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Bianchi Type I Cosmological Model with Variable Λ Terms in *f* (*R*, *T*) Gravity

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Abstract

Bianchi type-I cosmological model $in\, f(R,T)$ theory of gravity proposed by Harko *et. al.* (Phys. Rev. D 84:024020, 2011) have studied. We have explore that the universe starts with a big-bang at initial time and become isotropic at late times. Also we obtain physical and geometrical interpretation of the model.

Keywords: Bianchi type I, cosmological constant, universe, big bang, isotropic, variable.

1. Introduction

Now a day the cosmological constant problem is very interesting to researchers. The cosmological constant was originally given by Einstein in his field equations. To resolve the problem of the huge difference between the effective cosmological constant observed today and vacuum energy density predicted by quantum field theory, several mechanism have been proceed [1]. A possible way is to consider a varying cosmological term. Due to the coupling of the dynamic degree of freedom with the matter fields of the universe, Λ relaxes to its present small value through the expansion of the universe and creation of particles. From this point of view, the constant is small because the universe is so old.

Models with a dynamically decaying cosmological term representing the energy density of vacuum have been studied by several authors [2-4]. The observational and theoretical features suggest that the most natural candidate for the missing energy is the vacuum energy density or the cosmological constant Λ but selection of the cosmological constant as vacuum energy faces a serious fine-tuning problem, which demands that the value of Λ must be 120 orders of magnitude greater than its presently observed value.

One of the interesting and prospective version of modified gravity theories is the $f(R,T)$ gravity proposed by Harko et al. [5], where the gravitational lagrangian is given by an arbitrary function of the Ricci scalar R and of the trace of the stress-energy tensor T. They have obtained the

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gravitational field equations in the metric formalism, as well as, the equation of motion for test particles, which follow from the covariant divergence of the stress-energy tensor. The $f(R,T)$ gravity models can explain the time cosmic accelerated expansion of the universe. Recently, Chaubey and Sukla [6], Adhav [7], Samanta [8], Reddy et al. [9-11] have studied cosmological model in $f(R,T)$ gravity in different Bianchi type space-time.

In recent year Bianchi universes are playing important role in observational cosmology, since the WMAP data [12-14] seem to require an addition to the standard cosmological model with positive cosmological constant that bears a likeness to the Bianchi morphology [15-20]. According to this, the universe should reach to a slightly anisotropic special geometry in spite of the inflation, contrary to generic inflationary models [21-27] suggesting a non-trivial isotropization history of universe due to the presence of an anisotropic energy source. In order to explain the homogeneity and flatness of the presently observed universe, it is usually assumed that this undergone a period of exponential expansion [21, 23-25]. Mostly the expansion of the universe is described within the framework of the homogeneous and isotropic Friedman-Robertson-Walker (FRW) cosmology.

The reasons for this are purely technical. The simplicity of the field equations and the existence of analytical solutions in most of the cases have justified this over simplification for the geometry of space-time. However, there are no compelling physical reasons to assume the former before the inflationary period. To drop the assumption of homogeneity would make the problem intractable, which the isotropy of the space is something that can be relaxed and leads to anisotropy. Several authors [28-33] have studied particular case of anisotropic models and found that the scenario predicted by FRW model stand essentially unchanged even when large anisotropies were present before the inflationary period.

In this paper, we investigate the modified $f(R,T)$ gravity theory in a Bianchi type-I cosmology by considering variable Λ. For a specific choice of $f(R,T) = R + 2f(T)$, where $f(T) = -\lambda T$, λ an arbitrary constant, some exact solutions of the field equations have been generated explicitly. In sect.2, we give a basic formalism of $f(R,T)$ gravity, the field equations are presented in sect.3, where as the solutions are derived and discussed in sect.4, and finally follows the conclusion in sect. 5.

2. The basic formalism of $f(R,T)$ gravity

The $f(R,T)$ gravity [5] is a more generic extended theory of $f(R)$ gravity or more precisely general relativity which explains the coupling between matter and geometry in the Universe. The formalism of $f(R,T)$ model depends on a source term which is a function of Lagrangian matter density L_m. The action of $f(R,T)$ gravity is given by:

$$S = \frac{-1}{16\pi G}\int f(R,T)\sqrt{-g}\, d^4x + \int L_m \sqrt{-g} d^4x \tag{1}$$

where $f(R,T)$ is an arbitrary function of the Ricci Scalar R and the trace T of the energy-momentum tensor $T_{\mu\nu}$ i.e. $(T = g^{\mu\nu}T_{\mu\nu})$, and L_m corresponds to the matter Lagrangian density and g is the determinate of metric tensor $g_{\mu\nu}$.

