**Article**

# Connes' Idea about Inherent Time Evolution of Certain Algebraic Structures from TGD Viewpoint

Matti Pitkänen [1]

**Abstract**

Alan Connes has proposed that certain mathematical structures known as hyperfinite factors (HFFs) contain in their structure inherent time evolution.This time evolution is determined only modulo unitary automorphism analogous to a time evolution determined by Hamiltonian so that this time evolution seems to be too general for the purposes of a physicist. Zero energy ontology of TGD combined with adelic physics leads to a vision that the sequences of state function reductions implies a mathematical evolution in the sense that the extensions of rationals characterizing the space-time region increases gradually. This induces the increase of algebraic complexity implying time evolution as the analog of biological evolution. The dimension of extension corresponds to an effective Planck constant assumed to label dark matter as phases of ordinary matter. Therefore quantum coherence lengths increase in this evolution. This generalization of the idea of Connes is discussed in the framework provided by the recent view about TGD. In particular, the inclusion hierarchies of hyperfinite factors, the extension hierarchies of rationals, and fractal inclusion hierarchies of subalgebras of supersymplectic algebra isomorphic with the entire algebra are proposed to be more or less one and the same thing in TGD framework. The time evolution operator of Connes could corresponds to super-symplectic algebra (SSA) to the time evolution generated by $exp(iL_0\tau)$ so that the operator $\Delta$ of Connes would be identified as $\Delta = exp(L_0)$. This identification allows number theoretical universality if $\tau$ is quantized. Furthermore, one ends up with a model for the subjective time evolution by small state function reductions (SSFRs) for SSA with $SSA_n$ gauge conditions: the unitary time evolution for given SSFR would be generated by a linear combination of Virasoro generators not annihilating the states. This model would generalize the model for harmonic oscillator in external force allowing exact S-matrix.

## 1 Introduction

Jonathan Disckau asked me about what I think about the proposal of Connes represented in the summary of progress of noncommutative geometry in "Noncommutative Geometry Year 2000" [1] (see `https://arxiv.org/abs/math/0011193`) that certain mathematical structures have inherent time evolution coded into their structure.

I have written years ago about Connes's proposal. At that time I was trying to figure out how to understand the construction of scattering amplitudes in the TGD framework and the proposal of Connes looked attractive. Later I had to give up this idea. However, the basic idea is beautiful. One should only replace the notion of time evolution from a one-parameter group of automorphisms to something more interesting. Also time evolution as increasing algebraic complexity is a more attractive interpretation.

The inclusion hierarchies of hyperfinite factors (HFFs) - closely related to the work of Connes - are a key element of TGD and crucial for understanding evolutionary hierarchies in TGD. Is it possible that mathematical structure evolves in time in some sense? The TGD based answer is that quantum jump as a fundamental evolutionary step - moment of subjective time evolution - is a necessary new element. The sequence of moments of consciousness as quantum jumps would have an interpretation as hopping around in the space of mathematical structures leading to increasingly complex structures.

The generalization of the idea of Connes is discussed in this framework. In particular, the inclusion hierarchies of hyper-finite factors, the extension hierarchies of rationals, and fractal inclusion hierarchies

[1] Correspondence: Matti Pitkänen `http://tgdtheory.fi/`. Address: Rinnekatu 2-4 8A, 03620, Karkkila, Finland. Email: matpitka6@gmail.com.

of subalgebras of supersymplectic algebra isomorphic with the entire algebra are proposed to be more or less one and the same thing in TGD framework.

The time evolution operator of Connes could corresponds to super-symplectic algebra (SSA) to the time evolution generated by $exp(iL_0\tau)$ so that the operator $\Delta$ of Connes would be identified as $\Delta = exp(L_0)$. This identification allows number theoretical universality if $\tau$ is quantized. Furthermore, one ends up with a model for the subjective time evolution by small state function reductions (SSFRs) for SSA with $SSA_n$ gauge conditions: the unitary time evolution for given SSFR would be generated by a linear combination of Virasoro generators not annihilating the states. This model would generalize the model for harmonic oscillator in external force allowing exact S-matrix.

# 2 Connes proposal and TGD

In this section I develop in more detail the analog of Connes proposal in TGD framework.

## 2.1 What does Connes suggest?

One must first make clear what the automorphism of HFFs discovered by Connes is.

### 2.1.1 Tomita-Takesaki theory

Tomita-Takesaki theory is a vital part of the theory of factors. I have described the theory earlier [4, 6].
First some definitions.

