Article

Kantowski-Sachs Model with Modified Holographic Ricci Dark Energy in Self-Creation Theory of Gravitation

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Abstract

In this paper, a Kantowski-Sachs cosmological model is investigated in the presence of an anisotropic modified Ricci dark energy within the framework of Barber's second self-creation theory of gravitation. Exact solutions of the field equations are obtained by applying a special law of variation of Hubble's parameter which yield a negative constant value of deceleration parameter. The physical and kinematical features of the universe are discussed near the initial time singularity and at late-time.

Keywords: Kantowski-Sachs model, Holographic Ricci dark energy, Self-creation theory

1 Introduction

The recent cosmological observations of the type I_a supernovae (Riess et al [1,2]; Permulter et al. [3]), large scale structure (Tegmark et al. [4]), baryon oscillation (Eisenstein et al. [5]) etc have indicated that our present-day universe is not only expanding, but it is also accelerating. These observations seem to change the entire picture of our matter field universe. It has been observed that a kind of exotic fluid, known as dark energy (DE) with negative pressure, is responsible for this accelerated expansion. Many DE models have been proposed and extensively studied to explain this cosmic expansion (Copeland et al. [6]). The cosmological constant Λ is the simplest candidate of DE (Sahni and Starobinsky [7]; Padmanabhan [8]).

In recent years the investigation of holographic dark energy models to explain the present-day cosmic acceleration of the universe has attracted the attention of several researchers. Holographic DE model is based on the holographic principle, which states that in quantum gravity the entropy of a system scales not with its volume but with its surface area L^2. Motivated by this principle, Cohen et al. [9] suggested that the vacuum energy density is proportional to the Hubble's scale $L = H^{-1}$. Li [10] defined the energy density of holographic dark energy as

$$\rho_\Lambda = 3c^2 m_{pe}^2 L^{-2} \tag{1.1}$$

where L is the infrared (IR) cutoff radius, c is a constant and

$$m_{pe}^2 = \frac{1}{8\pi G} \tag{1.2}$$

is the Planck mass. The IR cutoff has been considered as the Hubble radius or future event horizon. Later, Gao et al. [11] assumed that the future event horizon is replaced by the inverse of the Ricci curvature (i,e., $L = R^{\frac{-1}{2}}$). In this case model is called the Ricci dark energy model. Granda and Oliveros [12] proposed a new holographic Ricci DE model with energy density ρ_Λ given by

$$\rho_\Lambda = 3m_{pe}^2(\alpha H^2 + \beta \dot{H}). \tag{1.3}$$

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Subsequently, Chen and Jing [13] modified this model with energy density given by

$$\rho_\Lambda = 3m_{pe}^2(\beta_1 H^2 + \beta_2 \dot{H} + \beta_3 \ddot{H} H^{-1}) \tag{1.4}$$

where an overdot indicates derivative with respect to cosmic time t. The model with energy density given by equation(1.4) is known as modified holographic Ricci DE model (MHRDE).

In recent years, the study of holographic Ricci DE models has drawn the attention of many researchers. Das and Sultana [13] MHRDE studied an anisotropic Bianchi type-II modified cosmological model in general relativity with hybrid expansion law. Santhi et al. [14] discussed an anisotropic Bianchi type-VI$_0$ MHRDE cosmological model with matter in Saez-Ballester scalar-tensor theory of gravitation. Santhi et al.[15] has obtained an anisotropic Bianchi type-III model filled with matter and MHRDE in general relativity following hybrid expansion law and a time-varying deceleration parameter. Rao et al. [16] investigated anisotropic and spatially homogeneous Bianchi type-V MHRDE cosmological model in the presence of matter in Barber's self-creation theory of gravitation. Further, Bhaskar Rao et al. [17] discussed a LRS Bianchi type-II model with dark matter and anisotropic MHRDE in Barber's second self-creation theory of gravitation. It deserves to mention that Reddy et al. [18] also discussed the dynamics of LRS Bianchi type-II anisotropic DE model in the presence of a massless scalar-meson field.

Motivated by above discussion, we investigated, in this paper, a Kantowski-Sachs cosmological model in the presence of dark energy matter and modified holographic Ricci dark energy in Barber's second self-creation theory of gravitation [19]. The paper is organized as follows: In Sect. 2, we present the metric and field equations. Sect.3 contains the solutions of field equations and resulting model of universe. The physical and kinematical parameters of the model are determined and their physical significance are highlighted in Sect. 4. The Sect. 5 deals with summary and conclusions.