Now using gravitational units $(c = 1)$ and the field equations of $f(R,T)$ gravity by varying the action (1) with respect to metric tensor have the form

$$f_R(R,T)R_{\mu\nu} - \frac{1}{2}f(R,T)g_{\mu\nu} + \left(g_{\mu\nu} \boxdot - \nabla_\mu\nabla_\nu\right)f_R(R,T)$$
$$= -8\pi G T_{\mu\nu} - f_T(R,T)T_{\mu\nu} - f_T(R,T)\Theta_{\mu\nu} \tag{2}$$

where $f_R = \frac{\partial f(R,T)}{\partial R}$, $f_T = \frac{\partial f(R,T)}{\partial T}$, $\boxdot = \nabla^\mu\nabla_\mu$

∇_μ is the covariant derirative and $f_R = f_R(R,T), f_T = f_T(R,T)$ and $R_{\mu\nu}$ is the Ricci tensor, and $T_{\mu\nu}$ is the energy momentum tensor given by

$$T_{\mu\nu} = \frac{-2}{\sqrt{-g}}\frac{\delta\left(\sqrt{-g}L_m\right)}{\delta g^{\mu\nu}} \tag{3}$$

In (2) where f_R and f_T denote the derivatives of $f(R,T)$ with respect to R and T respectively, ∇_μ is covariant derivative, $\boxdot = \nabla^\mu\nabla_\mu$ is the D' Alembert operator and $\Theta_{\mu\nu}$ is defined by

$$\Theta_{\mu\nu} = g^{\alpha\beta}\frac{\delta T_{\alpha\beta}}{\delta g^{\mu\nu}} \tag{4}$$

Using (3) and (4), we obtain

$$\Theta_{\mu\nu} = -2T_{\mu\nu} + g_{\mu\nu}L_m - 2g^{\alpha\beta}\frac{\partial^2 L_m}{\partial g^{\mu\nu}\partial g^{\alpha\beta}} \tag{5}$$

Contraction of equation (2)

$$f_R(R,T)R + 3 \boxdot f_R(R,T) - 2f(R,T)$$
$$= \{-8\pi G - f_T(R,T)\}T - f_T(R,T)\Theta \tag{6}$$

where $\Theta = g^{\mu\nu}\Theta_{\mu\nu}$. If we assume that the matter Lagrangian density L_m depends only on the matric tensor components $g_{\mu\nu}$ rather then its derivatives, then (3) reduces to the form

$$T_{\mu\nu} = g_{\mu\nu}L_m - 2\frac{\partial L_m}{\partial g^{\mu\nu}} \tag{7}$$

For a perfect fluid distribution, the energy-momentum tensor of the matter has the form

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + p g_{\mu\nu} \tag{8}$$

where ρ and p are the energy density and the pressure of the fluid respectively. Here u^{μ} is the four velocity vector satisfying $u^{\mu}u_{\mu} = -1$ and $u^{\mu}\nabla_{\mu}\nabla_{\nu} = 0$. Now using the fact that $L_m = -p$, (5) can be written as

$$\Theta_{\mu\nu} = -pg_{\mu\nu} - 2T_{\mu\nu} \tag{9}$$

The field equations (2) take the form

$$f_R(R,T)R_{\mu\nu} - \frac{1}{2}f(R,T)g_{\mu\nu} - (\nabla_\mu\nabla_\nu - \Box\, g_{\mu\nu})f_R(R,T) = -8\pi G T_{\mu\nu} + f_T(R,T)(T_{\mu\nu} + pg_{\mu\nu}) \tag{10}$$

We would like to mention here that Harko et al. [5] have considered three possible forms of the functional $f(R,T)$ as

$$f(R,T) = \begin{cases} R + 2f(T) \\ f_1(R) + f_2(T) \\ f_1(R) + f_2(R)f_3(T) \end{cases} \tag{11}$$

In the present study, we shall concentrate on the first form of $f(R,T)$, i.e., $f(R,T) = R + 2f(T)$ and choose $f(T) = -\lambda T$, where λ is an arbitrary constant. For this consideration, equation (10), becomes

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = -(8\pi G + 2\,\lambda)T_{\mu\nu} + \lambda(-T - 2p)g_{\mu\nu} \tag{12}$$

A comparison of (12) with Einstein's field equations

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = -8\pi G T_{\mu\nu} + \Lambda g_{\mu\nu} \tag{13}$$

