Exploration

On $M^8 - H$-duality, p-adic Length Scale Hypothesis & Dark Matter Hierarchy

Matti Pitkänen [1]

Abstract

$M^8 - H$ duality, p-adic length scale hypothesis and dark matter hierarchy as phases of ordinary matter with effective Planck constant $h_{eff} = nh_0$ are basic assumptions of TGD, which all reduce to number theoretic vision. In the sequel $M^8 - H$ duality, p-adic length scale hypothesis and dark matter hierarchy are discussed from number theoretic perspective. Several new results emerge. Strong form of holography (SH) allows to weaken strong form of $M^8 - H$ duality mapping space-time surfaces $X^4 \subset M^8$ to $H = M^4 \times CP_2$ that it allows to map only certain complex 2-D sub-manifolds of quaternionic space-time surface to H: SH allows to determine $X^4 \subset H$ from this 2-D data. Complex sub-manifolds are determined by conditions completely analogous to those determined space-time surface as quaternionic sub-manifold and only discrete set of them is obtained. M^8 duality allows to relate p-adic length scales L_p to differences for the roots of the polynomial defining the extension defining "special moments in the life of self" assignable causal diamond (CD) central in zero energy ontology (ZEO). Hence p-adic length scale hypothesis emerges both from p-adic mass calculations and $M^8 - H$ duality. It is proposed that the size scale of CD correspond to the largest dark scale nL_p for the extension and that the sub-extensions of extensions could define hierarchy of sub-CDs. Skyrmions are an important notion if nuclear and hadron physics. $M^8 - H$ duality suggests an interpretation of skyrmion number as winding number as that for a map defined by complex polynomial.

1 Introduction

This article is devoted to an attempt to summarize the recent understanding of p-adic length scale hypothesis and dark matter hierarchy. These considerations lead to more detailed proposals. In particular, a proposal for explicit form of dark scale is proposed.

1.1 p-Adic length scale hypothesis

In p-adic mass calculations [6] real mass squared is obtained by so called canonical identification from p-adic valued mass squared identified as analog of thermodynamical mass squared using p-adic generelization of thermodynamics assuming super-conformal invariance and Kac-Moody algebras assignable to isometries ad holonomies of $H = M^4 \times CP_2$. This implies that the mass squared is essentially the expectation value of sum of scaling generators associated with various tensor factors of the representations for the direct sum of super-conformal algebras and if the number of factors is 5 one obtains rather predictive scenario since the p-adic temperature T_p must be inverse integer in order that the analogs of Boltzmann factors identified essentially as p^{L_0/T_p}.

The p-adic mass squared is of form $Xp + O(p^2)$ and mapped to $X/p + O(1/p^2)$. For the p-adic primes assignable to elementary particles ($M_{127} = 2^{127} - 1$ for electron) the higher order corrections are in general extremely small unless the coefficient of second order contribution is larger integer of order p so that calculations are practically exact.

Elementary particles seem to correspond to p-adic primes near powers 2^k. Corresponding p-adic length - and time scales would come as half-octaves of basic scale if all integers k are allowed. For odd values of k one would have octaves as analog for period doubling. In chaotic systems also the generalization of

[1] Correspondence: Matti Pitkänen http://tgdtheory.fi/. Address: Rinnekatu 2-4 8A, 03620, Karkkila, Finland. Email: matpitka6@gmail.com.

period doubling in which prime $p = 2$ is replaced by some other small prime appear and there is indeed evidence for powers of $p = 3$ (period tripling as approach to chaos). Many elementary particles and also hadron physics and electroweak physics seem to correspond to Mersenne primes and Gaussian Mersennes which are maximally near to powers of 2.

For given prime p also higher powers of p define p-adic length scales: for instance, for electron the secondary p-adic time scale is .1 seconds characterizing fundamental bio-rhythm. Quite generally, elementary particles would be accompanied by macroscopic length and time scales perhaps assignable to their magnetic bodies or causal diamonds (CDs) accompanying them.

This inspired p-adic length scale hypothesis stating the size scales of space-time surface correspond to primes near half-octaves of 2. The predictions of p-adic are exponentially sensitive to the value of k and their success gives strong support for p-adic length scale hypothesis. This hypothesis applied not only to elementary particle physics but also to biology and even astrophysics and cosmology. TGD Universe could be p-adic fractal.

1.2 Dark matter as phases of ordinary matter with $h_{eff} = nh_0$

The identification of dark matter as phases of ordinary matter with effective Planck constant $h_{eff} = nh_0$ is second key hypothesis of TGD. To be precise, these phases behave like dark matter and galactic dark matter could correspond to dark energy in TGD sense assignable to cosmic strings thickened to magnetic flux tubes.

There are good arguments in favor of the identification $h = 6h_0$ [14, 22]. "Effective" means that the actual value of Planck constant is h_0 but in many-sheeted space-time n counts the number of symmetry related space-time sheets defining space-time surface as a covering. Each sheet gives identical contribution to action and this implies that effective value of Planck constant is nh_0.