2 Metric and field equations

We consider the metric of Kantowski-Sachs space-time in the form

$$ds^2 = dt^2 - A^2 dr^2 - B^2(d\theta^2 + \sin^2 d\phi^2) \tag{2.1}$$

where the scale factors A and B are functions of the cosmic time t only. The space-time has one transverse direction r and two equivalent longitudinal direction θ and ϕ.

The energy-momentum tensors of pressure less matter and holographic dark energy are respectively, given by

$$T_{ij} = \rho_m u_i u_j, \tag{2.2}$$

$$\overline{T}_{ij} = (\rho_\Lambda + p_\Lambda) u_i u_j - g_{ij} p_\Lambda \tag{2.3}$$

where ρ_m is matter energy density and ρ_Λ is the energy-density and p_Λ is the pressure of modified holographic Ricci dark energy. The Einstein field equations in Barber's theory of self-creation are

$$R_{ij} - \frac{1}{2} g_{ij} R = -\frac{8\pi}{\phi}\left(T_{ij}^m + T_{ij}^{de}\right), \tag{2.4}$$

$$\Box\phi = \phi_{;k}^{;k} = \frac{8\pi}{3}\mu(T + \bar{T}) \tag{2.5}$$

where μ is a coupling constant. The energy-density ρ_Λ of the MHRDE given by equation (1.4) takes the form

$$\rho_\Lambda = \frac{\phi}{8\pi}(\beta_1 H^2 + \beta_2 \dot{H} + \beta_3 \ddot{H} H^{-1}) \tag{2.6}$$

since $G = \phi^{-1}$ in Barber's theory of self-creation.
Now, by parametrization, we have, from equation (2.3)

$$\overline{T}^{\nu}_{\mu} = \mathrm{diag}(1, -\omega_r, -\omega_\theta, -\omega_\phi) = \mathrm{diag}[1, -\omega, -(\omega+\delta), -(\omega+\gamma)]\rho_\Lambda. \tag{2.7}$$

Here we have used the EoS parameter given by

$$\omega = \frac{p_\Lambda}{\rho_\Lambda} \tag{2.8}$$

and ω_r, ω_θ and ω_ϕ are the directional EoS parameters along r, θ and ϕ axes respectively. For the sake of simplicity we choose $\omega_r = \omega$ and the skewness parameters δ and γ are the deviations from ω for θ and ϕ axes respectively. Further, since $\bar{T}^2_2 = \bar{T}^2_3$ we have $\gamma = \delta$ and therefore equation(2.1) can be written in the form

$$\bar{T}^{\nu}_{\mu} = \mathrm{diag}[1, -\omega, -(\omega+\delta), -(\omega+\delta)]\rho_\Lambda. \tag{2.9}$$

In comoving coordinate system, the field equation (2.4) for the metric (2.1), with the help of equation(2.2), equation(2.5) and equation(2.9), give the following system of equations

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{1}{B^2} = -\frac{8\pi}{\phi}\omega_\Lambda\rho_\Lambda, \tag{2.10}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} = -\frac{8\pi}{\phi}(\omega_\Lambda + \delta)\rho_\Lambda, \tag{2.11}$$

$$2\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{1}{B^2} = \frac{8\pi}{\phi}(\rho_m + \rho_\Lambda). \tag{2.12}$$

$$\ddot{\phi} + \dot{\phi}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = \frac{8\pi}{3}(T + \overline{T}). \tag{2.13}$$

We now define some physical and kinematical parameters of cosmology that will be useful to solve the field equations and to discuss physical features of the universe. The spatial volume V and the average scale factor a are defined as

$$V = a^3 = AB^2. \tag{2.14}$$

The anisotropy parameter A_m of the expansion is given by

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 \tag{2.15}$$

where the mean Hubble parameter H is given as

$$H = \frac{1}{3}\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right) \tag{2.16}$$

and H_1, H_2, H_3 are respectively directional Hubble parameters in the direction r, θ and ϕ axes respectively defined as

$$H_1 = \frac{\dot{A}}{A}, H_2 = H_3 = \frac{\dot{B}}{B}. \tag{2.17}$$

The scalar expansion(θ) and shear scalar(σ) are given by

$$\theta = 3H = \left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right), \tag{2.18}$$

$$\sigma^2 = \left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)^2. \tag{2.19}$$

An important observational quantity is the deceleration parameter (q) defined by

$$q = -\frac{a\ddot{a}}{\dot{a}^2}. \tag{2.20}$$

The sign of q indicates whether the model inflates or not. The positive sign of q corresponds to a standard decelerating model and the negative sign indicates inflation.

