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Bulk Viscous Cosmological Model in *f*(*T*) Gravity by Hybrid Expansion Law

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Abstract

In this paper, the author studies plane symmetric cosmological model filled with bulk viscosity in the framework of Teleparallel Gravity by using Hybrid Expansion Law. Using depiction model of *f*(*T*) gravity, the behavior of accelerating universe is discussed. The physical and kinematical properties of the derived model have been discussed in detail.

Keywords: Plane symmetric, *f*(*T*) gravity, bulk viscous, hybrid expansion, cosmological model.

1. Introduction

Recent astrophysical data [1-4] strongly indicate that the universe is accelerating at present. A new kind of matter with positive energy density and with negative pressure dubbed as dark energy is one of the main candidate behind the acceleration of the present day universe. Dark energy occupies about 73% of the energy of our Universe; while dark matter about 23% and the usual baryonic matter 4%. On larger cosmological scales the modification in Einstein-Hilbert action may be an accurate description of a late time cosmic acceleration of the expanding universe. Among the various modifications of Einstein's theory, another one way to look at the theory beyond General Theory of Relativity (GTR) is the Teleparallel Gravity (TG) which is different from GTR (i.e. uses the Weitzenbock connection in place of the LeviCivita connection) which has no curvature but has torsion, responsible for the acceleration of the universe. Ferraro & Fiorini [5] provided models based on modified TG to inflation. In $f(T)$ gravity, the Teleparallel Lagrangian density described by the function of torsion scalar T in order to account for the late time cosmic acceleration [6-9] . Jamil et al. [10] resolved the Dark Matter (DM) problem in the light of $f(T)$ gravity. Setare and Houndjo [11] investigated particle creation in flat Friedman Robertson Walker universe in the framework of $f(T)$ gravity. $f(T)$ gravity has been extensively studied in the literature by several eminent researchers [12-19].

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Bulk viscosity is the simplest way to study entropy in cosmology. It arises any time a fluid expands rapidly and ceases to be in thermodynamic equilibrium. Eckart [20] developed the first relativistic theory of non-equilibrium thermodynamics to study the effect of bulk viscosity. Krori and Mukherjee [21] explored the evolution of Bianchi cosmologies with bulk viscosity and particle creation. Desikan [22] studied the effect of bulk viscosity for FRW models .Several Relativists considered the behavior of Bulk viscosity in different context [23 -36].

Motivated by above investigations, the paper deals with the investigations of bulk viscous for depiction model of $f(T)$ model by using plane symmetric universe with Hybrid expansion law .

2. Basic set up of $f(T)$ Gravity

Defining the action by generalizing the TG i.e. $f(T)$ theory as

$$S = \int \left[T + f(T) + L_{matter} \right] e\, d^4 x \quad . \tag{1}$$

Here, $f(T)$ denotes an algebraic function of the torsion scalar *T*. Making the functional variation of the action (1) with respect to the tetrads, we get the following equations of motion

$$S_{\mu}^{\nu\rho} \partial_{\rho} T f_{TT} + \left[e^{-1} e_{\mu}^{i} \partial_{\rho} \left(e e_{i}^{\alpha} S_{\alpha}^{\nu\rho} \right) + T^{\alpha}{}_{\lambda\mu} S_{\alpha}^{\nu\lambda} \right] \left(1 + f_T \right) + \frac{1}{4} \delta_{\mu}^{\nu} \left(T + f \right) = k^2 T_{\mu}^{\nu} , \tag{2}$$

The field equation (10) is written in terms of the tetrad and partial derivatives and appears very different from Einstein's equations. T_{μ}^{ν} is the energy momentum tensor, $f_T = \frac{df(T)}{dT}$ and by setting $f(T) = a_0 =$ constant this is dynamically equivalent to the General Relativity.

3. Metric and its field equations

The line element of Plane Symmetric space-time is given by

$$ds^2 = dt^2 - A^2 \left(dx^2 + dy^2 \right) - B^2 dz^2 , \tag{3}$$

where the metric potentials A and B be the functions of time *t* only.

The corresponding Torsion scalar is given by

$$T = -2 \left(2 \frac{\dot{A}}{A} \frac{\dot{B}}{B} + \frac{\dot{B}^2}{B^2} \right) . \tag{4}$$

The energy-momentum tensor T_{ij} for a Bulk Viscous fluid distribution is given by

$$T_{ij} = (\rho + \overline{p}) u_i u_j - \overline{p} g_{ij} , \tag{5}$$

where

$$\overline{p} = p - \eta u^i_{;i} = p - 3\eta H \,, \tag{6}$$

is the effective pressure , η is the coefficient of bulk viscosity , p is the isotropic pressure , ρ is the energy density , $3\eta H$ is usually known as bulk viscous pressure , H is Hubble's parameter and u^i is fluid four-velocity vector satisfying

$$u_i u^i = 1 \,. \tag{7}$$

In the co-moving co-ordinate system, we have from the above equations

$$T_1^1 = T_2^2 = T_3^3 = -\overline{p} \;\;,\;\; T_4^4 = \rho \,, T_i^j = 0 \,, i \neq j \,. \tag{8}$$