1.3 $M^8 - H$ duality

$M^8 - H$ duality ($H = M^4 \times CP_2$) [24] has taken a central role in TGD framework. $M^8 - H$ duality allows to identify space-time regions as "roots" of octonionic polynomials P in complexified M^8 - M^8_c - or as minimal surfaces in $H = M^4 \times CP_2$ having 2-D singularities.

Remark:O_c,H_c,C_c,R_c will be used in the sequel for complexifications of octonions, quaternions, etc.. number fields using commuting imaginary unit i appearing naturally via the roots of real polynomials.

The precise form of $M^8 - H$ duality has however remained unclear. Two assumptions are involved.

1. Associativity stating that the tangent or normal space of at the point of the space-time space-time surface M^8 is associative - that is quaternionic. There are good reasons to believe that this is true for the polynomial ansatz everywhere but there is no rigorous proof.

2. The tangent space of the point of space-time surface at points mappable from M^8 to H must contain fixed $M^2 \subset M^4 \subset M^8$ or an integrable distribution of $M^2(x)$ so that the 2-surface of M^4 determined by it belongs to space-time surface.

The strongest form of $M^8 - H$ duality states that $M^2(x)$ is contained to tangent spaces of X^4 at all points x. Strong form of holography (SH) states allows also the option for which this holds true only for 2-D surfaces - string world sheets and partonic 2-surfaces - therefore mappable to H and that SH allows to determined $X^4 \subset H$ from this data. In the following a realization of this weaker form of $M^8 - H$ duality is found. Note however that one cannot exclude the possibility that also associativity is true only at these surfaces for the polynomial ansatz.

1.4 Number theoretic origin of p-adic primes and dark matter

There are several questions to be answered. How to fuse real number based physics with various p-adic physics? How p-adic length scale hypothesis and dark matter hypothesis emerge from TGD?

The properties of p-adic number fields and the strange failure of complete non-determinism for p-adic differential equations led to the proposal that p-adic physics might serve as a correlate for cognition, imagination, and intention. This led to a development of number theoretic vision which I call adelic physics. A given adele corresponds to a fusion of reals and extensions of various p-adic number fields induced by a given extension of rationals.

The notion of space-time generalizes to a book like structure having real space-time surfaces and their p-adic counterparts as pages. The common points of pages defining is back correspond to points with coordinates in the extension of rationals considered. This discretization of space-time surface is in general finite and unique and is identified as what I call cognitive representation. The Galois group of extension becomes symmetry group in cognitive degrees of freedom. The ramified primes of extension are exceptionally interesting and are identified as preferred p-adic primes for the extension considered.

The basic challenge is to identify dark scale. There are some reasons to expect correlation between p-adic and dark scales which would mean that the dark scale would depend on ramified primes, which characterize roots of the polynomial defining the extensions and are thus not defined completely by extension alone. Same extension can be defined by many polynomials. The naive guess is that the scale is proportional to the dimension n of extension serving as a measure for algebraic complexity (there are also other measures). p-Adic length scales L_p would be proportional nL_p, p ramified prime of extension? The motivation would be that quantum scales are typically proportional to Planck constant. It turns out that the identification of CD scale as dark scale is rather natural.

2 New results about $M^8 - H$ duality

In the sequel some new results about $M^8 - H$ duality are deduced. Strong form of holography (SH) allows to weaken the assumptions making possible $M^8 - H$ duality. It would be enough to map only certain complex 2-D sub-manifolds of quaternionic space-time surface in M^8 to H: SH would allow to determine $X^4 \subset H$ from this 2-D data. Complex sub-manifolds would be determined by conditions completely analogous to those determined space-time surface as quaternionic sub-manifold and they form a discrete set.

2.1 Strong form of holography (SH)

Ordinary 3-D holography is forced by general coordinate invariance (GCI) and loosely states that the data at 3-D surfaces allows to determined space-time surface $X^4 \subset H$. In ZEO 3-surfaces correspond to pairs of 3-surfaces with members at the opposite light-like boundaries of causal diamond (CD) and are analogous to initial and final states of deterministic time evolution as Bohr orbit.

This poses additional strong conditions on the space-time surface.

1. The conjecture is that these conditions state the vanishing of super-symplectic Noether charges for a sub-algebra of super-symplectic algebra SC_n with radial conformal weights coming as n-multiples of those for the entire algebra SC and its commutator $[SC_n, SC]$ with the entire algebra: these conditions generalize super conformal conditions and one obtains a hierarchy of realizations.

   This hierarchy of minimal surfaces would naturally corresponds to the hierarchy of extensions of rationals with n identifiable as dimension of the extension giving rise to effective Planck constant. At the level of Hilbert spaces the inclusion hierarchies for extensions could also correspond to the inclusion hierarchies of hyper-finite factors of type I_1 [K10] so that $M^8 - H$ duality would imply beautiful connections between key ideas of TGD.