3 Solutions of field equations

We now obtain exact solutions of the equations (2.10)- (2.13) which is a system of four highly non-linear equations in seven unknowns A, B, ρ_m, ω, δ, ϕ and ρ_Λ. We obtain exact solutions of equations (2.10)-(2.13) by applying a special law of variation of Hubble parameter which leads to a negative constant value of deceleration parameter,

We define the deceleration parameter q to be a negative constant defined

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = \text{constant}. \tag{3.1}$$

For an accelerating model of the universe we take the constant as negative. Then integration of equation(3.1) leads the solution of the average scale factor given as

$$a = (c_1 t + c_2)^{\frac{1}{1+q}} \tag{3.2}$$

where c_1 and c_2 are constants of integration. Equation (3.2) implies that the condition of accelerated expansion is $1 + q > 0$.

Subtracting equation(2.10) from equation(2.11) and integrating the result, we get

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = \frac{X}{a^3} + \frac{1}{a^3}\int \left(\frac{1}{B^2} - \frac{8\pi}{\phi}\delta\rho_\Lambda\right)\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)^{-1} dt. \tag{3.3}$$

The integral term in equation(3.3) vanishes for

$$\delta = \frac{\phi}{8\pi B^2 \rho_\Lambda}. \tag{3.4}$$

Then from equations(3.3)-(3.4), we obtain

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = \frac{X}{a^3} = \frac{X}{(c_1 t + c_2)^{3/1+q}} \tag{3.5}$$

where X is constant of integration. From equation(2.14), we also have

$$\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B} = \frac{3c_1}{(c_1 t + c_2)}. \tag{3.6}$$

Solving equation(3.5) and equation(3.6) for $\frac{\dot{A}}{A}$ and $\frac{\dot{B}}{B}$, and integrating the results, we obtain the solutions of the scale factors A and B as follows

$$A = k_1(c_1t + c_2)^{\frac{1}{1+q}} \exp\left[\frac{2X(1+q)}{3c_1(q-2)}(c_1t + c_2)^{\frac{q-2}{1+q}}\right], \tag{3.7}$$

$$B = k_2(c_1t + c_2)^{\frac{1}{1+q}} \exp\left[-\frac{X(1+q)}{3c_1(q-2)}(c_1t + c_2)^{\frac{q-2}{1+q}}\right] \tag{3.8}$$

where k_1 and k_2 are integration constants which can be taken unity without loss of any generality.

Hence, the metric (2.1) becomes

$$ds^2 = dt^2 - (c_1t + c_2)^{\frac{2}{1+q}} \exp\left[\frac{4X(1+q)}{3c_1(q-2)}(c_1t + c_2)^{\frac{q-2}{1+q}}\right]dr^2 - (c_1t + c_2)^{\frac{2}{1+q}} \exp\left[-\frac{2X(1+q)}{3c_1(q-2)}(c_1t + c_2)^{\frac{q-2}{1+q}}\right](d\theta^2 + \sin^2\theta d\phi^2). \tag{3.9}$$

Using appropriate scale transformation of time coordinate the metric (3.9) of our solution can be written in the form

$$ds^2 = dt^2 - t^{\frac{2}{1+q}} \exp\left[\frac{4X(1+q)}{3(q-2)}t^{\frac{q-2}{1+q}}\right]dr^2 - t^{\frac{2}{1+q}} \exp\left[-\frac{2X(1+q)}{3(q-2)}t^{\frac{q-2}{1+q}}\right](d\theta^2 + \sin^2\theta d\phi^2) \tag{3.10}$$

where $q \neq 2$.

Now, to find the solution of the scalar function ϕ, we make use of the viable physical condition that [15, 16]

$$T + \bar{T} = 0. \tag{3.11}$$

Using equation(3.11) in equation(2.13) and integrating, we obtain

$$\phi = \frac{(1+q)}{(q-2)}\phi_0 t^{\frac{q-2}{1+q}}. \tag{3.12}$$

where ϕ_0 is an integration constant.