1. Let $\omega(x)$ be a faithful state of von Neumann algebra so that one has $\omega(xx^*) > 0$ for $x > 0$. Assume by Riesz lemma the representation of $\omega$ as a vacuum expectation value: $\omega = (\cdot\Omega, \Omega)$, where $\Omega$ is cyclic and separating state.

2. Let

$$L^{\infty}(\mathcal{M}) \equiv \mathcal{M} \quad , \quad L^2(\mathcal{M}) = \mathcal{H} \quad , \quad L^1(\mathcal{M}) = \mathcal{M}_* \quad , \tag{2.1}$$

   where $\mathcal{M}_*$ is the pre-dual of $\mathcal{M}$ defined by linear functionals in $\mathcal{M}$. One has $\mathcal{M}_*{}^* = \mathcal{M}$.

3. The conjugation $x \to x^*$ is isometric in $\mathcal{M}$ and defines a map $\mathcal{M} \to L^2(\mathcal{M})$ via $x \to x\Omega$. The map $S_0; x\Omega \to x^*\Omega$ is however non-isometric.

4. Denote by $S$ the closure of the anti-linear operator $S_0$ and by $S = J\Delta^{1/2}$ its polar decomposition analogous that for complex number and generalizing polar decomposition of linear operators by replacing (almost) unitary operator with anti-unitary $J$. Therefore $\Delta = S^*S > 0$ is positive self-adjoint and $J$ an anti-unitary involution. The non-triviality of $\Delta$ reflects the fact that the state is not trace so that hermitian conjugation represented by $S$ in the state space brings in additional factor $\Delta^{1/2}$.

5. What $x$ can be is puzzling to physicists. The restriction fermionic Fock space and thus to creation operators would imply that $\Delta$ would act non-trivially only vacuum state so that $\Delta > 0$ condition would not hold true. The resolution of puzzle is the allowance of tensor product of Fock spaces for which vacua are conjugates: only this gives cyclic and separating state. This is natural in ZEO.

The basic results of Tomita-Takesaki theory are following.

1. The basic result can be summarized through the following formulas

$$\Delta^{it} M \Delta^{-it} = \mathcal{M} \ \ , J\mathcal{M}J = \mathcal{M}' \ \ .$$

2. The latter formula implies that $\mathcal{M}$ and $\mathcal{M}'$ are isomorphic algebras. The first formula implies that a one parameter group of modular automorphisms characterizes partially the factor. The physical meaning of modular automorphisms is discussed in [2, 3] $\Delta$ is Hermitian and positive definite so that the eigenvalues of $log(\Delta)$ are real but can be negative. $\Delta^{it}$ is however not unitary for factors of type II and III. Physically the non-unitarity must relate to the fact that the flow is contracting so that hermiticity as a local condition is not enough to guarantee unitarity.

3. $\omega \to \sigma_t^\omega = Ad\Delta^{it}$ defines a canonical evolution -modular automorphism- associated with $\omega$ and depending on it. The $\Delta$:s associated with different $\omega$:s are related by a unitary inner automorphism so that their equivalence classes define an invariant of the factor.

Tomita-Takesaki theory gives rise to a non-commutative measure theory which is highly non-trivial. In particular the spectrum of $\Delta$ can be used to classify the factors of type II and III.

The definition of $\Delta^{it}$ reduces in eigenstate basis of $\Delta$ to the definition of complex function $d^{it}$. Note that is positive so that the logarithm of $d$ is real.

In TGD framework number theoretic universality poses additional conditions. In diagonal basis $e^{log(d)it}$ must exist. A simply manner to solve the conditions is $e = exp(m/r)$ existing p-adically for an extension of rational allowing $r$:th root of e. This requires also quantization of as a root of unity so that the exponent reduces to a root of unity.

### 2.1.2 Modular automorphisms

Modular automorphisms of factors are central for their classification.

1. One can divide the automorphisms to inner and outer ones. Inner automorphisms correspond to unitary operators obtained by exponentiating Hermitian Hamiltonian belonging to the factor and connected to identity by a flow. Outer automorphisms do not allow a representation as a unitary transformations although $log(\Delta)$ is formally a Hermitian operator.

2. The fundamental group of the type $\mathrm{II}_1$ factor defined as fundamental group group of corresponding $\mathrm{II}_\infty$ factor characterizes partially a factor of type $\mathrm{II}_1$. This group consists real numbers $\lambda$ such that there is an automorphism scaling the trace by $\lambda$. Fundamental group typically contains all reals but it can be also discrete and even trivial.

