Article

Some Characterizations on the Special Tubular Surfaces in Galilean Space

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Abstract

In this paper, the tubular surface generated by rectifying curves are explored. Also, using the Gaussian and mean curvatures of tubular surfaces, for the linear Weingarten surfaces and HK-quadric surfaces, harmonic surfaces some characterizations are given.

Keywords: Galilean space, tubular surfaces, rectifying curves, Weingarten surface, HK-quadric surface.

1. Introduction

In mathematics, a surface is a geometrical shape that corresponds a disfigured plane. The most common examples arise as boundaries of solid objects in ordinary three-dimensional Euclidean space. In the study of the differential geometry of surfaces, it is common to determine some surfaces satisfying curvature conditions. Also, the differential geometry of surfaces deals with the differential geometry of smooth surfaces with various additional structures for example a Riemannian metric. Surfaces have been extensively studied from various perspectives: extrinsically, relating to their embedding in Euclidean space and intrinsically, reflecting their properties determined solely by the distance within the surface as measured along curves on the surface.

A surface whose mean curvature is in functional relationship with its Gaussian curvature. Namely, a surface is said to be a Weingarten surface if there exists a relation, that does not depend on the parameters, between the mean curvature and the total curvature (or between the principal curvatures). Also, a surface is said to be a Weingarten if there is a smooth relation $U(k_1, k_2) = 0$ between two principle curvatures k_1 and k_2. If K and H denote the Gauss curvature and the mean curvatures, respectively, then $U(k_1, k_2) = 0$ implies a relation as $\Phi(K, H) = 0$. The existence of a non-trivial functional relation $\Phi(K, H) = 0$ on a surface, which is parameterized by a patch $x(w, v)$, is equivalent to the following Jacobian determinant [9],

$$\frac{\partial(K,H)}{\partial(w,v)} = 0, \qquad (1.1)$$

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Furthermore, if the equations $U = a_1 k_1 + a_2 k_2 - a_3$ or $\Phi = a_1 H + a_2 K - a_3$ hold, the surfaces are called linear Weingarten surfaces, where $a_i, i \in \mathbb{R}$ with $a_1^2 + a_2^2 \neq 0$. The following points are important for consideration:

1. If the constant $a_2 = 0$, a linear Weingarten surface reduces to a surface with constant Gaussian curvature.

2. If the constant $a_1 = 0$, a linear Weingarten surface reduces to a surface with constant mean curvature.

On the other hand, if a surface satisfies the following equation

$$a_1 H^2 + 2a_2 HK + a_3 K^2 = \text{constant}, a_1 \neq 0, \tag{1.2}$$

then the surface is said to be a HK −quadric surface, [6]. Also, the surfaces of revolution and surfaces with constant mean or constant Gaussian curvature are given as examples of Weingarten surfaces. Also, the linear Weingarten surfaces can be expressed as natural generalization of surfaces with constant Gaussian curvature or constant mean curvature, [9].

In [1], timelike tube surface around the spacelike curve with timelike and spacelike binormal vectors is studied in a three-dimensional Minkowski space E_1^3 by the authors. Moreover, Weingarten and linear Weingarten conditions are given for this surface with respect to their curvatures. The same studies and consequences about surfaces in G_3 are given by the authors in [2, 3]. In [8], the surfaces in Euclidean 3-space foliated by pieces of circles are studied and that satisfy a Weingarten condition of type $aH + bK = c$; $a, b, c \in \mathbb{R}$, H and K denote the mean curvature and the Gauss curvature respectively, by the author. In [12], a tube in a Euclidean 3-space satisfying some equation in terms of the Gaussian curvature, the mean curvature and the second Gaussian curvature are studied by the authors.

In [14], that parallel surfaces of a non-developable ruled surface are not ruled surfaces by using fundamental forms is studied. Also, that parallel surfaces of ruled Weingarten surface are Weingarten surface is shown by the authors. Furthermore, ruled Weingarten surfaces in the Galilean space are studied by the authors, in [15]. Weingarten surfaces are surfaces having a nontrivial funcional relation between their Gaussian and mean curvature. The same consequences about surfaces and curves in G_3 and pseudo Galilean space are given by the authors in [9, 10, 17]. Also, the some consequences about surfaces and curves in different ambiant spaces are investigated by the authors in [5, 6, 16].