Using equations (3) and (8), the field equations of the Teleparallel gravity can be written as

$$\left(T + f(T)\right) - 4\left(1 + f(T)\right)\left\{\frac{\dot{A}^2}{A^2} + 2\frac{\dot{A}}{A}\frac{\dot{B}}{B}\right\} = 2k^2\rho, \tag{9}$$

$$\begin{aligned} &4\left\{\frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} + \frac{\dot{A}}{A}\frac{\dot{B}}{B}\right\}\left(1 + f(T)\right) \\ &-16\frac{\dot{A}}{A}\left[\frac{\dot{A}}{A}\left(\frac{\ddot{B}}{B} - \frac{\dot{B}^2}{B^2}\right) + \left(\frac{\ddot{A}}{A} - \frac{\dot{A}^2}{A^2}\right)\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right)\right] f_{TT} - (T + f(T)) = -2k^2\overline{p} \end{aligned}, \tag{10}$$

where the overhead dot($\cdot$) denotes the derivative with respect to time t.

4. Solutions of the field equations

Here there are two highly non-linear differential equations with eight unknowns namely $A, B, f(T), p, \rho, T, \overline{p}, \eta$. The system is thus initially undetermined. Thus there is a need of extra physical conditions to solve the field equations completely.

The spatial volume for Plane Symmetric space-time is given by

$$V = a^3 = A^2 B \,. \tag{8}$$

The expansion scalar:

$$\theta = 3H \,. \tag{9}$$

The mean anisotropy parameter:

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2 . \tag{10}$$

The shear scalar:

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{3} H_i^2 - 3H^2\right) = \frac{3}{2} A_m H^2 , \tag{11}$$

where $H_i\left(i = 1, 2, 3\right)$ represent the directional Hubble parameters in the directions of r, θ, ϕ axes respectively. The hybrid expansion law, for the average scale factor, is given by

$$a(t) = A^2 B = a_1 t^{\alpha} e^{\beta t} , \tag{12}$$

where α, β are non-negative constants and a_1 is the present value of the scale factor. Equation (12) is known as the hybrid expansion law, which is a combination of a power law and an exponential function. It can be seen that $\alpha = 0$ provides power law cosmology while $\beta = 0$ gives exponential law cosmology. To solve the above set of highly nonlinear equations, the relation between the metric coefficients is considered as

$$A = B^m . \tag{13}$$

For a Baratropic fluid the combined effect of the proper pressure and the Baratropic bulk viscous pressure can be expressed as $\overline{p} = p - 3\eta H = \gamma\rho$, where $p = \gamma_0 \rho$, $0 \le \gamma_0 \le 1$ and m is arbitrary constant. Using equations (12) and (13), the metric potentials can be written as

$$B = \alpha_1 t^{\frac{3\alpha}{2m+1}} e^{\frac{3\beta t}{2m+1}}, \qquad \text{where } \alpha_1 = a_1^{\frac{3}{2m+1}} \tag{14}$$

$$A = \alpha_2 t^{\frac{3m\alpha}{2m+1}} e^{\frac{3m\beta t}{2m+1}} \qquad , \text{where } \alpha_2 = a_1^{\frac{3m}{2m+1}} . \tag{15}$$

This is a point type singularity since directional scale factors vanish at initial time.

5. Physical and Kinematical properties

The spatial volume is obtained as

$$V = a_1^3 t^{3\alpha} e^{3\beta t} . \tag{16}$$

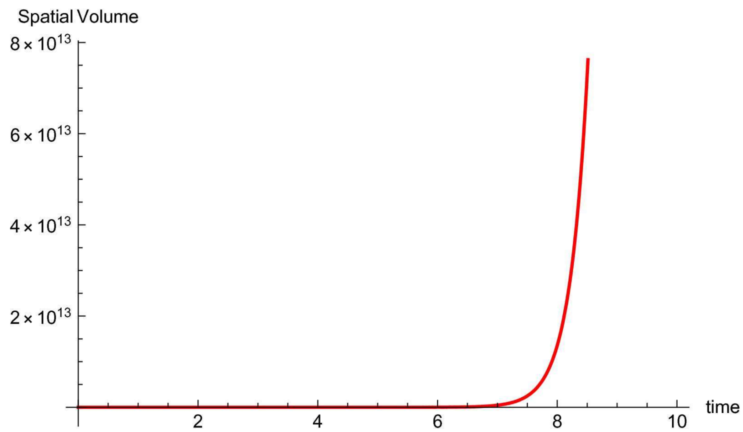

Figure 1: Spatial Volume vs time.