3. Factors of type III allow a one-parameter group of modular automorphisms, which can be used to achieve a partial classification of these factors. These automorphisms define a flow in the center of the factor known as flow of weights. The set of parameter values $\lambda$ for which $\omega$ is mapped to itself and the center of the factor defined by the identity operator (projector to the factor as a sub-algebra of $\mathcal{B}(\mathcal{H})$) is mapped to itself in the modular automorphism defines the Connes spectrum of the factor. For factors of type $III_\lambda$ this set consists of powers of $\lambda < 1$. For factors of type $III_0$ this set contains only identity automorphism so that there is no periodicity. For factors of type $\mathrm{III}_1$ Connes spectrum contains all real numbers so that the automorphisms do not affect the identity operator of the factor at all.

The modules over a factor correspond to separable Hilbert spaces that the factor acts on. These modules can be characterized by M-dimension. The idea is roughly that complex rays are replaced by the sub-spaces defined by the action of $\mathcal{M}$ as basic units. M-dimension is not integer valued in general. The so called standard module has a cyclic separating vector and each factor has a standard representation possessing antilinear involution $J$ such that $\mathcal{M}' = J\mathcal{M}J$ holds true (note that $J$ changes the order of the operators in conjugation). The inclusions of factors define modules having interpretation in terms of a finite measurement resolution defined by $\mathcal{M}$.

### 2.1.3 Objections against the idea of Connes

One can represent objections against this idea.

1. Ordinary time evolution in wave mechanics is a unitary automorphism, so that in this framework they would not have physical meaning but act as gauge transformations. If outer automorphisms define time evolutions, they must act as gauge transformations. One would have an analog of gauge field theory in HFF. This would be of course highly interesting: when I gave up the idea of Connes, I did not consider this possibility. Super-symplectic algebras having fractal structure is however extremely natural candidate for defining HFF and there is infinite number of gauge conditions possibly realizing the gauge conditions.

2. An automorphism is indeed in question so that the algebraic system would not be actually affected. Therefore one cannot say that HFF has inherent time evolution and time.

   However, one can represent in HFF dynamical systems obeying this inherent time evolution. This possibility is highly interesting as a kind of universal gauge theory.

3. The notion of time evolution itself is an essentially Newtonian concept: selecting a preferred time coordinate breaks Lorentz invariance. In TGD however time coordinate is replace by scaling parameter and the situation changes.

4. The proposal of Connes is not general enough if evolution is interpreted as an increase of complexity.

For these reasons I gave up the automorphism proposed by Connes as a candidate for defining time evolution giving rise to scattering amplitudes in TGD framework.

## 2.2 Two views about TGD

The two dual views about what TGD is described briefly in [15].

1. Physics as geometry of the world of "world of classical worlds" (WCW) identified as the space of space-time surfaces in $M^4 \times CP_2$ [5]. Twistor lift of TGD [7] implies that the space-time surfaces are minimal surfaces which can be also regarded as extermals of the Kähler action. This implies holography required by the general coordinate invariance in TGD framework.

2. TGD as generalized number theory forcing to generalize physics to adelic physics [9] fusing real physics as correlate of sensory experience and various p-adic physics as correlates of cognition. Now space-times are naturally co-associative surfaces in complexified $M^8$ (complexified octonions) defined as "roots" of octonionic polynomials determined by polynomials with rational coefficients [13, 14, 17]. Now holography extends dramatically: finite number of rational numbers/roots of rational polynomial/points of space-time region dictate it.

$M^8 - H$ duality relates these two views and is actually a generalization of Fourier transform and realizes generalization of momentum-position duality.

## 2.3 The notion of time evolution in TGD

Concerning various time evolutions in TGD, the general situation is now rather well understood.

There are two quantal time evolutions: geometric one assignable to single CD and and subjective time evolution which reflects the generalization of point-like particle to a 3-surface and the introduction of CD as 4-D perceptive field of particle in ZEO [12].

1. Geometric time evolution corresponds to the standard scattering amplitudes for which I have a general formula now in terms of zero energy ontology (ZEO) [16, 13, 14, 17]. The analog of S-matrix corresponds to entanglement coefficients between members of zero energy state at opposite boundaries of causal diamond (CD).

2. Subjective time evolution of conscious entity corresponds to a sequence of "small" state function reductions (SSFRs) as moments of consciousness: each SSFR is preceded by an analog of unitary time evolution, call it $U$. SSFRs are the TGD counterparts of "weak" measurements.

   $U(t)$ is generated by the scaling generator $L_0$ scaling light-like radial coordinate of light-cone boundary and is a generalization of corresponding operator in superconformal and string theories and defined for super-symplectic algebras acting as isometries of the world of classical worlds (WCW) [17]. $U(t)$ is not the exponential of energy as a generator of time translation as in QFTs but an exponential of the mass squared operator and corresponds to the scaling of radial light-like coordinate $r$ of the light-like boundary of CD: $r$ is analogous to the complex coordinate $z$ in conformal field theories.