2. Preliminaries

Classical context of the Euclidean space is the origin of results, which could be transferred to some other geometries. One way of defining new geometries is through Cayley-Klein spaces. They are expressed as projective spaces, $P_n R$, with an absolute figure, which is a subset of $P_n R$ originating as sequence of quadrics and planes 1. The projective space, $P_n R$, has invariants as the absolute figure definitition for the subgroup of projectivities named as the Cayley-Klein space group of movements. By means of the absolute figure, metric connections are defined and they are invariant under the group of movements.

The scalar product of the vectors $U=(u_1,u_2,u_3)$, $V=(v_1,v_2,v_3)$ in G_3 is defined as,

$$\langle U,V\rangle=\begin{cases}u_1v_1, & \text{if } u_1 \neq 0 \text{or} v_1 \neq 0\\ u_2v_2+u_3v_3, & \text{if } u_1 =0, v_1 =0\end{cases} \tag{2.1}$$

The cross product in Galilean space is given as

$$U\times V=\begin{cases}(0,v_1u_3-v_3u_1,v_2u_1-v_1u_2), & \text{if} u_1 \neq 0 \text{or} v_1 \neq 0\\ (v_3u_2-v_2u_3,0,0), & \text{if} u_1 =0, v_1 =0\end{cases} \tag{2.2}$$

Let $\beta: I\subset\mathbb{R}\to G_3$ be a curve given by $\beta(s)=(s,y(s),z(s))$. The vectors of the Frenet-Serret frame are defined by

$$t(s)=\beta'(s)=(1,y'(s),z'(s));n(s)=\frac{t'(s)}{\kappa(s)};b(s)=\frac{n'(s)}{\tau(s)},$$

where the real valued function $\kappa(s)=\| t'(s)\|$ is given as the first curvature of the curve β, the second curvature function is defined as $\tau(s)=\| n'(s)\|$. For the curve in G_3, Frenet-Serret equations are written as follows

$$t'=\kappa n, n'=\tau b, b'=-\tau n. \tag{2.3}$$

Let the equation of a surface $\Theta=\Theta(s,v)$ in G_3 be given by

$$\Theta(s,v)=(x(s,v),y(s,v),z(s,v)). \tag{2.4}$$

Then the unit isotropic normal vector field η on $\Theta(s,v)$ becomes as

$$\eta=\frac{\Theta_{,1}\times\Theta_{,2}}{\|\Theta_{,1}\times\Theta_{,2}\|}, \tag{2.5}$$

where the partial differentiations with respect to s and v will be denoted as follows

$$\Theta_{,1}=\frac{\partial\Theta(s,v)}{\partial s};\Theta_{,2}=\frac{\partial\Theta(s,v)}{\partial v}. \tag{2.6}$$

On the other hand, the isotropic unit vector δ on the tangent plane of the surface is given by

$$\delta=\frac{x_{,2}\Theta_{,1}-x_{,1}\Theta_{,2}}{w}, \tag{2.7}$$

where $x_{,1}=\frac{\partial x(s,v)}{\partial s}$, $x_{,2}=\frac{\partial x(s,v)}{\partial v}$ and $w=\|\Theta_{,1}\times\Theta_{,2}\|$.

Let us define

$$g_1=x_{,1},g_2=x_{,2},g_{ij}=g_ig_j;g^1=\frac{x_{,2}}{w};g^2=\frac{x_{,1}}{w};g^{ij}=g^ig^j;i,j=1,2 \tag{2.8}$$

$$h_{11}=\langle\Theta^*_{,1},\Theta^*_{,1}\rangle,h_{12}=\langle\Theta^*_{,1},\Theta^*_{,2}\rangle;h_{22}=\langle\Theta^*_{,2},\Theta^*_{,2}\rangle, \tag{2.9}$$

where $\Theta^*_{,1}$ and $\Theta^*_{,2}$ are the projections of the vectors $\Theta_{,1}$ and $\Theta_{,2}$ onto the yz-plane, respectively. The first fundamental form ds^2 of the surface $\Theta(s,v)$ is given as, [7, 11],

$$ds^2=ds_1^2+ds_2^2=(g_1ds+g_2dv)^2+\varepsilon(h_{11}ds^2+2h_{12}dsdv+h_{22}dv^2), \tag{2.10}$$

where

$$\varepsilon = \begin{cases} 0, & dw\colon dv_1 \text{non} - \text{isotropic} \\ 1, & dw\colon dv_1 \text{isotropic} \end{cases} \tag{2.11}$$