2. Second conjecture is that the preferred extremals (PEs) are extremals of both the volume term and Kähler action term of the action resulting by dimensional reduction making possible the induction of twistor structure from the product of twistor spaces of M^4 and CP_2 to 6-D S^2 bundle over X^4 defining the analog of twistor space. These twistor spaces must have Kähler structure since action for 6-D surfaces is Kähler action - it exists only in these two cases [1] so that TGD is unique.

Strong form of holography (SH) is a strengthening of 3-D holography. Strong form of GCI requires that one can use either the data associated either with

- light-like 3-surfaces defining partonic orbits as surfaces at which signature of the induced metric changes from Euclidian to Minkowskian or
- the space-like 3-surfaces at the ends of CD to determine space-time surface as PE (in case that it exists).

This suggests that the data at the intersections of these 2-surfaces defined by partonic 2-surfaces might be enough for holography. A slightly weaker form of SH is that also string world sheets intersecting partonic orbits along their 1-D boundaries is needed and this form seems more realistic.

SH allows to weaken strong form of $M^8 - H$ duality mapping space-time surfaces $X^4 \subset M^8$ to $H = M^4 \times CP_2$ that it allows to map only certain complex 2-D sub-manifolds of quaternionic space-time surface to H: SH allows to determine $X^4 \subset H$ from this 2-D data. Complex sub-manifolds are determined by conditions completely analogous to those determined space-time surface as quaternionic sub-manifold and only discrete set of them is obtained.

2.2 Space-time as algebraic surface in M_c^8 regarded as complexified octonions

The octonionic polynomial giving rise to space-time surface as its "root" is obtained from ordinary real polynomial P with rational coefficients by algebraic continuation. The conjecture is that the identification in terms of roots of polynomials of even real analytic functions guarantees associativity and one can formulate this as rather convincing argument [17, 18, 19]. Space-time surface X_c^4 is identified as a 4-D root for a H_c-valued "imaginary" or "real" part of O_c valued polynomial obtained as an O_c continuation of a real polynomial P with rational coefficients, which can be chosen to be integers. These options correspond to complexified-quaternionic tangent- or normal spaces. For $P(x) = x^n + ..$ ordinary roots are algebraic integers. The real 4-D space-time surface is projection of this surface from M_c^8 to M^8. One could drop the subscripts "$_c$" but in the sequel they will be kept.

M_c^4 appears as a special solution for any polynomial P. M_c^4 seems to be like a universal reference solution with which to compare other solutions.

One obtains also brane-like 6-surfaces as 6-spheres as universal solutions. They have M^4 projection, which is a piece of hyper-surface for which Minkowski time as time coordinate of CD corresponds to a root $t = r_n$ of P. For monic polynomials these time values are algebraic integers and Galois group permutes them.

One cannot exclude rational functions or even real analytic functions in the sense that Taylor coefficients are octonionically real (proportional to octonionic real unit). Number theoretical vision - adelic physics [20], suggests that polynomial coefficients are rational or perhaps in extensions of rationals. The real coefficients could in principle be replaced with complex numbers $a + ib$, where i commutes with the octonionic units and defines complexifiation of octonions. i appears also in the roots defining complex extensions of rationals.

2.3 How do the solutions assignable to the opposite boundaries of CD relate to each other?

CD has two boundaries. The polynomials associated with them could be different in the general formulation discussed in [**?**]chaostgd,SSFRGalois but they could be also same. How are the solutions associated with opposite boundaries of CD glued together in a continuous manner?

1. The polynomials assignable to the opposite boundaries of CD are allowed to be polynomials of o *resp.* $(o - T)$: here T is the distance between the tips of CD.

2. CD brings in mind the realization of conformal invariance at sphere: the two hemispheres correspond to powers of z and $1/z$: the condition $z = \overline{1/z}$ at unit circle is essential and there is no real conjugation. How the sphere is replaced with 8-D CD which is also complexified. The absence of conjugation looks natural also now: could CD contain a 3-surface analogous to the unit circle of sphere at which the analog of $z = \overline{1/z}$ holds true? If so, one has $P(o, z) = P(1/o, z)$ and the solutions representing roots fo $P(o, z)$ and $P(1/o, z)$ can be glued together.

   Note that $1/o$ can be expressed as $\overline{o}/o\overline{o}$ when the Minkowskian norm squared $\overline{o}o$ is non-vanishing and one has polynomial equation also now. This condition is true outside the boundary of 8-D light-cone, in particular near the upper boundary of CD.

   The counter part for the length squared of octonion in Minkowskian signature is light-one proper time coordinate $a^2 = t^2 - r^2$ for M^8_+. Replacing o which scaled dimensionless variable $o_1 = o/(T/2)$ the gluing take place along $a = T/2$ hyperboloid.