   Also "big" SFRs (BSFRs) are possible and correspond to "ordinary" SFRs and in TGD framework mean death of self in the universal sense and followed by reincarnation as time reversed subjective time evolution [10].

3. There is also classical time evolution at the level of space-time surfaces. Here the assumption that $X^4$ belongs to $H = M^4 \times CP_2$ defines Minkowski coordinates of $M^4$ as almost unique space-time coordinates of $X^4$ is the $M^4$ projection of $X^4$ is 4-D. This generalizes also to the case of $M^8$. Symmetries make it possible to identify an essentially a unique time coordinate.

   This means enormous simplification. General coordinate invariance is a marvellous symmetry but it leads to the problem of specifying space-time coordinates that is finding preferred coordinates. This seems impossible since 3-metric is dynamical. $M^4$ provides a fixed reference system and the problem disappears. $M^4$ is dynamical by its Minkowskian signature and one can speak about classical signals.

4. There is also classical time evolution for the induced spinor fields. At the level of $H$ the spinor field is a superposition of modes of the massless Dirac operator (massless in 8-D sense). This spinor field is free and second quantized. Second quantization of induced spinor trivializes and this is absolutely crucial for obtaining scattering amplitudes for fermions and avoiding the usual problems for quantization of fermions in curved background.

   The induced spinor field is a restriction of this spinor field to the space-time surface and satisfies modified Dirac equation automatically. There is no need for second quantization at the level of space-time surface and propagators etc.... are directly calculable. This is an enormous simplification.

There are therefore as many as 4 time evolutions and subjective time evolution by BSFRs and possibly also by SSFRs is a natural candidate for time evolution as genuine evolution as emergence of more complex algebraic structures.

### 2.4 Could the inherent time evolution of HFF have a physical meaning in TGD after all?

The idea about inherent time evolution defined by HFF itself as one parameter group of outer automorphisms is very attractive by its universality: physics would become part of mathematics.

1. The idea does not make sense in the TGD framework if the time coordinate is non-Lorentz invariant linear Minkowski coordinate.

2. It is also clear that one cannot assign the outer automorphism to the S-matrix as a generalization of the S-matrix of particle physics defined by the entanglement coefficients of zero energy states. Therefore I gave up the idea of Connes when considering it for the first time.

However, TGD inspired theory of consciousness as a generalization of quantum measurement theory has evolved since then and the situation is different now.

The sequence of SSFRs defines subjective time evolution having no counterpart in QFTs. Each SSFR is preceded by a unitary time evolution, which however corresponds to the scaling of the light-like radial coordinate of the light-cone boundary [17] rather than time translation. Hamiltonian is replaced with the scaling generator $L_0$ acting as Lorentz invariant mass squared operator so that Lorentz invariance is not lost.

Could the time evolution assignable to $L_0$ correspond to the outer automorphism of Connes when one poses an infinite number of gauge conditions making inner automorphisms gauge transformations? The connection of Connes proposal with conformal field theories and with TGD is indeed suggestive.

1. Conformally invariant systems obey infinite number of gauge conditions stating that the conformal generators $L_n$, $n > 0$, annihilate physical states and carry vanishing Noether charges.

   These gauge conditions bring in mind the condition that infinitesimal inner automorphisms do not change the system physically. Does this mean that Connes outer automorphism generates the time evolution and inner automorphisms act as gauge symmetries? One would have an analog of gauge field theory in HFF.

2. In TGD framework one has an infinite hierarchy of systems satisfying conditions analogous to the conformal gauge conditions. The generators of the super-symplectic algebra (SCA) acting as isometries of the "world of classical worlds" (WCW) are labelled by non-negative conformal weight $n$ and it has infinite hierarchy of algebras $\mathrm{SCA}_k$ isomorphic to it with conformal weights given by k-multiple of those of the entire algebra, $k = 1, 2, ....$.

   Gauge conditions state for $SCA_k$ that the generators of $SCA_k$ and its commutator with SCA annihilate physical states. The interpretation is in terms of a hierarchy of improving measurement resolutions with degrees of freedom below measurement resolution acting like gauge transformations.

   The Connes automorphism would "see" only the time evolution in the degrees of freedom above measurement resolution and as $k$ increases, their number would increase.

### 2.5 Three views about finite measurement resolution

Evolution could be seen physically as improving finite measurement resolution: this applies to both sensory experience and cognition. There are 3 views about finite measurement resolution (FMR) in TGD.