Yields $\Lambda = \Lambda(T) = -\lambda(T + 2p)$ and $-8\pi G = -(8\pi G + 2\,\lambda)$. Therefore in $f(R,T)$ theory of gravity, the field equations with G and Λ can be expressed as

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = -(8\pi G + 2\,\lambda)T_{\mu\nu} + \Lambda g_{\mu\nu} \tag{14}$$

It can clearly be seen from equation (14), which follows from equation (12), that the usual energy conservation law does not hold in $f(R,T)$ theory. The left side of equation (14) has zero divergence through the Bianchi identities. Hence the right side must also have zero divergence. This leads to the following equation

$$-8\pi G_{,\mu}(3p-\rho)+(-8\pi G+2\,\lambda)T^{\nu}_{\mu;\nu}+\Lambda_{,\mu}=0$$

From this equation, it may be seen that the usual conservation law $T^{\nu}_{\mu};\nu=0$ can hold in $f(R,T)$ theory, providing that

$$8\pi G,\mu(3p-\rho)-\Lambda_{,\mu}=0$$

We shall return to this point later. Several authors, [34, 35] have pointed out that in $f(R,T)$ theory, it is possible to either have the usual energy-momentum conservation law or not.

3. Line Element and Field Equations

The gravitational field for a spatially homogeneous and anisotropic Bianchi type-I space- time is given by the line element

$$ds^2=A^2dx^2+B^2dy^2+C^2dz^2-dt^2 \tag{15}$$

Where A, B, C are metric functions of cosmic time t only.
we have considered the equation of state of the form

$$p=\omega\rho \tag{16}$$

For the Bianchi type- I space then the equation (15) and the field equations (14) in $f(R,T)$ gravity for $8\pi G=1$ yield the following dynamical equations

$$\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{B}\dot{C}}{BC}=\Lambda-(1+2\,\lambda)\mathrm{p} \tag{17}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{C}}{AC}=\Lambda-(1+2\,\lambda)\mathrm{p} \tag{18}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}=\Lambda-(1+2\,\lambda)\mathrm{p} \tag{19}$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{A}\dot{C}}{AC}=\Lambda+(1+2\,\lambda)\rho \tag{20}$$

where dot $(\cdot)$ indicates the derivative with respect to cosmic time t.

The spatial volume (V), average scale factor (a) and the generalized Hubble parameter (H) for the Bianchi-I universe are given by

$$V = ABC \tag{21}$$

$$a = (ABC)^{\frac{1}{3}} = V^{\frac{1}{3}} \tag{22}$$

$$H = \frac{1}{3}\left(H_x + H_y + H_z \right) \tag{23}$$

The directional Hubble parameters in the x, y *and* z respectively may be defined as

$$H_x = \frac{\dot{A}}{A},\ H_y = \frac{\dot{B}}{B},\ and\ H_z = \frac{\dot{C}}{C}$$

Equation (22) and (23) provide us with an important relation

$$H = \frac{\dot{a}}{a} = \frac{1}{3}(H_x + H_y + H_z) \tag{24}$$

The expressions for the expansion scalar (θ) and shear scalar (σ) can be introduced as

$$\theta = U^{\mu}_{;\mu} \tag{25}$$

And

$$\sigma^2 = \frac{1}{2}\sigma_{\mu\nu}\sigma^{\mu\nu} \tag{26}$$

Where $\sigma^{\mu\nu}$ is the usual shear tensor. For the Bianchi-I metric, the expressions for θ and σ takes the form

$$\theta = 3\frac{\dot{a}}{a} \tag{27}$$

And

$$\sigma = \frac{\sqrt{k}}{\sqrt{3}a^3} \tag{28}$$

where k is constant.

From equations (17) and (18) we have

$$\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} - \frac{\dot{C}}{C}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) = 0$$

which, can be written as

$$\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)^{\cdot} + \left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)\frac{(ABC)^{\cdot}}{ABC} = 0$$

This integrates to

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = \frac{k_1}{ABC}$$

This can be written, from equation (22), as

$$\frac{\dot{A}}{A}-\frac{\dot{B}}{B}=\frac{k_1}{a^3} \tag{29}$$

A similar calculation with equations (18) and (19) leads to

$$\frac{\dot{B}}{B}-\frac{\dot{C}}{C}=\frac{k_2}{a^3} \tag{30}$$

Equations (29) and (30) integrate to give

$$A=m_1 a\, exp\left[\frac{2k_1+k_2}{3}\int\frac{dt}{a^3}\right] \tag{31}$$

$$B=m_2 a\, exp\left[\frac{k_2-k_1}{3}\int\frac{dt}{a^3}\right] \tag{32}$$

$$C=m_3 a\, exp\left[-\frac{k_1+2k_2}{3}\int\frac{dt}{a^3}\right] \tag{33}$$

Where k_1, k_2 and m_1, m_2, m_3 are arbitrary constants of integration satisfying $m_1 m_2 m_3 = 1$. We define the anisotropy parameter $(\overline{A})$ and deceleration parameter(q) for the model as