One has algebraic holomorphy with respect to o but also anti-holomorphy is possible. What could these two options correspond to? Could the space-time surfaces assignable to self and its time-reversal relate by octonionic conjugation $o \to \overline{o}$ relating two Fock vacuums annihilated by fermionic annihilation *resp.* creation operators?

In [29, 30] the possibility that the sequence of SSFRs or BSFRs could involve iteration of the polynomial defining space-time surface - actually different polynomials were allowed for two boundaries. There are 3 options: each SSFR would involve the replacement $Q = P \circ .. \circ P \to P \circ Q$, the replacement occurs only when new "special moments in the life of self" defined by the roots of P as $t = r_n$ balls of cd, or the replacement can occur in BSFR when the metabolic resources do not allow to continue the iteration (the increase of h_{eff} during iteration increases the needed metabolic feed).

The iteration is compatible with the proposed picture. The assumption $P(0) = 0$ implies that iterates of P contain also the roots of P as roots - they are like conserved genes. Also the 8-D light-cone boundary remains invariant under iteration. Even more general function decompositions $P \to Q \to P$ are consistent with the proposed picture.

2.4 Brane-like solutions

One obtains also 6-D brane-like solutions to the equations.

1. In general the zero loci for imaginary or real part are 4-D but the 7-D light-cone δM^8_+ of M^8 with tip at the origin of coordinates is an exception [17, 18, 19]. At δM^8_+ the octonionic coordinate o is light-like and one can write $o = re$, where 8-D time coordinate and radial coordinate are related by $t = r$ and one has $e = (1 + e_r)/\sqrt{2}$ such that one as $e^2 = e$.

   Polynomial $P(o)$ can be written at δM^8_+ as $P(o) = P(r)e$ and its roots correspond to 6-spheres S^6 represented as surfaces $t_M = t = r_N$, $r_M = \sqrt{r_N^2 - r_E^2} \leq r_N$, $r_E \leq r_N$, where the value of Minkowski time $t = r = r_N$ is a root of $P(r)$ and r_M denotes radial Minkowski coordinate. The points with distance r_M from origin of $t = r_N$ ball of M^4 has as fiber 3-sphere with radius $r = \sqrt{r_N^2 - r_E^2}$. At the boundary of S^3 contracts to a point.

2. These 6-spheres are analogous to 6-D branes in that the 4-D solutions would intersect them in the generic case along 2-D surfaces X^2. The boundaries $r_M = r_N$ of balls belong to the boundary of M^4 light-cone. In this case the intersection would be that of 4-D and 3-D surface, and empty in the generic case (it is however quite not clear whether topological notion of "genericity" applies to octonionic polynomials with very special symmetry properties).

3. The 6-spheres $t_M = r_N$ would be very special. At these 6-spheres the 4-D space-time surfaces X^4 as usual roots of $P(o)$ could meet. Brane picture suggests that the 4-D solutions connect the 6-D branes with different values of r_n.

   The basic assumption has been that particle vertices are 2-D partonic 2-surfaces and light-like 3-D surfaces - partonic orbits identified as boundaries between Minkowskian and Euclidian regions of space-time surface in the induced metric (at least at H level) - meet along their 2-D ends X^2 at these partonic 2-surfaces. This would generalize the vertices of ordinary Feynman diagrams. Obviously this would make the definition of the generalized vertices mathematically elegant and simple.

   Note that this does not require that space-time surfaces X^4 meet along 3-D surfaces at S^6. The interpretation of the times t_n as moments of phase transition like phenomena is suggestive. ZEO based theory of consciousness suggests interpretation as moments for state function reductions analogous to weak measurements ad giving rise to the flow of experienced time.

4. One could perhaps interpret the free selection of 2-D partonic surfaces at the 6-D roots as initial data fixing the 4-D roots of polynomials. This would give precise content to strong form of holography (SH), which is one of the central ideas of TGD and strengthens the 3-D holography coded by ZEO alone in the sense that pairs of 3-surfaces at boundaries of CD define unique preferred extremals. The reduction to 2-D holography would be due to preferred extremal property realizing the huge symplectic symmetries and making $M^8 - H$ duality possible as also classical twistor lift.

   I have also considered the possibility that 2-D string world sheets in M^8 could correspond to intersections $X^4 \cap S^6$? This is not possible since time coordinate t_M constant at the roots and varies at string world sheets.

   Note that the compexification of M^8 (or equivalently octonionic E^8) allows to consider also different variants for the signature of the 6-D roots and hyperbolic spaces would appear for $(\epsilon_1, \epsilon_i, .., \epsilon_8)$, $epsilon_i = \pm 1$ signatures. Their physical interpretation - if any - remains open at this moment.

5. The universal 6-D brane-like solutions S^6_c have also lower-D counterparts. The condition determining X^2 states that the C_c-valued "real" or "imaginary" for the non-vanishing Q_c-valued "real" or "imaginary" for P vanishes. This condition allows universal brane-like solution as a restriction of O_c to M^4_c (that is CD_c) and corresponds to the complexified time=constant hyperplanes defined by the roots $t = r_n$ of P defining "special moments in the life of self" assignable to CD. The condition for reality in R_c sense in turn gives roots of $t = r_n$ a hyper-surfaces in M^2_c.