The mean Hubble parameter is found out to be

$$H = \left(\beta + \frac{\alpha}{t} \right). \tag{17}$$

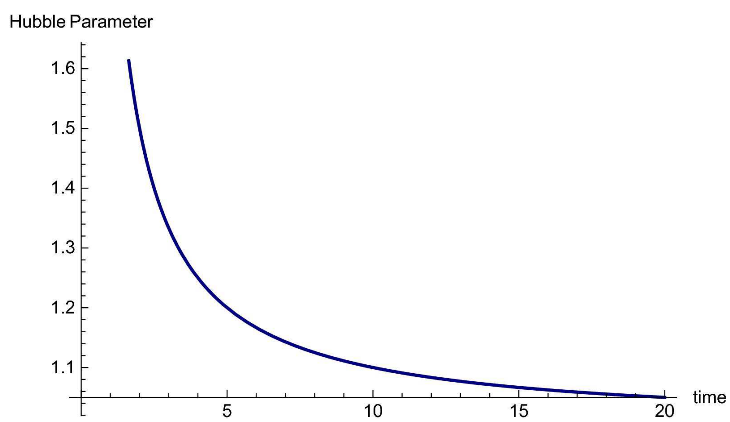

Figure 2: Hubble Parameter vs time.

The Expansion scalar is obtained as

$$\theta = 3\left(\beta + \frac{\alpha}{t} \right). \tag{18}$$

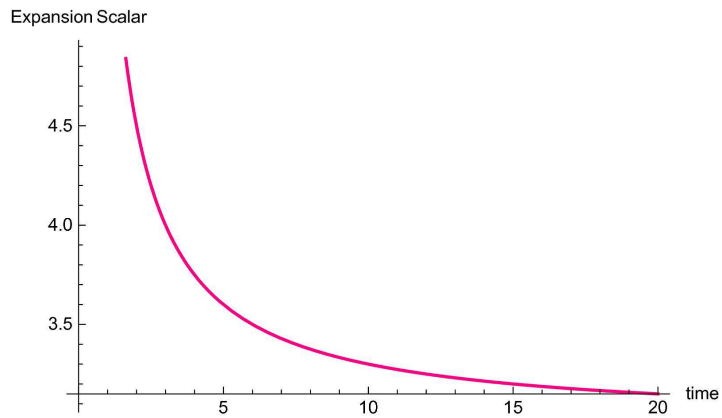

Figure 3. Scalar Expansion vs time.

The mean anisotropic parameter and shear scalar are obtained as

$$A_m = \frac{3(2m^2+1)}{(2m+1)^2}. \tag{19}$$

$$\sigma^2 = \frac{9(2m^2+1)}{2(2m+1)^2}\left(\beta + \frac{\alpha}{t}\right)^2. \tag{20}$$

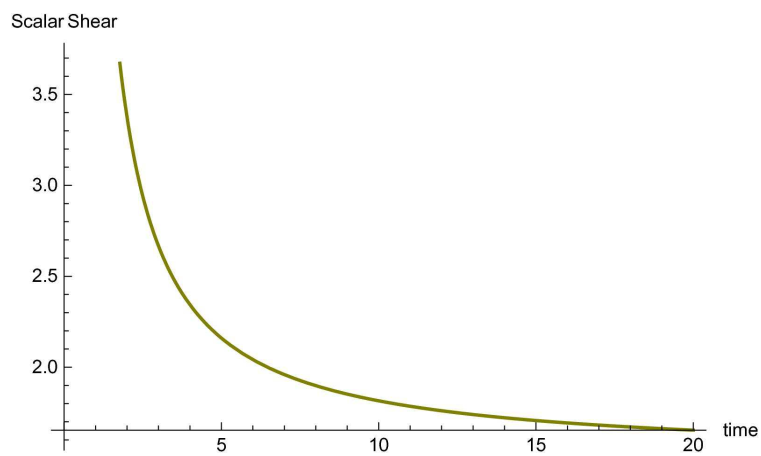

Figure 4. Shear Scalar vs time.