In this case, the coefficients of ds^2 are denoted by g_{ij}^*. The function can be represented in terms of g_i and h_{ij} as follows

$$w^2 = g_1^2 h_{22} - 2g_1 g_2 h_{12} + g_2^2 h_{11}.$$

The Gaussian curvature and the mean curvature of a surface are defined by means of the second fundamental form L_{ij} coefficients, which are the normal components of $\Theta_{,i,j}(i,j = 1,2)$. Namely,

$$\Theta_{,i,j} = \sum_{k=1}^{2} \Gamma_{ij}^k \Theta_{,k} + L_{ij}\eta, \tag{2.12}$$

where Γ_{ij}^k is the Christoffel symbols of the surface and L_{ij} are given as

$$L_{ij} = \frac{1}{g_1}\left\langle g_1 \Theta^*_{,i,j} - g_{i,j}\Theta^*_{,1}, \eta \right\rangle = \frac{1}{g_2}\left\langle g_2 \Theta^*_{,i,j} - g_{i,j}\Theta^*_{,2}, \eta \right\rangle. \tag{2.13}$$

From this, the Gaussian curvature K and the mean curvature H of the surface are given as,

$$K = \frac{L_{11}L_{22} - L_{12}^2}{w^2}, H = \frac{g_2^2 L_{11} - 2g_1 g_2 L_{12} + g_1^2 L_{22}^2}{w^2}, \tag{2.14}$$

[7, 11, 13].

Definition 1 *A vector $x = (x_1, x_2, x_3)$ is called a non-isotropic if $x_1 \neq 0$. All unit isotropic vectors are of the form $x = (1, x_2, x_3)$. For isotropic vectors, $x_1 = 0$ hold, [7].*

Proposition 1 *Δf Laplacian of the differentiable function given by $f\colon U \subset \mathbb{R}^2 \to \mathbb{R}$ is defined as*

$$\Delta f = \left(\frac{d^2 f}{du^2}\right) + \left(\frac{d^2 f}{dv^2}\right), (u, v) \in U.$$

If $\Delta f = 0$ then the function f is harmonic in U, [4].

3. The Special Tubular Surfaces Generated by Rectifying Curves in Galilean 3-Space

In this work, the tubular surface generated by rectifying curves are examined, and using the Gaussian and mean curvatures of the special tubular surfaces, the conditions being linear, Weingarten surfaces and HK −quadric surface are expressed.

3.1 Characterization of isotropic rectifying curves in G_3

In this subsection, using the position components of vectors' curvature functions, the rectifying curves in G_3 can be described.

Theorem 1 *Let $\beta\colon I \subset \mathbb{R} \to G_3$ be a regular isotropic curve with curvatures $\kappa(w) \geq 0, \tau$ in G_3. Then, β is a rectifying curve if and only if the position vector of β satisfies the vector equation*

$$\beta(w) = (w + c)\overleftarrow{t} + \frac{\kappa(w)(w+c)}{\tau}\overleftarrow{b} \text{ and } \tau(w) = \frac{\kappa(w)(w+c)}{d}, c, d \in \mathbb{R}_0. \quad (3.1)$$

Proof. Assume that $\beta(w)$ is an rectifying curve with the curvature functions $\kappa(w)$, $\tau(w)$ in G_3 as follows

$$\beta(w) = \Sigma_0\overleftarrow{t} + \Sigma_1\overleftarrow{b}, \quad (3.2)$$

for some differentiable functions $\Sigma_0(w)$, $\Sigma_1(w)$ and differentiating (3.2) with respect to w and using (2.3), one can obtain

$$\overleftarrow{t} = \Sigma_0'\overleftarrow{t} + (\Sigma_0\kappa - \Sigma_1\tau)\overleftarrow{n} + \Sigma_1'\overleftarrow{b}. \quad (3.3)$$