4 Physical and Kinematical parameters

The spatial volume of the model is

$$V = t^{\frac{1}{1+q}}. \tag{4.1}$$

The directional Hubble parameter on r, θ and ϕ axes are obtained as

$$H_1 = \frac{1}{(1+q)}\frac{1}{t} + \frac{2X}{3t^{\frac{3}{1+q}}}, \tag{4.2}$$

$$H_2 = H_3 = \frac{1}{(1+q)}\frac{1}{t} - \frac{X}{3t^{\frac{3}{1+q}}}. \tag{4.3}$$

The mean Hubble parameter, scalar expansion and shear scalar are respectively obtained as

$$H = \frac{1}{(1+q)t}, \tag{4.4}$$

$$\theta = \frac{3}{(1+q)t}, \tag{4.5}$$

$$\sigma^2 = \frac{2X^2}{3t^{\frac{6}{1+q}}}. \tag{4.6}$$

The mean anisotropy parameter A_m has the value given by

$$A_m = \frac{2X^2}{9t^{\frac{6}{1+q}}}. \tag{4.7}$$

Now, from equation(2.6) and equation(4.4), we obtain the energy density of modified holographic Ricci dark energy as

$$\rho_\Lambda = \frac{\phi_0 \left[3\beta_1 - 3\beta_2(1+q) + 2\beta_3(1+q)^2\right]}{8\pi(1+q)(q-2)t^{\frac{4+q}{1+q}}}. \tag{4.8}$$

From the field equations (2.10)- (2.12) and the equations (3.10) and (4.8), we obtain

$$\omega_\Lambda = -\frac{\phi_0(1+q)}{8\pi t^{\frac{2-q}{1+q}}\rho_\Lambda}\left[\frac{X^2}{3t^{\frac{6}{1+q}}} + \frac{(1-2q)}{(1+q)^2t^2} + t^{\frac{-2}{1+q}}\exp\left[\frac{2X(1+q)}{3(q-2)}t^{\frac{q-2}{1+q}}\right]\right], \tag{4.9}$$

$$\rho_m = \frac{\phi_0(1+q)}{8\pi(q-2)t^{\frac{2-q}{1+q}}}\left[\frac{3}{(1+q)^2t^2} - \frac{X^2}{3t^{\frac{6}{1+q}}} + t^{\frac{-2}{1+q}}\exp\left[\frac{2X(1+q)}{3(q-2)}t^{\frac{q-2}{1+q}}\right]\right] - \frac{3\phi_0\left[\beta_1 - \beta_2(1+q) + 2\beta_3(1+q)^2\right]}{8\pi(1+q)(q-2)t^{\frac{4+q}{1+q}}}, \tag{4.10}$$

$$\delta = \frac{\phi_0}{\rho_\Lambda}t^{\frac{-2}{1+q}}\exp\left[\frac{2X(1+q)}{3(q-2)}t^{\frac{q-2}{1+q}}\right]. \tag{4.11}$$

Equation(4.1) shows that the spatial volume is zero as the time $t = 0$ and it takes infinitely large values as $t \to \infty$. The expansion scalar, shear scalar and mean anisotropy parameter are infinite as $t \to 0$ and decrease with the increase of time. Thus, the universe begins evolving with zero volume at $t = 0$ with an infinite rate of expansion and the expansion rate slows down for later times and ultimately tends to zero as $t \to \infty$. Hence, the model has a big-bang type singularity at the instant $t = 0$.

The anisotropy parameter of expansion is infinite for earlier time of evolution of the universe and decreases with time and ultimately approaches to zero for large time. The isotropy condition $\frac{\sigma}{\theta}$ tends to zero as $t \to \infty$ of the expansion of the universe is also satisfied. Hence the universe becomes isotropic at late time which is consistent with present day universe.

The physical quantities ρ_Λ, ω, δ and ρ_m are well behaved for $0 \leq t < \infty$. The DE density is positive which decreases with cosmic time. The behaviour of skewness parameter shows that the dark energy always remains anisotropic throughout the evolution of the universe and ultimately vanishes for very large time. The matter energy density , being infinite at $t = 0$, vanishes time $t \to \infty$.

5 Conclusion

In this paper, we have investigated a Kantowski-Sachs cosmological model in the presence of an anisotropic modified holographic Ricci dark energy within the framework of Barber's second theory of self-creation.

Exact solutions of Einstein's field equations are presented using a special law of variation of the mean Hubble parameter that yields a negative constant deceleration parameter. Using these solutions a DE model is presented and the kinematical parameters corresponding to the model are determined and their physical significance are discussed. It is observed that our model evolves from the initial singularity at $t = 0$ and expands with accelerated expansion. The physical behaviours of the universe near the initial singularity and at late-time have been discussed. The behavior of the model is in good agreement with the modern cosmological observations.

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