The deceleration parameter is

$$q = \frac{\alpha}{(\beta t+\alpha)^2} - 1. \tag{21}$$

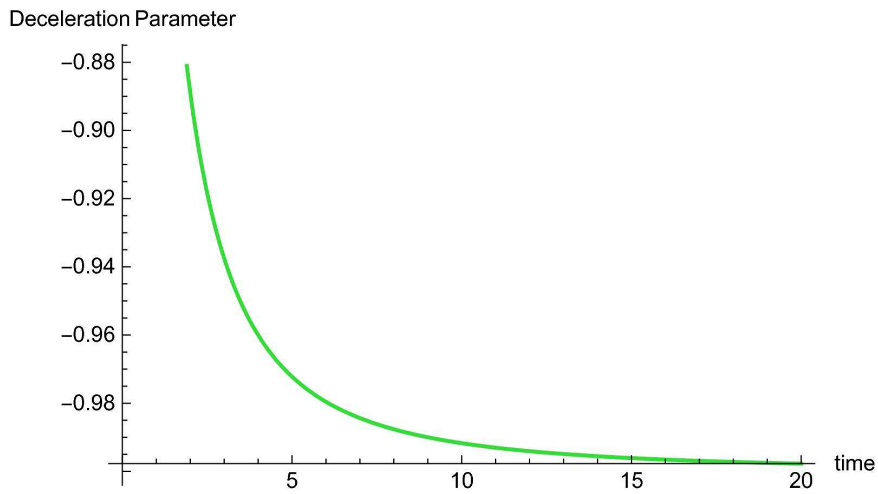

Figure 5. Deceleration Parameter vs time.

Energy density (ρ) of the universe becomes

$$\rho = \left\{ \frac{324\, m^2 (m+2)^2}{(2m+1)^4} \left(\beta + \frac{\alpha}{t} \right)^4 - \frac{36\,(2m+m^2)}{(2m+1)^2} \left(\beta + \frac{\alpha}{t} \right)^2 \right\} \tag{22}$$

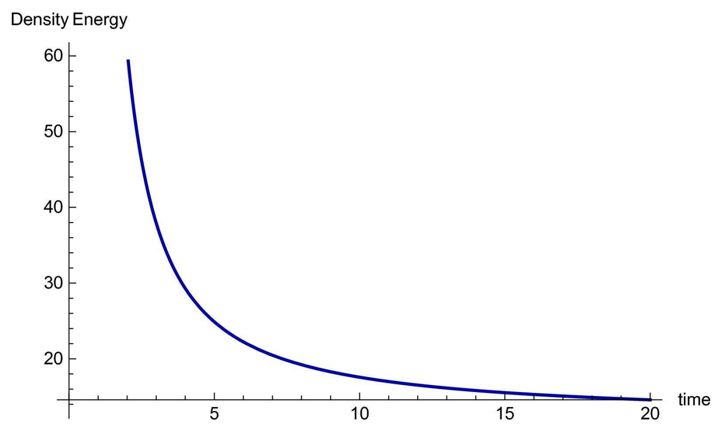

Figure 6. Energy Density vs time.

The isotropic pressure, effective pressure and bulk viscous coefficient respectively are found to be

$$p = \gamma_0 \left\{ \frac{324\, m^2 (m+2)^2}{(2m+1)^4} \left(\beta + \frac{\alpha}{t} \right)^4 - \frac{36\,(2m+m^2)}{(2m+1)^2} \left(\beta + \frac{\alpha}{t} \right)^2 \right\} \tag{23}$$

$$\overline{p} = \gamma \left\{ \frac{324\, m^2 (m+2)^2}{(2m+1)^4} \left(\beta + \frac{\alpha}{t} \right)^4 - \frac{36\,(2m+m^2)}{(2m+1)^2} \left(\beta + \frac{\alpha}{t} \right)^2 \right\} \tag{24}$$

$$\eta = \frac{1}{3H}(\gamma_0 - \gamma)\left\{\frac{324m^2(m+2)^2}{(2m+1)^4}\left(\beta + \frac{\alpha}{t}\right)^4 - \frac{36(2m+m^2)}{(2m+1)^2}\left(\beta + \frac{\alpha}{t}\right)^2\right\}. \quad (25)$$

Figure 1 depicts that at an initial epoch, the spatial volume vanishes. The Hubble parameter tends to zero i.e. $H \to 0$ as $t \to \infty$ (Figure 2) [37-41]. It is observe that expansion scalar is infinite at $t = 0$ as shown in Figure.3 [42-45]. The shear scalar diverges at an initial epoch as depicted in Figure 4 and tends to zero as $t \to \infty$ [46-49]. The anisotropy parameter tends to a constant, which means that the anisotropy in the universe is maintained throughout. Figure 5 represents the deceleration parameter evolution in time. As $q \approx -1$, the universe is accelerating. It is observed from figure 6 that the energy density decreases as the universe expands [50-58].

6. Conclusions

In this paper, an expansion of the universe in $f(T)$ gravity is explored. An exact solution of the Plane Symmetric space-times using bulk viscous with the help of Hybrid Expansion Law has been investigated. The deceleration parameter appears with negative sign which implies accelerating expansion of the universe. The shear scalar is also infinite at an initial epoch and becomes zero as time increases. It is observed that energy density is positive decreasing function of time and converges to zero as $t \to \infty$.

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