Exposing the inner product t, n, b of the both side in (3.3), respectively, one can have

$$\Sigma_0' = 1; \Sigma_0\kappa - \Sigma_1\tau = 0; \Sigma_1' = 0. \quad (3.4)$$

Using (3.4) and making necessary calculations, one may get,

$$\Sigma_0 = c + w; \Sigma_1 = d \text{or} \Sigma_1 = \frac{\kappa(w)(w+c)}{\tau}. \quad (3.5)$$

Thus, one can find the position vector as,

$$\beta(w) = (c + w)\overleftarrow{t} + d\overleftarrow{b} = (w + c)\overleftarrow{t} + \frac{\kappa(w)(w+c)}{\tau}\overleftarrow{b}.$$

3.2 The mathematical approach on tube surfaces with rectifying curve in G_3

In this section, the tubular surfaces generated by rectifying curve are investigated according to mathematical approach. A canal surface is expressed as the envelope of a setting out sphere with exchanging radius, which is described by the orbit $\beta(w(s))$ (spine curve) with its center and a radius function ρ in addition to its parametrized through Frenet frame of the spine curve $\beta(w(s))$. If the radius function ρ is a constant, then the canal surface is called as a tube. Let one denotes by ρ the vector connecting the point from the parametrized curve $\beta(w(s))$ with the point from the surface, one can have the position vector R of a point on the surface as

$$R = \beta(w(s)) + \rho, i \in \mathbb{N}, \quad (3.6)$$

and since ρ lies in the Euclidean normal plane of the curve $\beta(w(s))$, the points at a distance A_1 from a point of $\beta(w(s))$ form an Euclidean circle in G_3, [3]. Thus, it can be written as $\rho = A_1(\cos v_1\vec{n} + \sin v_1\vec{b})$, where v_1 is the Euclidean angle between the isotropic vectors $\overleftarrow{n}$ and $\overleftarrow{\rho}$.

Let $\Theta(w, v_1)$ be the tube surface generated by rectifying curve and let $\beta: I \subset \mathbb{R} \to G_3$ be a regular isotropic curve with curvatures $\kappa(w) \geq 0, \tau$ in G_3. Then, the tube surface generated by rectifying curve is parametrized as

$$\Theta(w, v_1) = \beta(w(s)) + A_1(\cos v_1(s)\overleftarrow{n} + \sin v_1(s)\overleftarrow{b}), \quad (3.7)$$

where angle v_1 lies between the isotropic vectors $\overleftarrow{n}$ and $\overleftarrow{R} = A_1$. Clearly, one can get,

$$\Theta(w,v_1)=(w+c)\overleftarrow{t}+A_1\cos v_1\overleftarrow{n}+(d+A_1\sin v_1)\overleftarrow{b} \quad (3.8)$$

or

$$\Theta(w,v_1)=(w+c)\overleftarrow{t}+A_1\cos v_1\overleftarrow{n}+(\frac{\kappa(w)(w+c)}{\tau}+A_1\sin v_1)\overleftarrow{b}. \quad (3.9)$$

Then, one can get partial derivatives of $\Theta(w,v_1)$ with respect to w and v_1 as follows

$$\Theta_w=\overleftarrow{t}+\big((w+c)\kappa-\tau(d+A_1\sin v_1)\big)\overleftarrow{n}+\tau A_1\cos v_1\overleftarrow{b}=N_w, \quad (3.10)$$
$$\Theta_{v_1}=A_1(-\sin v_1\overleftarrow{n}+\cos v_1\overleftarrow{b})=A_1N_{v_1}; \quad (3.11)$$

It follows that the vector cross product of them is obtained as

$$\Theta_w\times\Theta_{v_1}=-A_1\cos v_1\overleftarrow{n}-A_1\sin v_1\overleftarrow{b}; \quad (3.12)$$
$$\|\Theta_w\times\Theta_{v_1}\|=A_1. \quad (3.13)$$