2.5 Explicit realization of $M^8 - H$ duality

$M^8 - H$ duality allows to map space-time surfaces in M^8 to H so that one has two equivalent descriptions for the space-time surfaces as algebraic surfaces in M^8 and as minimal surfaces with 2-D singularities in H satisfying an infinite number of additional conditions stating vanishing of Noether charges for super-symplectic algebra actings as isometries for the "world of classical worlds" (WCW). Twistor lift allows variants of this duality. M^8_H duality predicts that space-time surfaces form a hierarchy induced by the hierarchy of extensions of rationals defining an evolutionary hierarchy. This forms the basis for the number theoretical vision about TGD.

$M^8 - H$ duality makes sense under 2 additional assumptions to be considered in the following more explicitly than in earlier discussions.

1. Associativity condition for tangent-/normal space is the first essential condition for the existence of $M^8 - H$ duality and means that tangent - or normal space is quaternionic.

2. The tangent space of space-time surface and thus space-time surface itself must contain a preferred $M^2_c \subset M^4_c$ or more generally, an integrable distribution of tangent spaces $M^2_c(x)$ and similar distribution of their complements $E^2c(x)$. The string world sheet like entity defined by this distribution is 2-D surface $X^2_c \subset X^4_c$ in R_c sense. $E^2_c(x)$ would correspond to partonic 2-surface.

One can imagine two realizations for this condition.

Option I: Global option states that the distributions $M_c^2(x)$ and $E_c^2(x)$ define slicing of X_c^4.

Option II: Only discrete set of 2-surfaces satisfying the conditions exist, they are mapped to H, and strong form of holography (SH) applied in H allows to deduce space-time surfaces in H. This would be the minimal option.

That the selection between these options is not trivial is suggested by following.

1. For massless extremals (MEs, topological light rays) parameterized by light-like vector vector k defining $M^2 \subset M^2 \times E^2 \subset M^4$ at each point and by space-like polarization vector ϵ depending on single transversal coordinate of E^2 [12].

2. CP_2 coordinates have an arbitrary dependence on both $u = k \cdot m$ and $w = \epsilon \cdot m$ and can be also multivalued functions of u and w. Single light-like vector k is enough to identify M^2. CP_2 type extremals having metric and Kähler form of CP_2 have light-like geodesic as M^4 projection defining M^2 and its complement E^2 in the normal space.

3. String like objects $X^2 \times Y^2 \subset M^4 \times CP_2$ are minimal surfaces and X^2 defines the distribution of $M^2(x) \subset M^4$. Y^2 ddefines the complement of this distribution.

   Option I is realized in all 3 cases. It is not clear whether M^2 can depend on position in the first 2 cases and also CP_2 point in the third case. It could be that only a discrete set of these string world sheets assignable to wormhole contacts representing massless particles is possible (**Option II**).

How these conditions would be realized?

1. The basic observation is that X^2c can be fixed by posing to the non-vanishing H_c-valued part of octonionic polynomial P condition that the C_c valued "real" or "imaginary" part in C_c sense for P vanishes. M_c^2 would be the simplest solution but also more general complex sub-manifolds $X_c^2 \subset M_c^4$ are possible. This condition allows only a discrete set of 2-surfaces as its solutions so that it works only for **Option II**.

   These surfaces would be like the families of curves in complex plane defined by $u = 0$ an $v = 0$ curves of analytic function $f(z) = u + iv$. One should have family of polynomials differing by a constant term, which should be real so that $v = 0$ surfaces would form a discrete set.

2. As found, there are also classes special global solutions for which the choice of M_c^2 is global and does not depend on space-time point. The interpretation would be in terms of modes of classical massless fields characterized by polarization and momentum. If the identification of M_c^2 is correct, these surfaces are however unstable against perturbations generating discrete string world sheets and orbits of partonic 2-surfaces having interpretation space-time counterparts of quanta. That fields are detected via their quanta was the revolutionary observation that led to quantum theory. Could quantum measurement induce the instability decomposing the field to quanta at the level of space-time topology?

3. One can generalize this condition so that it selects 1-D surface in X_c^2. By assuming that R_c-valued "real" or "imaginary" part of quaternionic part of P at this 2-surface vanishes. one obtains preferred M_c^1 or E_c^1 containing octonionic real and preferred imaginary unit or distribution of the imaginary unit having interpretation as complexified string. Together these kind 1-D surfaces in R_c sense would define local quantization axis of energy and spin. The outcome would be a realization of the hierarchy $R_c \rightarrow C_c \rightarrow H_c \rightarrow O_c$ realized as surfaces.

