m_1^2}{3}\left[p^2+5p+6\right] \ , \ p\geq 0 \ . \tag{2.4}$$

$(p,p+3)$ instead of (p,p) reflects the fact that leptons couple to 3-multiple of Kähler gauge potential. Right-handed neutrino has however vanishing total coupling.

Left handed solutions are obtained by operating with CP_2 Dirac operator on right handed solutions with one exception: the action of the Dirac operator on the covariantly constant right handed neutrino ($(p,p)=(0,0)$ state) annihilates it.

The mass splitting between charged leptons and neutrinos is given by

$$\Delta m^2(L,\nu) = m^2(L,p) - m^2(\nu,p) = m_1^2(p+2) = 3\Delta m^2(D,U) \ , \tag{2.5}$$

and is 3 times larger than the corresponding mass splitting. The mass splitting for leptons as states of type UUD and UDD is however different. If mass squared is additive as assumed in p-adic mass calculations one has $\Delta m^2(UDD,UUD) = \Delta m^2(D,U)$. The condition that the mass splitting for lepton states is the same as predicted by the identification as 3-quark states requires that the scale factor m_1^2 for 3 quarks states is 3 times larger than for quarks:

$$m_1^2(L) = 3m_1^2(q) \ . \tag{2.6}$$

2.3 Additivity of mass squared for quarks does not give masses of lepton modes

It would be natural that the same values for the leptons as 3-quark composites are same as for leptons as fundamental fermions. It is interesting to see whether the additivity of the mass squared values conforms with this hypothesis.

The sums of mass squared values for UUD (charged lepton) and UDD (neutrino) type states are given by

$$m^2(UUD) = 2m(U)^2 + m(D)^2 = 3p^2 + 10p + 8 \ ,$$
$$m^2(UDD) = 2m(D)^2 + m(U)^2 = 3p^2 + 11p + 10 \ . \tag{2.7}$$

These mass squared values are not consistent with the values proportional to the mass squared values proportional to p^2+5p+6 for L and to $p(p+2)$ for neutrinos. Covariantly constant right handed neutrino is not possible as a 3-quark state and this conforms with empirical facts.

The working hypothesis that mass squared is additive can be of course given up and a more general condition could be formulated in terms of four-momenta:

$$\begin{aligned} & p_1(U) + p_2(U) + p(D))^2 \\ & = 2m(U)^2 + m(D)^2 + 2\sum[p_1(U)\cdot p_2(U) + (p_1(U)+p_2(U))\cdot p(D)] = km(L)^2 \ , \\ & (p(U) + p_1(D) + p_2(D))^2 \\ & = m(U)^2 + 2m(D)^2 + 2\sum[p_1(D)\cdot p_2(D) + (p_1(D)+p_2(D))\cdot p(U)] = km(\nu)^2 \ . \end{aligned} \tag{2.8}$$

k is proportionality constant. These condition give single constraint in the 9-dimensional 3-fold Cartesian power of 3-D mass shells. The constraint is rather mild.

2.4 Can one obtain observed leptons and avoid leptonic Δ?

The antisymmetry of the wave function under exchange of quark states gives a strong constraint and fixes the allowed states. Does one obtain states with the quantum numbers of observed leptons as color singlets, and can one avoid the leptonic analogue of Δ?

1. For ordinary leptons complete color antisymmetry would require a complete symmetry under permutations of spin-ew quantum numbers: there are four states altogether. Antisymmetrization would be completely analogous to that occurring for baryons as 3-quark states and would require that fundamental leptons are antisymmetric color singlets.

2. The standard quark model picture natural for strong isospin does not conform with spin-ew symmetries and the resulting states need not allow an interpretation as effective modes of fundamental leptonic spinors. For $SU(2)_L \times U(1)_R$ the situation changes since right-handed helicities are $SU(2)_L$ singlets. The states of form $U_L D_R U_R$ (L_R) and $D_L D_R U_R$ (ν_R) could correspond to right-handed leptons and states of form $U_L D_R U_R$ (L_L) and $D_L D_R U_R$ (ν_L) to left-handed leptons.

3. The manipulation of Yang Tableaux (`https://cutt.ly/Ik9SGuU`) allow to see when a color singlet is contained in all 3-fold tensor products - that is $3\otimes 3\times 3$, $3\times 3\times \overline{6}$, $3\times\overline{6}\times\overline{6}$, and $\overline{6}\times\overline{6}\times\overline{6}$ - formed from the representations 3 and $\overline{6}$.

   One has $3\otimes 3 = \overline{3} + 6$ and $\overline{6}\otimes\overline{6} = 6 + 15_1 + 15_2$. Both $\overline{3}\otimes 3 = 1\oplus 2\times 8\oplus 10$ and $\overline{6}\otimes 6 = 1\oplus 8\oplus 27$ contain singlet and octet.

   Therefore both $3\otimes 3\times 3$ (UUU) and $\overline{6}\times\overline{6}\times\overline{6}$ (DDD) contain 1 and 8. $3\otimes 3\otimes\overline{6}$ (UUD corresponding to charged lepton) contains $6\otimes\overline{6}$ and therefore both 1 and 8. However, $3\otimes\overline{6}\otimes\overline{6}$ (neutrino as UDD) contains neither singlet nor octet.

4. The singlet contained in $\overline{6} \otimes 6$ should be also antisymmetric under the permutations of the color partial waves of quarks in 6. The singlet state has representation of the form $B_{KLM}A^KA^LA^M$, where $A^K = A^K_{rs}q^rq^s$ is the representation of $\overline{6}$ in terms of color triplet q^i. The tensor G_{KLM} should be antisymmetric. Since the singlet comes from Yang diagram as a vertical column, which corresponds to an anti-symmetric representation of S_3t, it seems that it is indeed antisymmetric.

   If this is the case, UUU and DDD singlets are indeed antisymmetric with respect to the exchange of quarks, and the state in spin-ew degrees of freedom can be totally symmetric.

5. As found, $\overline{6} \otimes 3 \times 3)$ (charged lepton as UUD) contains both 1 and 8 and 1 is antisymmetric as a full vertical column in the Yang diagram. If charged lepton corresponds to 1 it is analogous to proton in these degrees of freedom.

   $\overline{6} \otimes \overline{6} \times 3)$ (neutrino as DDU) contains neither 1 nor 8. In both cases an entanglement between color and spin-ew degrees of freedom is implied.

   Remark: Baryonic quarks reside at distinct partonic 2-surfaces and allow separate color neutralization by SSA or SKMA generators and are color triplets so that the standard picture about color confinement prevails in the baryonic sector.

6. If the 3-quark state is not a color octet, the operators needed to cancel the negative conformal weight must consist of at least two SSA or SKMA operators, which are color octets. UUD contains 8 and 1 but UDD does not.

   For neutrinos which cannot be color octets or singlets, at least 2 color octet generators are required to neutralize the color. For color singlet charged lepton this is not needed since p-adic thermodynamics allows a massless ground state. The difference charged leptons and neutrinos might relate to the fact that the long p-adic length scales for neutrinos are so long as compared to those for charged leptons.

   As has become clear, the neutral Δ type state UDD is not possible since color singlet and octet are not allowed and the neutralization of the negative conformal weight using at least two color generators as in the case of neutrino. Also for other components of Δ color singlet-ness requires at least two generators whereas octet requires only one generator. For color octets a complete symmetry in spin-ew degrees of freedom is not possible.

The conclusion is that charged lepton and charged components of Δ allow for color singlet completely symmetric wave function in spin-ew degrees of freedom unentangled from color. Neutrino and neutral Δ require entanglement between color and spin-ew degrees of freedom.

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