Article

Bianchi type VI_0 Dark Energy Models with Constant Deceleration Parameter in Self-Creation Cosmology

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Abstract

We have constructed dark energy cosmological model in an anisotropic Bianchi type VI_0 space time with a variable equation of state (EoS) parameter ω in Barber's second self- creation theory of gravitation. The models are obtained using the special law of variation of Hubble's parameter that yields a constant value of deceleration parameter. Models have been derived using above conditions. Many physical and kinematical properties have also been discussed.

Keywords: Bianchi type VI_0, dark energy, deceleration, anisotropic, Hubble, parameter.

Introduction

Recent observations on expansion phenomenon of the universe indicate that the universe is currently experiencing a phase of accelerated expansion. This was first observed from high red shift supernova Ia (Riess et al. [1-2], Perlmutter et al. [3], Astier et al. [4], Spergel et al.[5] etc.) and same was confirmed by cross checks from the cosmic microwave background radiation (Bennett et al. [6], Abazajian et al.[7-9], Hawkins et al. [10] etc.). The current accelerating expansion of the universe attributed to the fact that our universe is dominated by an unknown dark energy an exotic energy with negative pressure. The simplest dark energy candidate is the vacuum energy density which is mathematically equivalent to the cosmological constant Λ. As per Copeland et al. [11] "fine tuning" and the cosmic "coincidence" are the two well-known difficulties of the cosmological constant problems.

There are several alternative theories for the dynamical DE scenario which have been proposed by scientists to interpret the accelerating universe. Wang and Tegmark [12] have shown that the universe is actually undergoing an acceleration with repulsive gravity of some strange energy-form i.e. DE at work. Dark energy is a mysterious substance with negative pressure and accounts for nearly 70% of total matter-energy of universe, but has no clear explanation. Karami et al. [13] introduced a polytropic gas model of DE as an alternative model to explain the accelerated expansion of the universe. Gupta and Pradhan [14] proposed a new candidate known as cosmological nuclear-energy as a possible candidate for the dark energy.

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Bianchi types I-IX cosmological models are important in the sense that these are homogeneous and anisotropic, from which the process of isotropization of the universe is studied through the passage of time. Moreover, from the theoretical point of view anisotropic universe have a greater generality than FRW isotropic models. The simplicity of the field equations made Bianchi space-times useful in constructing models of spatially homogeneous and anisotropic cosmologies. Considerable works have been done in obtaining various Bianchi type cosmological models and their inhomogeneous generalization. Bianchi type-VI_0 space-time is of special interest in anisotropic cosmology. Barrow [15] pointed out that Bianchi type-VI_0 models of the universe give a better explanation of some of the cosmological problems like primordial helium abundance and they also isotropize in a special sense.

Looking to the importance of Bianchi type-VI_0 universes, many authors [16-20] have studied it in different context. Shri Ram [21, 22] has presented Bianchi type-VI_0 cosmological models filled with dust and perfect fluid in modified Brans-Dicke theory respectively. Adhav et al. [23] studied Bianchi type-VI_0 cosmological models with anisotropic dark energy. Abdussattar and Prajapati [24] obtained a class of bouncing non-singular FRW models by constraining the deceleration parameter in the presence of an interacting dark energy represented by a time-varying cosmological constant. They have also discussed the role of deceleration parameter and interacting dark energy in singularity avoidance. Bisabr [25] has shown that an accelerating expansion is possible in a spatially flat universe for large values of the Brans-Dicke parameter consistent with the local gravity experiments. Yadav and Saha [26] studied DE models with variable equation of state (EoS) parameter.

Recently, Saha and Yadav [27] presented a general relativistic cosmological model with time-dependent DP in LRS Bianchi type-II space-time which can be described by isotopic and variable EoS parameter. Many researchers have made various attempts to find physical and geometrical properties of such cosmological models [28-33]. In this paper, We present general relativistic cosmological models with constant and time-dependent DP in Bianchi type-VI_0 space-time which can be described by isotropic constant and variable EoS parameters. This paper is organized as follows: We present the metric and field equations in Sect.2. In Sect.3, we obtain the solutions of the field equations representing Bianchi type-VI_0 cosmological models with perfect fluid by imposing the condition that the shear scalar is proportional to expansion scalar. We also discuss the physical and kinematical behaviors of the cosmological models with dark energy. Concluding remarks are given in Sect.4.