   This option could be made possible by SH. SH states that preferred extremals are determined by data at 2-D surfaces of X^4. Even if the conditions defining X_c^2 have only a discrete set of solutions, SH at the level of H could allow to deduce the preferred extremals from the data provided by the images of these 2-surfaces under $M^8 - H$ duality. Associativity and existence of $M^2(x)$ would be required only at the 2-D surfaces.

4. I have proposed that physical string world sheets and partonic 2-surfaces appear as singularities and correspond to 2-D folds of space-time surfaces at which the dimension of the quaternionic tangent space degenerates from 4 to 2 [25, 12]. This interpretation is consistent with a book like structure with 2-pages. Also 1-D real and imaginary manifolds could be interpreted as folds or equivalently books with 2 pages.

   For the singular surfaces the dimension quaternionic tangent or normal space would reduce from 4 to 2 and it is not possible to assign CP_2 point to the tangent space. This does not of course preclude the singular surfaces and they could be analogous to poles of analytic function. Light-like orbits of partonic 2-surfaces would in turn correspond to cuts.

5. What could the normal space singularity mean at the level of H? Second fundamental form defining vector basis in normal space is expected to vanish. This would be the case for minimal surfaces.

   (a) String world sheets with Minkowskian signature (in M^4 actually) are expected to be minimal surfaces. In this case T matters and string world sheets could be mapped to H by M^8-H duality and SH would work for them.

   (b) The light-like orbits of partonic 2-surfaces with Euclidian signature in H would serve as analogs of cuts. N is expected to matter and partonic 2-surfaces should be minimal surfaces. Their branching of partonic 2-surfaces is thus possible and would make possible (note the analogy with the branching of soap films) for them to appear as 2-D vertices in H.

   The problem is to identify the pre-images of partonic 2-surfaces in M^8. The light-likeness of the orbits of partonic 2-surfaces (induced 4-metric changes its signature and degenerates to 3-D) should be important. Could light-likeness in this sense define the pre-images partonic orbits in M^8?

Remark: It must be emphasized that SH makes possible M^8-H correspondence assuming that also associativity conditions hold true only at partonic 2-surfaces and string world sheets. Thus one could give up the conjecture that the polynomial ansatz implies that tangent or normal spaces are associative. Proving that this is the case for the tangent/normal spaces of these 2-surfaces should be easier.

2.6 Does M^8-H duality relate hadron physics at high and low energies?

During the writing of this article I realized that M^8-H duality has very nice interpretation in terms of symmetries. For $H=M^4\times CP_2$ the isometries correspond to Poincare symmetries and color $SU(3)$ plus electroweak symmetries as holonomies of CP_2. For octonionic M^8 the subgroup $SU(3)\subset G_2$ is the sub-group of octonionic automorphisms leaving fixed octonionic imaginary unit invariant - this is essential for M^8-H duality. $SU(3)$ is also subgroup of $SO(6)\equiv SU(4)$ acting as rotation on $M^8=M^2\times E^6$. The subgroup of the holonomy group of $SO(4)$ for E^4 factor of $M^8=M^4\times E^4$ is $SU(2)\times U(1)$ and corresponds to electroweak symmetries. One can say that at the level of M^8 one has symmetry breaking from $SO(6)$ to $SU(3)$ and from $SO(4)=SU(2)\times SO(3)$ to $U(2)$.

This interpretation gives a justification for the earlier proposal that the descriptions provided by the old-fashioned low energy hadron physics assuming $SU(2)_L\times SU(2)_R$ and acting acting as covering group for isometries $SO(4)$ of E^4 and by high energy hadron physics relying on color group $SU(3)$ are dual to each other.

2.7 Skyrmions and M^8-H duality

I received a link (`https://tinyurl.com/ycathr3u`) to an article telling about research (`https://tinyurl.com/yddwhr2o`) carried out for skyrmions, which are very general condensed matter quasiparticles. They were found to replicate like DNA and cells. I realized that I have not clarified myself the possibility of skyrmions on TGD world and decided to clarify my thoughts.

2.7.1 What skyrmions are?

Consider first what skyrmions are.

1. Skyrmions are topological entities. One has some order parameter having values in some compact space S. This parameter is defined in say 3-ball such that the parameter is constant at the boundary meaning that one has effectively 3-sphere. If the 3rd homotopy group of S characterizing topology equivalence classes of maps from 3-sphere to S is non-trivial, you get soliton-llike entities, stable field configurations not deformable to trivial ones (constant value). Skyrmions can be assigned to space S which is coset space $SU(2)_L \times SU(2)_R/SU(2)_V$, essentially S^3 and are labelled by conserved integer-valued topological quantum number.

2. One can imagine variants of this. For instance, one can replace 3-ball with disk. $SO(3) = S^3$ with 2-sphere S^2. The example considered in the article corresponds to discretized situation in which one has magnetic dipoles/spins at points of say discretized disk such that spins have same direction about boundary circle. The distribution of directions of spin can give rise to skyrmion-like entity. Second option is distribution of molecules which do not have symmetry axis so that as rigid bodies the space of their orientations is discretized version of $SO(3)$. The field would be the orientation of a molecule of lattice and one has also now discrete analogs of skyrmions.