#### 2.5.1 Hyper finite factors (HFFs) and FMR

HFFs are an essential part of Connes's work and I encountered them about 15 years ago or so [8, 6].

The inclusions of hyper-finite factors HFFs provide one of the three - as it seems equivalent - ways to describe finite measurement resolution (FMR) in TGD framework: the included factor defines an analog for gauge degrees of freedom which correspond to those below measurement resolution.

### 2.5.2 Cognitive representations and FMR

Another description for FMR in the framework of adelic physics would be in terms of cognitive representations [11]. First some background about $M^8 - H$ duality.

1. There are number theoretic and geometric views about dynamics. In algebraic dynamics at the level of $M^8$, the space-time surfaces are roots of polynomials. There are no partial differential equations like in the geometric dynamics at the level of $H$.
2. The algebraic "dynamics" of space-time surfaces in $M^8$ is dictated by co-associativity, which means that the normal space of the space-time surface is associative and thus quaternionic. That normal space rather than tangent space must be associative became clear last year [13, 14].
3. $M^8 - H$ duality maps these algebraic surfaces in $M^8$ to $H = M^4 \times CP_2$ and the one obtains the usual dynamics based on variational principle giving minimal surfaces which are non-linear analogs for the solutions of massless field equations. Instead of polynomials the natural functions at the level of $H$ are periodic functions used in Fourier analysis [17].

At level of complexified $M^8$ cognitive representation would consist of points of co-associative space-time surface $X^4$ in complexified $M^8$ (complexified octonions), whose coordinates belong to extension of rationals and therefore make sense also p-adically for extension of p-adic numbers induced by extension of rationals. $M^8 - H$ duality maps the cognitive representations to $H$.

Cognitive representations form a hierarchy: the larger the extension of rationals, the larger the number of points in the extension and in the unique discretization of space-time surface. Therefore also the measurement resolution improves.

The surprise was that the cognitive representations which are typically finite, are for the "roots" of octonionic polynomials infinite [13, 14]. Also in this case the density of points of cognitive representation increases as the dimension of extensions increases.

The understanding of the physical interpretation of $M^8 - H$ duality increased dramatically during the last half year.

1. $X^4$ in $M^8$ is highly analogous to momentum space (4-D analog of Fermi ball one might say) and $H$ to position space. Physical states correspond to discrete sets of points - 4-momenta - in $X^4$. This is just the description used in particle physics for physical states. Time and space in this description are replaced by energy and 4-momentum. At the level of $H$ one space-time and classical fields and one talks about frequencies and wavelengths instead of momenta.
2. $M^8 - H$ duality is a generalization of Fourier transform. Hitherto I have assumed that the space-time surface in $M^8$ is mapped to $H$. The momentum space interpretation at the level of $M^8$ however requires that the image must be a superposition of translates of the image in plane wave with some momentum: only the translates inside some bigger CD are allowed - this means infrared cutoff.

   The total momentum as sum of momenta for two half-cones of CD in $M^8$ is indeed well-defined. One has a generalization of a plane wave over translational degrees of freedom of CD and restricted to a bigger CD.

   At the limit of infinitely large size for bigger CD, the result is non-vanishing only when the sum of the momenta for two half-cones of CD vanishes: this corresponds to conservation of 4-momentum as a consequence of Poincare invariance rather than assumption as in the earlier approach [17].

   This generalizes the position-momentum duality of wave mechanics lost in quantum field theory. Point-like particle becomes a quantum superposition of space-time surfaces inside the causal diamond (CD). Plane wave is a plane wave for the superposition of space-time surfaces inside CD having the cm coordinates of CD as argument.

#### 2.5.3 Inclusion hierarchy of supersymplectic algebras and FMR

The third inclusion hierarchy allowing to describe finite measurement resolution is defined by supersymplectic algebras acting as the isometries of the "world of classical worlds" (WCW) consisting of space-time surfaces are preferred extremals ("roots" of polynomials in $M^8$ and minimal surfaces satisfying infinite-D set of additional "gauge conditions" in H).

At a given level of hierarchy generators with conformal weight larger than n act like gauge generators as also their commutators with generators with conformal weight smaller than n correspond to vanishing Noether charges. This defines "gauge conditions".

To sum up, there are therefore 3 hierarchies allowing to describe finite measurement resolution and they must be essentially equivalent in TGD framework.

### 2.6 Three evolutionary hierarchies

There are three evolutionary hierarchies: hierarchies of extensions of extensions of... ofrationals...; inclusions of inclusions of .... of HFFs, and inclusions of isomorphic super symplectic algebras.