Metric and Field Equations

We consider Bianchi VI_0 metric in the form

$$ds^2 = -dt^2 + A^2(t)dx^2 + e^{-2qx}B^2(t)dy^2 + e^{2qx}C^2(t)dz^2 \tag{1}$$

where $A, B \text{ and } C$ are cosmic-scale functions and q is non-zero constant.
The Einstein's field equations are given by

$$R_i^j - \frac{1}{2}Rg_i^j = -8\pi\phi^{-1}T_i^j \tag{2}$$

$$\phi^{k}_{;k} = \frac{8\pi}{3}\eta T' \tag{3}$$

where $\phi^{k}_{;k}$ is invariant d'Alembertian and the contracted tensor T' is the trace of energy momentum tensor describing all non-gravitational and non-scalar field matter and energy. Here η is a coupling constant to be determined from experiments. Because of the homogeneity condition imposed by the metric, the scalar field ϕ will be a function of t only.

The simplest generalization of the EoS parameter of a perfect fluid may be determine the EoS parameter separately on each spatial axis by presenting the diagonal form of the energy momentum tensor.

Thus the energy momentum tensor T_i^j is given by

$$T^i_j = diag\left[T^1_1, T^2_2, T^3_3, T^4_4\right]$$

It can be parameterized as

$$T^i_j = diag\left[-p_x, -p_y, -p_z, \rho\right]$$

$$T^i_j = diag\left[-\omega_x, -\omega_y, -\omega_z, 1\right]\rho$$

$$T^i_j = diag\left[-\omega, -(\omega+\delta), -(\omega+\gamma), 1\right]\rho \tag{4}$$

where ρ is energy density of the fluid. p_x, p_y and p_z are the pressures and ω_x, ω_y and ω_z are the directional EoS parameters along x, y and z axes respectively. We have parameterized the deviation from isotropy by setting $\omega_x = \omega$, where ω is deviation free EoS parameter of the fluid and then introducing skewness parameter. δ and γ are deviation from ω along y and z axes respectively.

In a commoving coordinate system, field equations (2) and (3) for the metric (1) in case of (4), yield the following system of equations

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} + \frac{q^2}{A^2} = \frac{8\pi\rho\omega}{\phi} \tag{5}$$

$$\frac{C_{44}}{C} + \frac{A_{44}}{A} + \frac{A_4 C_4}{AC} - \frac{q^2}{A^2} = \frac{8\pi\rho(\omega+\delta)}{\phi} \tag{6}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} - \frac{q^2}{A^2} = \frac{8\pi\rho(\omega+\gamma)}{\phi} \tag{7}$$

$$\frac{A_4 B_4}{AB} + \frac{B_4 C_4}{BC} + \frac{A_4 C_4}{AC} - \frac{q^2}{A^2} = -\frac{8\pi\rho}{\phi} \tag{8}$$

$$\frac{B_4}{B} - \frac{C_4}{C} = 0 \tag{9}$$

$$\phi_{44} + \phi_4\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) = \frac{8\pi}{3}\lambda(3\omega+\delta+\gamma-1)\rho \tag{10}$$

where the sub-indices 4 denotes ordinary differentiation with respect to t.

Solutions of Field Equations

Integrating (9) we obtain

$$C = lB \tag{11}$$

where l is constant of integration.
From equations (6) and (11), we have

$$\frac{B_{44}}{B} + \frac{A_{44}}{A} + \frac{B_4 A_4}{BA} - \frac{q^2}{A^2} = \frac{8\pi\rho\left(\omega+\delta\right)}{\phi} \tag{12}$$

From equations (7) and (12), we obtain

$$\delta = \gamma \tag{13}$$

Hence skewness in y and z directions are equal.