From previous equations, by using (3.12) and (3.13), the unit isotropic normal vector η of $\Theta(w,v_1)$ is given as follows $\eta=-\cos v_1\overleftarrow{n}-\sin v_1\overleftarrow{b}$. Furthermore, from (2.7), one can obtain $\delta=\frac{-\Omega^1_{v_1}}{A_1}=\sin v_1\overleftarrow{n}-\cos v_1\overleftarrow{b}$. Since, $\overleftarrow{n}$ and $\overleftarrow{b}$ are the isotropic vectors, the Galilean frenet frame, usage leads to,

$$x(w,v_1)=w+c; x_w=1=g_1; x_{v_1}=1=g_2; g_{11}=1, g_{12}=0, g_{22}=0; \quad (3.14)$$
$$g^1=0, g^2=\frac{-1}{A_1}; h_{11}=1, h_{12}=0, h_{22}=A_1^2. \quad (3.15)$$

After the substitution of (3.14) and (3.15) into (2.10), the coefficients of the first fundamental form of the tubular surface can be obtained with the Galilean frenet frame in Galilean space as

$$I=dw^2+\varepsilon(dw^2+A_1^2dv_1^2)\text{or}I=2dw^2+A_1^2dv_1^2; \varepsilon=1. \quad (3.16)$$

If one wants to calculate the second fundamental form of $\Phi(w,v_1)$, it is then necessary have to compute the following equations

$$\begin{aligned}\Theta_{ww}&=(2\kappa+(w+c)\kappa'-\tau'(d+A_1\sin v_1)-\tau^2A_1\cos v_1)\overleftarrow{n}\\&+(\tau\kappa(w+c)-\tau^2(d+A_1\sin v_1)+\tau'A_1\cos v_1)\overleftarrow{b},\\ \Theta_{v_1v_1}&=A_1(-\cos v_1\overleftarrow{n}-\sin v_1\overleftarrow{b}); \Theta_{wv_1}=-\tau A_1\cos v_1\overleftarrow{n}-\tau A_1\sin v_1\overleftarrow{b}. \end{aligned} \quad (3.17)$$

The coefficients of the second fundamental form are calculated from (2.13) and (3.14), (3.17), as follows

$$L_{11}=\big(-2\kappa(w)+\tau'd-\kappa'(w)(w+c)\big)\cos v_1+\tau^2A_1;$$
$$L_{22}=A_1; L_{12}=\tau A_1. \quad (3.18)$$

Thus, the Gaussian curvature K and the mean curvature H are expressed as

$$K = \frac{-\kappa(w)\cos v_1}{A_1}(\tau d = \kappa.(w + c)), \tag{3.19}$$

$$H = \frac{1}{2A_1}. \tag{3.20}$$

and from $\tau(w) = \frac{(w+c)\kappa(w)}{d}$ and (3.19), (3.20), the curvatures of the rectifying curve are obtain as

$$\kappa(w) = \frac{-K}{2H\cos v_1}; \tau(w) = \frac{-(w+c)K}{2dH\cos v_1}.$$

Hence, the following theorem can be given.

Theorem 2 *Let Θ be a tubular surface by generated a rectifying curve in G_3. Then, the curvatures of the rectifying curve are given as*

$$\kappa(w) = \frac{-K}{2Hcosv_1}; \tau(w) = \frac{-(w+c)K}{2dHcosv_1}.$$

Theorem 3 *A tubular surface Θ by generated a rectifying curve in G_3 is also a $\Phi(K,H)$-Weingarten surface.*

Proof. From definition of the Weingarten surface, taking derivative of K and H with respect to w and v_1 yields,

$$K_w = \frac{(\tau_{ww}(w)d - \kappa_{ww}(w)(w+c) - 3\kappa_w(w))\cos v_1}{A_1}, H_w = 0;$$
$$K_{v_1} = \frac{-(\tau_w(w)d - (w+c)\kappa_w(w) - 2\kappa(w))\sin v_1}{A_1}, H_{v_1} = 0.$$

Since $\frac{(w+c)\kappa(w)}{d} = \tau(w)$, for a rectifying curve, one can write

$$K_w = \frac{-\kappa_w(w)\cos v_1}{A_1}, H_w = 0; K_{v_1} = \frac{\kappa(w)\sin v_1}{A_1}, H_{v_1} = 0. \tag{3.21}$$

Furthermore, if the tubular surface Θ generated by a rectifying curve in G_3 satisfies the equation $\Phi(X,Y) = 0$, then the surface is called as $\Phi(K,H)$-Weingarten surface. Therefore, by using (3.21), we get

$$\Phi(K,H) = \frac{\partial(K,H)}{\partial(w,v_1)} = K_w H_{v_1} - K_{v_1} H_w = 0,$$

and we say that the surface Θ is a $\Phi(K,H)$-Weingarten surface.