#### 2.6.1 Extensions of rationals

The extensions of rationals become algebraically increasingly complex as their dimension increases. The co-associative space-time surfaces in $M^8$ are "roots" of real polynomials with rational coefficients to guarantee number theoretical universality and this means space-time surfaces are characterized by extension of rationals.

Each extension of rationals defines extensions for p-adic number fields and entire adele. The interpretation is as a cognitive leap: the system's intelligence/algebraic complexity increases when the extension is extended further.

The extensions of extensions of .... define hierarchies with Galois groups in certain sense products of extensions involved. Exceptional extensions are those which do not allow this decomposition. In this case Galois group is a simple group. Simple groups are primes of finite groups and correspond to elementary particles of cognition. Kind of fundamental, non-decomposable ideas. Mystic might speak of pure states of consciousnesswith no thoughts.

In the evolution by quantum jumps the dimension of extension increases in statistical sense and evolution is unavoidable. This evolution is due to subjective time evolution by quantum jumps, something which is in spirit with Connes proposal but replaces time evolution by a sequence of evolutionary leaps.

#### 2.6.2 Inclusions of HFFs as a hierarchy

HFFs are fractals. They have infinite inclusion hierarchies in which sub-HFF isomorphicto entire HFFs is included to HFF.

Also the hierarchies of inclusions define evolutionary hierarchies: HFF which is isomorphic with original becomes larger and in some sense more complex than the included factor. Also now one has sequences of inclusions of inclusions of.... These sequences would correspond to sequences for extensions of extensions... of rationals. Note that the inclusion hierarchy would be the basic object: not only single HFF in the hierarchy.

#### 2.6.3 Inclusions of supersymplectic algebras as an evolutionary hierarchy

The third hierarchy is defined by the fractal hierarchy of sub-algebras of supersymplectic algebra isomorphic to the algebra itself. At a given level of hierarchy generators with conformal weight larger than n correspond to gauge degrees of freedom. As $n$ increases the number of physical degrees of freedom above measurement resolution increases which means evolution. This hierarchy should correspond rather concretely to that for the extensions of rationals. These hierarchies would be essentially one and the same thing in the TGD Universe.

### 2.7 TGD based model for subjective time development

The understanding of subjective time development as sequences of SSFRs preceded by unitary "time" evolution has improved quite considerably recently [17]. The idea is that the subjective time development as a sequence of scalings at the light-cone boundary generated by the vibrational part $\hat{L}_0$ of the scaling generator $L_0 = p^2 - \hat{L_0}$ ($L_0$ annihilates the physical states). Also p-adic mass calculations use $\hat{L}_0$ .

For more than 10 years ago [4, 6], I considered the possibility that Connes time evolution operator that he assigned with thermo-dynamical time could have a significant role in the definition of S-matrix in standard sense but had to give up the idea.

It however seems that for super-symplectic algebra $\hat{L}_0$ generates an outer automorphism since the algebra has only generators with conformal with $n > 0$ and its extension to included also generators with $n \leq 0$ is required to introduce $L_0$: since $L_0$ contains annihilation operators, it indeed generates outer automorphism in SCA. The two views could be equivalent! Whereas Connes considered thermo-dynamical time evolution, in TGD framework the time evolution would be subjective time evolution by SSFRs.

1. The guess would be that the exponential of the scaling operator $L_0$ gives the time evolution. The problem is that $L_0$ annihilates the physical states. The solution of the problem would be the same as in p-adic thermodynamics. $L_0$ decomposes as $L_0 = p^2 - \hat{L_0}$ and the vibrational part $\hat{L}_0$ this gives mass spectrum as eigenvalues of $p^2$. The thermo-dynamical state in p-adic thermodynamics is $p^{\hat{L}_0\beta}$. This operator exists p-adically in the p-adic number field defined by prime $p$.

2. Could unitary subjective time development involve the operator $exp(i2\pi L_0\tau)$ $\tau = log(T/T_0)$? This requires $T/T_0 = exp(n/m)$ guaranteeing that exponential is a root of unity for an eigenstate of $L_0$. The scalings are discretized and scalings come as powers of $e^{1/m}$. This is possible using extensions of rationals generated by a root of $e$. The unique feature of p-adics is that $e^p$ is ordinary p-adic number. This alone would give periodic time evolution for eigenstates of $L_0$ with integer eigenvalues $n$.

### 2.8 $SSA$ and $SSA_n$

Supersymplectic algebra $SSA$ has fractal hierarchies of subalgebras $SSA_n$. The integers in a given hierarchy are of forn $n_1, n_1n_2, n_1n_2n_3, ...$ and correspond naturally to hierarchies of inclusions of HFFs. Conformal weights are positive: $n > 0$. For ordinary conformal algebras also negative weights are allowed. Yangians have only non-negative weights. This is of utmost importance.