Now the equations (5) to (10) reduce to

$$2\frac{B_{44}}{B} + \frac{B_4^2}{B^2} + \frac{q^2}{A^2} = \frac{8\pi\rho\omega}{\phi} \tag{14}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} - \frac{q^2}{A^2} = \frac{8\pi\rho\left(\omega+\gamma\right)}{\phi} \tag{15}$$

$$2\frac{A_4 B_4}{AB} + \frac{B_4^2}{B^2} - \frac{q^2}{A^2} = \frac{8\pi\rho}{\phi} \tag{16}$$

$$\phi_{44} + \phi_4\left(\frac{A_4}{A} + 2\frac{B_4}{B}\right) = \frac{8\pi}{3}\lambda\left(3\omega + 2\gamma - 1\right)\rho \tag{17}$$

Equations (14) to (17) are four independent equations in A, B, ρ, ω, ϕ and γ (six unknowns), hence we need two more conditions to solve these equations.

(i) Firstly, we apply the law of variation for Hubble parameter given by Berman(1983), which yields a constant value of deceleration parameter

$$H = kR^{-n}$$

or

$$H = k\left(ABC\right)^{-n/3} \tag{18}$$

where $k > 0$ and $n \geq 0$ are constants.
From equation (18) we have

$$\frac{1}{3}\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) = k\left(ABC\right)^{-n/3} \tag{19}$$

which on integration gives

$$ABC = \left(k_1 t + k_2\right)^{3/n} \text{ for } n \neq 0 \tag{20}$$

where k_1 and k_2 are constants of integration.

The deceleration parameter so obtained is

$$q = n - 1 \tag{21}$$

(ii) Secondly we assume expansion(θ) is proportional to shear(σ), which leads to

$$A = \beta B^m \tag{22}$$

From equations (11), (20) and (22) we get

$$A = \left(\frac{\beta}{l}\right)^{m/m+1} \left(k_1 t + k_2\right)^{3m/n(m+1)} \tag{23}$$

$$B = \left(\frac{1}{\beta l}\right)^{1/m+1} \left(k_1 t + k_2\right)^{3/n(m+1)} \tag{24}$$

$$C = l\left(\frac{1}{\beta l}\right)^{1/m+1} \left(k_1 t + k_2\right)^{3/n(m+1)} \tag{25}$$

Hence the metric (1) takes the form

$$ds^2 = -dt^2 + \left(\frac{\beta}{l}\right)^{2m/m+1} \left(k_1 t + k_2\right)^{6m/n(m+1)} dx^2 + e^{-2qx}\left(\frac{1}{\beta l}\right)^{2/m+1} \left(k_1 t + k_2\right)^{6/n(m+1)} dy^2 + e^{2qx} l^2 \left(\frac{1}{\beta l}\right)^{2/m+1} \left(k_1 t + k_2\right)^{6/n(m+1)} dz^2 \tag{26}$$

Using suitable transformations we can obtain the metric in the following form

$$ds^2 = -\frac{dT^2}{k_1^2} + \left(\frac{\beta}{l}\right)^{2m/m+1} T^{6m/n(m+1)} dX^2 + e^{-2qX}\left(\frac{1}{\beta l}\right)^{2/m+1} T^{6/n(m+1)} dY^2 + e^{2qX} l^2 \left(\frac{1}{\beta l}\right)^{2/m+1} T^{6/n(m+1)} dZ^2 \tag{27}$$

Where $T = k_1 t + k_2, X = x, Y = y, Z = z$

Physical and Kinematical Properties

The expressions of the Scalar Expansion (θ), Hubble parameter (H), Shear(σ) and average anisotropy parameter (A_m) for the model (27) are given as

$$\theta = 3H \tag{28}$$

or

$$\theta = \frac{3k_1(m+2)}{n(m+1)T} \tag{29}$$

Components of Hubble Parameter are

$$H_1 = \frac{A_4}{A} = \frac{3k_1 m}{n(m+1)T} \tag{30}$$

$$H_2 = \frac{B_4}{B} = \frac{3k_1}{n(m+1)T} \tag{31}$$

$$H_3 = \frac{C_4}{C} = \frac{3k_1}{n(m+1)T} \tag{32}$$

And Hubble parameter is obtained as

$$H = \frac{k_1(m+2)}{n(m+1)T} \tag{33}$$

Shear (σ^2) is given as

$$\sigma^2 = \frac{1}{4}\left[\frac{9k_1^2 m(m-4)}{n^2(m+1)^2 T^2}\right] \tag{34}$$