$$\overline{A}=\frac{1}{3}\sum_{i=3}^{3}\left(\frac{H_i-H}{H}\right)^2 \tag{34}$$

and

$$q=-\frac{a\ddot{a}}{\dot{a}^2}=-\left(\frac{\dot{H}+H^2}{H^2}\right) \tag{35}$$

The above equations (17)-(20) can also be written as

$$3H^2=(1+2\lambda)\rho+\sigma^2+\Lambda \tag{36}$$

$$H^2(2q-1)=(1+2\lambda)p+\sigma^2-\Lambda \tag{37}$$

4. Solution of field equations

Now equations (16)-(20) are a system of five independent equations in six unknowns, $viz, A, B, C, \rho, p\ and\ \Lambda$, respectively. To obtain the exact solution we need one extra condition. The phenomenological Λ decay scenarios have been considered by a number of authors. Chen & Wu [36] considered $\Lambda \propto a^{-2}$ (a is the scale factor of the Robertson-Walker metric). Hoyle et al. [37] considered $\Lambda \propto a^{-3}$ while $\Lambda \propto a^{-m}$ (a is a scale factor and m is a constant) was considered by Olson & Jordan [38]; Pavon [39]; Maia & Silva [34]; Silveira & Waga [41, 42] and Bloomfield Torres & Waga [43]; Overduin & Cooperstock [44]; have been explained by

$$\Lambda \propto aH\frac{dH}{da}$$

it means $$\Lambda = \alpha a H\frac{dH}{da} \tag{38}$$

where α is constant.

By using equation (16) in equation (36) and (37), we obtain

$$2\dot{H} + 3(1+\omega)H^2 + (1-\omega)\sigma^2 - (1+\omega)\Lambda = 0 \tag{39}$$

Substituting the value of σ and Λ from equation (28) and (38) in equation (39), we obtain

$$\dot{H}[2-\alpha(1+\omega)] + 3(\omega+1)H^2 + \frac{k(1-\omega)}{3a^6} = 0 \tag{40}$$

on integrating

$$H = \frac{\sqrt{\frac{k(\omega-1)}{[\omega-1+\alpha(1+\omega)]}}}{3\left[\sqrt{\frac{k(\omega-1)}{[\omega-1+\alpha(1+\omega)]}}t+c\right]} \tag{41}$$

Again integrating equation (41), we get

$$a = \left[\sqrt{\frac{k(\omega-1)}{[\omega-1+\alpha(1+\omega)]}}\,t + c\right]^{\frac{1}{3}} \tag{42}$$

where c is constant of integration. The solutions of metric potentials for the above value of scale factor are

$$A = m_1\left[\sqrt{\frac{k(\omega-1)}{[\omega-1+\alpha(1+\omega)]}}\,t + c\right]^{\frac{1}{3}}\left(c\sqrt{\frac{k(\omega-1)}{[\omega-1+\alpha(1+\omega)]}}\,t\right)^{\frac{(2k_1+k_2)}{3\sqrt{\frac{k(\omega-1)}{[\omega-1+\alpha(1+\omega)]}}}} \tag{43}$$

$$B = m_2\left[\sqrt{\frac{k(\omega-1)}{[\omega-1+\alpha(1+\omega)]}}\,t + c\right]^{\frac{1}{3}}\left(c\sqrt{\frac{k(\omega-1)}{[\omega-1+\alpha(1+\omega)]}}\,t\right)^{\frac{(k_2-k_1)}{3\sqrt{\frac{k(\omega-1)}{[\omega-1+\alpha(1+\omega)]}}}} \tag{44}$$

$$C = m_3\left[\sqrt{\frac{k(\omega-1)}{[\omega-1+\alpha(1+\omega)]}}\,t + c\right]^{\frac{1}{3}}\left(c\sqrt{\frac{k(\omega-1)}{[\omega-1+\alpha(1+\omega)]}}\,t\right)^{\frac{-(k_1+2k_2)}{3\sqrt{\frac{k(\omega-1)}{[\omega-1+\alpha(1+\omega)]}}}} \tag{45}$$