$SSA_n$ with generators have radial light-like conformal weights coming as multiples of n. $SSA_n$ annihilates physical states and $[SSA_n, SSA]$ does the same. Hence the generators with conformal weight larger than n annihilate the physical states.

What about generators with conformal weights smaller than $n$? At least a subset of them need not annihilate the physical states. Since $L_n$ are superpositions of creation operators, the idea that analogs of coherent states could be in question.

It would be nice to have a situation in which $L_n$, $n < m$ commute. $[L_k, L_l] = 0$ effectively for $k+l \geq m$.

The simplest way to obtain a set of effectively commuting operators is to take the generators $L_k$, $[m/2] < k < m$, where $[m/2]$ is nearest integer larger than $m/2$.

This raises interesting questions.

1. Could the Virasoro generators $O(\{c_k\}) = \sum_{k\in[m/2],m]} c_kL_k$ as linear combinations of creation operators generate a set of coherent states as eigenstates of their Hermitian conjugates.

2. Some facts about coherent states are in order.

(a) When one adds to quantum harmonic oscillator Hamiltonian oscillator a time dependent perturbation which lasts for a finite the vacuum state evolves to an oscillator vacuum whose position is displacemented. The displacement is complex and is a Fourier component of the external force $f(t)$ corresponding to the harmonic oscillator frequency $\omega$. Time evolution picks up only this component.

(b) Coherent state property means that the state is eigenstate of the annihilation creation operator with eivengeu $\alpha = -ig(\omega)$ where $g(omega) = \int f(u)exp(-i\omega u)du$ is Fourier transform of $f(t)$.

(c) Coherent states are not orthogonal and form an overcomplete set. The overlaps of coherent states are proportional to a Gaussian depending on the complex parameters characterizing them. One can however develop any state in terms of coherent states as a unique expansion since one can represent unitary in terms of coherent states.

(d) Coherent state obtained from the vacuum state by time evolution in presence of $f(t)$ by a unitary displacement operator $D(\alpha) = exp(\alpha a^\dagger - \overline{\alpha} a)$. (`https://en.wikipedia.org/wiki/Displacement_operator`).

The displacement operator is a unitary operator and in the general case the displacement is complex. The product of two displacement operators would be apart from a phase factor a displacement operator associated with the sum of displacements.

(e) Harmonic oscillator coherent states are indeed maximally classical since wave packets have minimal width in both q and p space. Furthermore, the classical expectation values for $q$ and $p$ obey classical equations of motion.

These observations raise interesting questions about how the evolution by SSFRs could be modelled.

(a) Instead of harmonic oscillator in q-space, one would have time evolution in the space of scalings of causal diamond parameterized by the scaling parameter $\tau = log(T/T_0)$, where $T$ can be identified as the radial light-like coordinate of light-cone boundary.

The analogs of harmonic oscillator states would be defined in this space and would be essentially wave packets with ground state minimizing the width of the wave packet.

(b) The role of harmonic oscillator Hamiltonian in absence of external force would be taken by the generator $\hat{L}_0$ ($L_0 = p^2 - \hat{L}_0$ acts trivially) and gives rise to mass squared quantization. The situation would be highly analogous to that in p-adic thermodynamics. The role of $\omega$ would be taken by the minimal conformal weight $h_{min}$ such that the eigenvalues of $L_0$ are its multiples. It seems that this weight must be equal to $h_{min} = 1$.

The commutations of $\hbar L_0$ with $L_k$, $k > 0$ would be as for $L_0$ so what the replacement should not affect the situation.

(c) The scaling parameter $\tau$ is analogous to the spatial coordinate $q$ for the harmonic oscillator. Can one identify the analog of the external force $f(t)$ acting during unitary evolution between two SSFRs? Or is it enough to use only the analog of $g(\omega \to h_{min} = 1)$ - that is the coefficients $C_k$.

To identify $f(t)$, one needs a time coordinate $t$. This was already identified as $\tau$. This one would have $q = t$, which looks strange. The space in which time evolution is the space of scalings and the time evolutions are scalings and thus time evolution means translation in this space. The analog for this would be Hamiltonian $H = i\hbar d/dq$.

Number theoretical universality allows only the values of $\tau = r/s$ whose exponents give roots of unity. Also $exp(n\tau)$ makes sense p-adically for these values. This would mean that the Fourier transform defining $g$ would become discrete and be sum over the values $f(\tau = r/s)$.