Anisotropy parameter is obtained as

$$A_m = \frac{1}{3}\left[\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2\right] \tag{35}$$

For the model (27) it is given as

$$A_m = \frac{1}{27}\left(3m^2 - 8m + 6\right) \tag{36}$$

From equation (16) we have

$$\frac{\rho}{\phi} = -\frac{1}{8\pi}\left[\frac{2A_4B_4}{AB} + \frac{B_4^2}{B^2} - \frac{q^2}{A^2}\right] \tag{37}$$

Or

$$\frac{\rho}{\phi} = -\frac{1}{8\pi}\left[\frac{18k_1^2 m}{n^2(m+1)^2T^2} + \frac{9k_1^2}{n^2(m+1)^2T^2} - \frac{q^2}{\left(\frac{\beta}{l}\right)^{2m/m+1} T^{6m/n(m+1)}}\right] \tag{38}$$

From equation (14), we get

$$\omega = -\left[\frac{\frac{6k_1^2(3-nm-n)}{n^2(m+1)^2T^2} + \frac{9k_1^2}{n^2(m+1)^2T^2} + \frac{q^2}{\left(\frac{\beta}{l}\right)^{2m/m+1} T^{6m/n(m+1)}}}{\frac{18k_1^2 m}{n^2(m+1)^2T^2} + \frac{9k_1^2}{n^2(m+1)^2T^2} - \frac{q^2}{\left(\frac{\beta}{l}\right)^{2m/m+1} T^{6m/n(m+1)}}}\right] \tag{39}$$

From equation (15), we obtain

$$\gamma = \frac{\left[\frac{B_{44}}{B} - \frac{A_{44}}{A} - \frac{A_4 B_4}{AB} + \frac{B_4^2}{B^2} + 2\frac{q^2}{A^2}\right]}{\left[\frac{2A_4 B_4}{AB} + \frac{B_4^2}{B^2} - \frac{q^2}{A^2}\right]} \tag{40}$$

or

$$\gamma = \left[\frac{\frac{6k_1^2(3-nm-n)}{n^2(m+1)^2 T^2} - \frac{3k_1^2 m(n+nm-9m)}{n^2(m+1)^2 T^2} + \frac{9k_1^2 m}{n^2(m+1)^2 T^2} - \frac{q^2}{\left(\frac{\beta}{l}\right)^{2m/m+1} T^{6m/n(m+1)}}}{\frac{18k_1^2 m}{n^2(m+1)^2 T^2} + \frac{9k_1^2}{n^2(m+1)^2 T^2} - \frac{q^2}{\left(\frac{\beta}{l}\right)^{2m/m+1} T^{6m/n(m+1)}}}\right] \tag{41}$$

From equation (13), we get

$$\delta = \left[\frac{\frac{6k_1^2(3-nm-n)}{n^2(m+1)^2 T^2} - \frac{3k_1^2 m(n+nm-9m)}{n^2(m+1)^2 T^2} + \frac{9k_1^2 m}{n^2(m+1)^2 T^2} - \frac{q^2}{\left(\frac{\beta}{l}\right)^{2m/m+1} T^{6m/n(m+1)}}}{\frac{18k_1^2 m}{n^2(m+1)^2 T^2} + \frac{9k_1^2}{n^2(m+1)^2 T^2} - \frac{q^2}{\left(\frac{\beta}{l}\right)^{2m/m+1} T^{6m/n(m+1)}}}\right] \tag{42}$$