The directional Hubble parameters $H_x, H_y, and\ H_z$ are given by

$$H_x = \frac{\sqrt{\frac{k(\omega-1)}{[\omega-1+\alpha(1+\omega)]}}}{3\left[\sqrt{\frac{k(\omega-1)}{[\omega-1+\alpha(1+\omega)]}}t+c\right]} + \frac{(2k_1+k_2)}{3\left[\sqrt{\frac{k(\omega-1)}{[\omega-1+\alpha(1+\omega)]}}t+c\right]} \tag{46}$$

$$H_y = \frac{\sqrt{\frac{k(\omega-1)}{[\omega-1+\alpha(1+\omega)]}}}{3\left[\sqrt{\frac{k(\omega-1)}{[\omega-1+\alpha(1+\omega)]}}t+c\right]} + \frac{(k_2-k_1)}{3\left[\sqrt{\frac{k(\omega-1)}{[\omega-1+\alpha(1+\omega)]}}t+c\right]} \tag{47}$$

$$H_z = \frac{\sqrt{\frac{k(\omega-1)}{[\omega-1+\alpha(1+\omega)]}}}{3\left[\sqrt{\frac{k(\omega-1)}{[\omega-1+\alpha(1+\omega)]}}t+c\right]} - \frac{(k_1+2k_2)}{3\left[\sqrt{\frac{k(\omega-1)}{[\omega-1+\alpha(1+\omega)]}}t+c\right]} \tag{48}$$

The spatial volume V, expansion scalar θ, shear σ^2 and deceleration parameter q takes the form

$$V = \left[\sqrt{\frac{k(\omega-1)}{[\omega-1+\alpha(1+\omega)]}}t + c\right] \tag{49}$$

$$\theta = 3H = \frac{\sqrt{\frac{k(\omega-1)}{[\omega-1+\alpha(1+\omega)]}}}{\left[\sqrt{\frac{k(\omega-1)}{[\omega-1+\alpha(1+\omega)]}}t+c\right]} \tag{50}$$

$$\sigma^2 = \frac{k^2}{\left[\sqrt{\frac{k(\omega-1)}{[\omega-1+\alpha(1+\omega)]}}t+c\right]^2} \tag{51}$$

$$q = 2 \tag{52}$$

$$\rho = \frac{1}{\lambda(1+\omega)}\left[\frac{k(\omega-1)}{3[\omega-1+\alpha(1+\omega)\left[\sqrt{\frac{k(\omega-1)}{[\omega-1+\alpha(1+\omega)]}}t+c\right]^2} - \frac{\left(k_1^2+k_2^2+k_1k_2\right)}{3\left[\sqrt{\frac{k(\omega-1)}{[\omega-1+\alpha(1+\omega)]}}t+c\right]^2}\right] \tag{54}$$

$$p = \frac{\omega}{\lambda(1+\omega)}\left[\frac{k(\omega-1)}{3[\omega-1+\alpha(1+\omega)\left[\sqrt{\frac{k(\omega-1)}{[\omega-1+\alpha(1+\omega)]}}t+c\right]^2} - \frac{\left(k_1^2+k_2^2+k_1k_2\right)}{3\left[\sqrt{\frac{k(\omega-1)}{[\omega-1+\alpha(1+\omega)]}}t+c\right]^2}\right] \tag{55}$$

The cosmological parameter Λ

$$\Lambda = \frac{-3\alpha k(\omega-1)}{[\omega-1+\alpha(1+\omega)\left[\sqrt{\frac{k(\omega-1)}{[\omega-1+\alpha(1+\omega)]}}t+c\right]^2} \tag{56}$$

The critical density ρ_c

$$\rho_c = \frac{k(\omega-1)}{3[\omega-1+\alpha(1+\omega)\left[\sqrt{\frac{k(\omega-1)}{[\omega-1+\alpha(1+\omega)]}}t+c\right]^2} \tag{57}$$

$$\Omega = 2 - \frac{[\omega-1+\alpha(1+\omega)]}{3(\omega-1)} \tag{58}$$

$$\rho_\Lambda = \frac{-3\alpha k(\omega-1)}{[\omega-1+\alpha(1+\omega)\Big[\sqrt{\frac{k(\omega-1)}{[\omega-1+\alpha(1+\omega)]}}t+c\Big]^2} \qquad (59)$$

We have seen that at initially pressure , energy density,expansion scalar and cosmological constant are infinite and all become zero asymptotically .The model becomes isotropic at late times.

5. Conclusion

Bianchi type-I cosmological model $in\, f(R,T)$ theory of gravity have studied . We have explore that the universe starts with a big-bang at initial time and become isotropic at late times. All the cosmological parameters posses the behavior that are very closed to recent observations.

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