Theorem 4 *If Θ is a linear Weingarten surface in G_3, for $a_1 \neq 0, v_1 \neq \frac{(2n+1)\pi}{2}, n \in \mathbb{N}$, the tubular surface Θ generated by the rectifying curve is a linear Weingarten surface, while it is also a flat surface reduced to a cylindrical surface with constant Gaussian curvature.*

Proof. Let Θ be a linear Weingarten surface in G_3, then from the definition of the Weingarten surface, by using the equations $K = \frac{-\kappa(w)\cos v_1}{A_1}, H = \frac{1}{2A_1}$, one can reach to

$$a_1K + a_2H = a_3$$
$$a_1.\frac{-\kappa(w)\cos v_1}{A_1} + a_2.\frac{1}{2A_1} = a_3$$
$$-\kappa(w)2a_1.\cos v_1 + (a_2 - a_3 2A_1) = 0.$$

From the definition of the linear independent of vectors,

$$-\kappa(w)2a_1.\cos v_1 = 0 \text{ and } (a_1 - a_3 2A_1) = 0$$

and by using previous equations, from $A_1 = \frac{-\kappa(w)\cos v_1}{K}$, one have

$$\frac{a_2}{2a_3} = A_1; \kappa(w) = 0 \Rightarrow \frac{a_2}{2a_3} = \frac{-\kappa(w)\cos v_1}{K} \; and \; a_2 = 0. \tag{3.22}$$

Hence, the surface Θ is a cylinder and for the constant $a_2 = 0$, the surface Θ is reduced to a cylinder surface with constant Gaussian curvature. Therefore, from $\kappa(w) = 0$ and $K = \frac{-\kappa(w)\cos v}{A}$ equations, $K = 0$ is found that the surface is flat.

Theorem 5 *Let Θ be a tube surface with rectifying curve in G_3. For the parameter*

$$v_1 = arc\cos\left[\frac{a_2/2a_3}{\kappa(w)}\right] = arc\cos\left[\left(\frac{a_2}{a_3}\right)\frac{H}{K}\cos v_1\right],$$

the surface Θ is a HK −quadric surface.

Proof. Assume that the tubular surface Θ is HK −quadric surface. From definition of the HK −quadric surface, after taking necessary differentials it is possible to have calculations, we get

$$a_1HH_w + a_2(H_wK + HK_w) + a_3KK_w = 0.$$

Then, by using the equations $K_w = \frac{(\tau_{ww}(w)d - \kappa_{ww}(w)(w+c) - 3\kappa_w(w))\cos v_1}{A_1}$ and $H_w = 0$, one can obtain

$$a_2HK_w + a_3KK_w = 0,$$
$$(d\tau_{ww} - (w+c)\kappa_{ww} - 3\kappa_w)\cos v_1\big(a_2 + 2a_3\cos v_1(d\tau_w - (w+c)\kappa_w - 2\kappa)\big) = 0.$$

Later from the previous equation, one gets

$$(\tau_{ww}d - \kappa_{ww}(w+c) - 3\kappa_w) = 0$$

or

$$\big(a_2 + 2a_3\cos v_1(\tau_w d - \kappa_w(w+c) - 2\kappa)\big) = 0.$$

For a rectifying curve in G_3, since $(w+c)\kappa(w) = \tau(w)d$, one can get

$$(a_2 - \kappa 2a_3\cos v_1) = 0.$$

Hence, since $\kappa(w)=\frac{-K}{2H\cos v_1}$, the parameter v_1 can be written as

$$v_1 = arc\cos\left[\frac{a_2/2a_3}{\kappa(w)}\right] = arc\cos\left[\left(\frac{a_2}{a_3}\right)\frac{H}{K}\cos v_1\right]. \tag{3.23}$$

Therefore, one can state that the tubular surface Θ generated by the rectifying curve is HK −quadric surface.