(d) What happens if one replaces $\hat{L}_0$ with $L_0$. In this case one would have the replacement of $\omega$ with $h_{vac} = 0$. Also the analog of Fourier transform with zero frequency makes sense. $\hat{L_0} = p^2 - L_0$

is the most natural choice for the Hamiltonian defining the time evolution operator but is trivial. Could $\Delta^{i\tau}$ describe the inherent time evolution. It would be outer automorphism since it is not defined solely in terms of SCA. So: could one have $\Delta = exp(\hat{L_0})$ so that $\Delta^{i\tau}$ coincide with $exp(i\hat{L_0}\tau)$? This would mean the identification

$$\Delta = exp(\hat{L_0}) \ ,$$

which is a positive definite operator. The exponents coming from $exp(iL_0\tau)$ can be number theoretically universal if $\tau = log(T/T_0)$ is a rational number implying $T/T_0 = exp(r/s)$, which is possible number theoretically) and the extension of rationals contains some roots of $e$.

(e) Could one have $\Delta = \hat{L_0}$? Also now that positivity condition would be satisfied if SSA conformal weights satisfy $n > 0$.

The problem with this operation is that it is not number theoretically universal since the exponents $exp(ilog(n)\tau)$ do not exist p-adically without introducing infinite-D extension of p-adic number making $log(n)$ well-defined.

What is however intriguing is that the "time" evolution operator $\Delta^{i\tau}$ in the eigenstate basis would have trace equal to $Tr(\Delta^{i\tau})\sum d(n)n^{i\tau}$, where $d(n)$ is the degeneracy of the state. This is a typical zeta function: for Riemann Zeta one has $d(n) = 1$.

For $\Delta = exp(L_0)$ option $Tr(\Delta^{i\tau}) = \sum d(n)exp(in\tau)$ exists for $\tau = r/s$ if $r$:th root of $e$ belongs to the extension of p-adics.

To sum up, one would have Gaussian wave packet as harmonic oscillator vacuum in the space of scaled variants of CD. The unitary time evolution associated with SSFR would displace the peak of the wave packet to a larger scalings. The Gaussian wave function in the space of scaled CDs has been proposed earlier.

Could this time evolution make sense and be even realistic?

1. The analogs of harmonic oscillator states are defined in the space of scalings as Gaussians and states obtained from them using oscillator operators. There would be a wave function in the moduli space of CDs analogous to a state of harmonic oscillator.

2. SSFR following the time evolutions would project to an eigenstate of harmonic oscillator having in general displaced argument. The unitary displacement operator $D$ should commute with the operators having the members of zero energy states at the passive boundary of CD as eigenstates. This poses strong conditions. At least number theoretic measurements could satisfy these conditions.

3. SSFRs are identified as weak measurements as near as possible to classical measurements. Time evolution by the displacement would be indeed highly analogous to classical time evolution for theeharmonic oscillator.

4. The unitary displacement operator corresponds to the arbitrary external force on the harmonic oscillator and it seems that it would be selected in SSFR for the unitary evolution after SSFR. This means fixing the coefficients $C_k$ in the operator $\sum C_k L_k$.

What is the subjective "time" evolution operator when in the case of $SSA_n$?

1. The scaling analog of the unitary displacement operator $D$ as $D = \sum exp(\sum C_k L_k - \overline{C}_k L_{-k})$ is highly suggestive and would take the oscillator vacuum to a coherent state. Coefficients $C_k$ would be proportional to $\tau$. There would be a large number of choices for the unitary displacement operator. One can also consider complex values of $\tau$ since one has complexified $M^8$.

2. There should be a normalization for the coefficients: without this it is not possible to talk about a special value of $\tau$ does not make sense. For instance, the sum of their moduli squared could be equal to 1. This would give interpretation as a quantum state in the degrees of freedom considered. The width of the Gaussian would increase slowly during the unitary time evolution and be proportional to $log(T/T_0)$.

   The width of the Gaussian would increase slowly as a function of $T$ during the unitary time evolution and be proportional to $log(T/T_0)$. The condition that $c_k$ are proportional the same complex number times $\tau$ is too strong.

3. The arbitrariness in the choice of $C_k$ would bring in a kind of non-determinism as a selection of this superposition. The ability to engineer physical systems is in conflict with the determinism of classical physics and also difficult to understand in standard quantum physics. Could one interpret this choice as an analog for engineering a Hamiltonian as in say quantum computation or build-up of an electric circuit for some purpose? Could goal directed action correspond to this choice?

   If so engineerable degrees of freedom would correspond to intermediate degrees of freedom associated with $L_k$, $[m/2] \leq k \leq m$. They would be totally absent for $k = 1$ and this would correspond to a situation analogous to the standard physics without any intentional action.

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