3. More generally, skyrmions emerge naturally in old-fashioned hadron physics, where $SU(2)_L \times SU(2)_R/SU(2)_V$ involves left-handed, right-handed and vectorial subgroups of $SO(4) = SU(2)_L \times SU(2)_R$. The realization would be in terms of 4-component field (π, σ), where π is charged pion with 3 components - axial vector - and σ which is scalar. The additional constraint $\pi \cdot \pi + \sigma^2 = constant$ defines 3-sphere so that one has field with values in S^3. There are models assigning this kind of skyrmion with nucleon, atomic nuclei, and also in the bag model of hadrons bag can be thought of as a hole inside skyrmion. These models seem to have something to do with reality so that a natural question is whether skyrmions might appear in TGD.

2.7.2 Skyrmion number as winding number

In TGD framework one can regard space-time as 4-surface in either octonionic M^8_c, c refers here to complexification by an imaginary unit i commuting with octonions, or in $M^4 \times CP_2$. For the solution surfaces M^8 has natural decomposition $M^8 = M^2 \times E^6$ and E^6 has $SO(6)$ as isometry group containing subgroup $SU(3)$ having automorphisms of octonions as subgroup leaving M^2 invariant. $SO(6) = SU(4)$ contains $SU(3)$ as subgroup, which has interpretation as isometries of CP_2 and counterpart of color gauge group. This supports $M^8 - H$ duality.

The map $S^3 \rightarrow S^3$ defining skyrmion could be taken as a phenomenological consequence of $M^8 - H$ duality implying the old-fashioned description of hadrons involving broken $SO(4)$ symmetry (PCAC) and unbroken symmetry for diagonal group $SO(3)_V$ (CCV). The analog of (π, σ) field could correspond to a B-E condensate of pions (π, σ).

The obvious question is whether the map $S^3 \rightarrow S^3$ defining skyrmion could have a deeper interpretation in TGD framework. I failed to find any elegant formulation. One could however generalize and ask whether skyrmion like entities characterize by winding number are predicted by basic TGD.

1. In the models of nucleon and nuclei the interpretation of conserved topological skyrmion number is as baryon number. This number should correspond to the homotopy class of the map in question, essentially winding number. For polynomials of complex number degree corresponds to winding number. Could the degree $n = h_{eff}/h_0$ of polynomial P having interpretation as effective Planck constant and measure of complexity - kind of number theoretic IQ - be identifiable as skyrmion number? Could it be interpreted as baryon number too?

2. For leptons regarded as local 3 anti-quark composites in TGD based view about SUSY [27] the same interpretation would make sense. It seems however that the winding number must have both signs. Degree is n is however non-negative.

   Here complexification of M^8 to M^8_c is essential. One an allow both holomorphic and anti-holomorphic continuations of real polynomials P (with rational coefficients) using complexification defined by commutative imaginary unit i in M^8_c so that one has polynomials $P(z)$ *resp.* $P(\overline{z})$ in turn algebraically continued to complexified octonionic polynomials $P(z,o)$ *resp.* $P(\overline{z},o)$.

   Particles *resp.* antiparticles would correspond to the roots of octonionic polynomial $P(z,o)$ resp. $P(\overline{z},o)$ meaning space-time geometrization of the particle-antiparticle dichotomy and would be conjugates of each other. This could give a nice physical interpretation to the somewhat mysterious complex roots of P.

2.7.3 More detailed formulation

To make this formulation more detailed on must ask how 4-D space-time surfaces correspond to 8-D "roots" for the "imaginary" ("real") part of complexified octonionic polynomial as surfaces in M^8_c.

1. Equations state the simultaneous vanishing of the 4 components of complexified quaternion valued polynomial having degree n and with coefficients depending on the components of O_c, which are regarded as complex numbers $x + iy$, where i commutes with octonionic units. The coefficients of polynomials depend on complex coordinates associated with non-vanishing "real" ("imaginary") part of the O_c valued polynomial.

2. To get perspective, one can compare the situation with that in catastrophe theory in which one considers roots for the gradient of potential function of behavior variables x^i. Potential function is polynomial having control variables as parameters. Now behavior variable correspond "imaginary" ("real") part and control variables to "real" ("imaginary") of octonionic polynomial.

   For a polynomial with real coefficients the solution divides to regions in which some roots are real and some roots are complex. In the case of cusp catastrophe one has cusp region with 3-D region of the parameter defined by behavior variable x and 2 control parameters with 3 real roots, the region in which one has one real root. The boundaries for the projection of 3-sheeted cusp to the plane defined by control variables correspond to degeneration of two complex roots to one real root.

   In the recent case it is not clear whether one cannot require the M^8_c coordinates for space-time surface to be real but to be in $M^8 = M^1 + iE^7$.