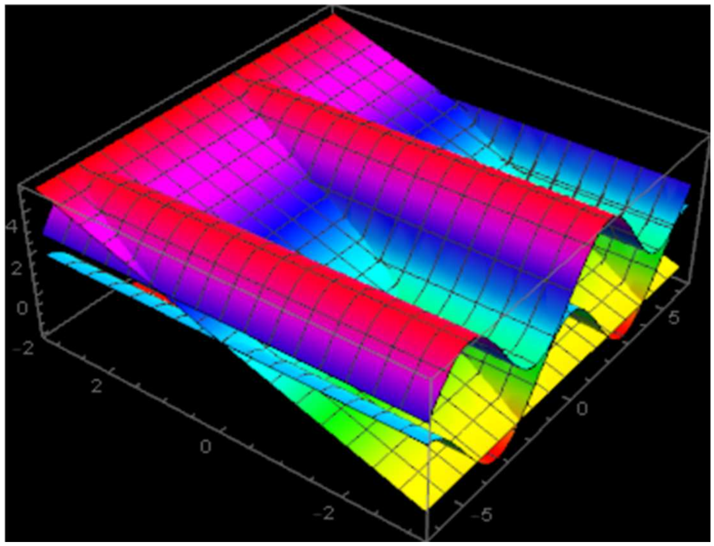

Figure 1. The Weingarten tube surface formed by the rectifying curve in Galilean space

Theorem 6 *The tubular surface Θ generated by a rectifying curve in G_3 is a harmonic ⇔ the following expression is provided*

$$v = k\pi, k \in \mathbb{Z} \text{ or } v = \arccos\left(\frac{-1}{A}\frac{\partial^2\left(\frac{(w+c)\kappa(w)}{\tau(w)}\right)}{\partial w^2}\right).$$

Proof. Let the tube surface Θ be formed by the rectifying curve β in G_3. Also, in order for its Θ^i, $i=1,2,3$ coordinates functions to be harmonic, it is necessary to provide $\Delta\Theta^i=0$ equality from the definition. So, by making the necessary calculations in the following expression,

$$\Theta(w,v) = \left(w+c, A\cos v, \frac{\kappa(w)(w+c)}{\tau(w)} + A\sin v\right)$$
$$\Theta(w,v) = (\Theta^1, \Theta^2, \Theta^3),$$

the following equations can be written

$$\Delta\Theta^1_w = \frac{\partial^2\Theta^1}{\partial w^2} = 0, \Delta\Theta^1_v = \frac{\partial^2\Theta^1}{\partial v^2} = 0$$
$$\Delta\Theta^1 = 0;$$
$$\Delta\Theta^2_w = \frac{\partial^2\Theta^2}{\partial w^2} = 0, \Delta\Theta^2_v = \frac{\partial^2\Theta^2}{\partial v^2} = -A\sin v$$
$$\Delta\Theta^2 = -A\sin v = 0;$$

$$\Delta\Theta^3_w = \frac{\partial^2\Theta^3}{\partial w^2} = \frac{\partial^2\left(\frac{(w+c)\kappa(w)}{\tau(w)}\right)}{\partial w^2}, \Delta\Theta^3_v = \frac{\partial^2\Theta^3}{\partial v^2} = A\cos v$$

$$\Delta\Theta^3 = \frac{\partial^2\left(\frac{(w+c)\kappa(w)}{\tau(w)}\right)}{\partial w^2} + A\cos v = 0.$$

Hence, from the previous equations, respectively, one gets

$$-A\sin v = 0 \Rightarrow v = k\pi, k \in \mathbb{Z}$$

or

$$\frac{\partial^2\left(\frac{(w+c)\kappa(w)}{\tau(w)}\right)}{\partial w^2} + A\cos v = 0 \Rightarrow v = \arccos\left(\frac{-1}{A}\frac{\partial^2\left(\frac{(w+c)\kappa(w)}{\tau(w)}\right)}{\partial w^2}\right).$$

4. Conclusion

In this paper, the tube surfaces generated by rectifying curves are examined and some certain results according to the curvatures of the surfaces are presented in detail. Moreover, using the Gaussian and mean curvatures of tube surfaces generated by rectifying curve, it is possible to try and to express the conditions being linear Weingarten surfaces, $HK-$quadric surface and harmonic. The authors are currently working on the properties of these tubular and canal surfaces with a view to devising suitable metric in 3-Galilean and 4-Galilean spaces by adapting the type of conservation laws considered in the paper.

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