From equations (17), (38) and (39) and using the condition $n = \frac{6m}{2(2m+1)}$, we get

$$T^2\phi_{44} + Tp_1\phi_4 - p_2\phi = 0 \tag{43}$$

where

$p_1 = \frac{3(m+2)k_1}{n(m+1)}$ and

$$p_2 = \frac{3k_1^2(3-nm-n)}{n^2(m+1)^2} - \frac{3k_1^2 m(n+nm-9m)}{n^2(m+1)^2} + \frac{18k_1^2 m}{n^2(m+1)^2} + \frac{9k_1^2}{n^2(m+1)^2} - \frac{q^2}{\left(\frac{\beta}{l}\right)^{2m/m+1}}$$

On solving equation (43) we get

$$\phi = c_1 T^{\frac{1}{2}\left(-i\sqrt{p_1}\sqrt{-\frac{(p_1-1)^2}{p_2}-4} - p_1 + 1\right)} + c_2 T^{\frac{1}{2}\left(i\sqrt{p_1}\sqrt{-\frac{(p_1-1)^2}{p_2}-4} - p_1 + 1\right)} \tag{44}$$

Energy density (ρ) is given as

$$\rho = -\frac{1}{8\pi}\left[\frac{18k_1^2 m}{n^2(m+1)^2 T^2} + \frac{9k_1^2}{n^2(m+1)^2 T^2} - \frac{q^2}{\left(\frac{\beta}{l}\right)^{2m/m+1} T^{6m/n(m+1)}}\right] \mathrm{X} \tag{45}$$

$$\left[c_1 T^{\frac{1}{2}\left(-i\sqrt{p_1}\sqrt{-\frac{(p_1-1)^2}{p_2}-4}-p_1+1\right)} + c_2 T^{\frac{1}{2}\left(i\sqrt{p_1}\sqrt{-\frac{(p_1-1)^2}{p_2}-4}-p_1+1\right)}\right]$$

The spatial volume (V) is evaluated as

$$V = R^3 = ABC = \frac{1}{(\beta l)^{\frac{m}{3(m+1)}}} T^{m+2/n(m+1)} \tag{46}$$

Hence we can observe that $V \to 0 \ as\, T \to 0$ and $V \to \infty \ as\ T \to \infty$ with $\frac{m+2}{n(m+1)} > 0$ but $V \to \infty\, as\, T \to 0 \ \ and \ \ V \to 0 \ as\, T \to \infty$ with $\frac{m+2}{n(m+1)} < 0$.

Now

$$\frac{\sigma}{\theta} = \frac{1}{2}\left[\frac{m\sqrt{m-4}}{m+2}\right] \tag{47}$$

Jerk parameter $\{j(t)\}$ for the model (27) is given as

$$j(t) = \frac{1}{H^3}\frac{\dddot{R}}{R} \tag{48}$$

or

$$j(t) = \frac{[(m+2) - n(m+1)][(m+2) - 2n(m+1)]n(m+1)}{(m+2)^3} T^{3-3n(m+1)} \tag{49}$$

Conclusion

In this paper we have presented Bianchi type VI_0 Dark Energy Models with Constant Deceleration Parameter in Self Creation Cosmology. The model obtained in this paper is useful in self-creation cosmology to study the large scale dynamics of physical space.

It is also observed that when $T \to 0$ the physical quantities for the model (27) like scalar expansion(θ), shear scalar(σ) and Hubble parameter (H) diverge for $m \neq -1, n \neq 0$. Thus the model (27) starts with a big-bang at T=0. The scalar field (ϕ) is given by equation (44). we can observe that $V \to 0 \ as\, T \to 0$ and $V \to \infty \ as\ T \to \infty$ with $\frac{m+2}{n(m+1)} > 0$ but

$V \to \infty \; as \, T \to 0 \quad and \quad V \to 0 \; as \, T \to \infty$ with $\frac{m+2}{n(m+1)} < 0$. Anisotropy Parameter is obtained is constant for the model (27). The model (27) has a POINT TYPE singularity at T=0 when $m \neq -1, n \neq 0$.

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