3. Allowing complex roots gives 8-D space-time surfaces. How to obtain real 4-D space-time surfaces?

   (a) One could project space-time surfaces to real $M^8 = M^1 + iE^7$ to obtain 4-D real space-time surfaces. In time direction the real part of root is accepted and is same for the root and its conjugate. For E^7 this would mean that imaginary part is picked up.

   (b) If one allows only real roots, the complex conjugation proposed to relate fermions and anti-fermions would be lost.

4. One can select for 4 complex M^8_c coordinates X^k of the surface and the remaining 4 coordinates Y^k can be formally solved as roots of n:th degree polynomial with dynamical coefficients depending on X^k and the remaining Y^k. This is expected to give rise to preferred extremals with varying dimension of M^4 and CP_2 projections.

5. It seems that all roots must be complex.

(a) The holomorphy of the polynomials with respect to the complex M_c^8 coordinates implies that the coefficients are complex in the generic point M_c^8. If so, all 4 roots are in general complex but do not appear as conjugate pairs. The naive guess is that the maximal number of solutions would be n^4 for a given choice of M^8 coordinates solved as roots. An open question is whether one can select subset of roots and what happens at $t = r_n$ surfaces: could different solutions be glued together at them.

(b) Just for completeness one can consider also the case that the dynamical coefficients are real - this is true in the E^8 sector and whether it has physical meaning is not clear. In this case the roots come as real roots and pairs formed by complex root and its conjugate. The solution surface can be divided into regions depending on the character of 4 roots. The n roots consist of complex root pairs and real roots. The members or complex root pairs are mapped to same point in E^8.

2.7.4 Could skyrmions in TGD sense replicate?

What about the observation that condensed matter skyrmions replicate? Could this have analog at fundamental level?

1. The assignment of conserved topological quantum number to the skyrmion is not consistent with replication unless the skyrmion numbers of outgoing states sum up to that of the initial state. If the system is open one can circumvent this objection. The replication would be like replication of DNA in which nucleotides of new DNA strands are brought to the system to form new strands.

2. It would be fascinating if all skyrmions would correspond to space-time surfaces at fundamental M^8 level. If so, skyrmion property also in magnetic sense could be induced by from a deeper geometric skyrmion property of the MB of the system. The openness of the system would be essential to guarantee conservation of baryon number. Here the fact that leptons and baryons have opposite baryon numbers helps in TGD framework. Note also ordinary DNA replication could correspond to replication of MB and thus of skyrmion sequences.

3 About p-adic length scale hypothesis and dark matter hierarchy

It is good to introduce first some background related to p-adic length scale hypothesis discussed in chapters of [11] and dark matter hierarchy discussed in chapters [9, 10], in particular in chatper [13].

3.1 General form of p-adic length scale hypothesis

The most general form of p-adic length scale hypothesis does not pose conditions on allowed p-adic primes and emerges from p-adic mass calculations [5, 6, 7]. It has two forms corresponding to massive particles and massless particles.

1. For massive particles the preferred p-adic mass calculations based on p-adic thermodynamics predicts the p-adic mass squared m^2 to be proportional to p or its power- the real counterpart of m^2 is proportional to $1/p$ or its power. In the simplest case one has

$$m^2 = \frac{X}{p}\frac{\hbar}{L_0} \ ,$$

where L_0 is apart from numerical constant the length R of CP_2 geodesic circle. X is a numerical constant not far from unity. $X \geq 1$ is small integer in good approximation. For instance for electron one has $x = 5$.

By Uncertainty Principle the Compton length of particle is characterizing the size of 3-surfaces assignable to particle are proportional to $\sqrt{p}$:

$$L_c(m) = \frac{\hbar}{m} = \sqrt{\frac{1}{X}} L_p \ , \quad L_p = \sqrt{p} L_0 = \quad .$$

Here L_p is p-adic length scale and corresponds to minimal mass for given p-adic prime. p-Adic length scale would be would characterize the size of the 3-surface assignable to the particle and would correspond to Compton length.

2. For massless particles mass vanishes and the above picture is not possible unless there is very small mass coming from p-adic thermodynamics and determined by the size scale of CD - this is quite possible. The preferred time/spatial scales p-adic energy- equivalently 3-momentum are proportional to p-adic prime p or its power. The real energy is proportional to $1/p$. At the imbedding space level the size of scale causal diamond (CD) [26] would be proportional to p: $L = T = pL_0$, $L_0 = T_0$ for $c = 1$. The interpretation in terms of Uncertainty Principle is possible.

   There would be therefore two levels: space-time level and imbedding space level . At the space-time level the primary p-adic length scale would be proportional to $\sqrt{p}$ whereas the p-adic length scale at imbedding space-time would correspond to secondary p-adic length scale proportional to p. The secondary p-adic length scales would assign to elementary new physics in macroscopic scales. For electron the size scale of CD would be about .1 seconds, the time scale associated with the fundamental bio-rhythm of about 10 Hz.

3. A third piece in the picture is adelic physics [20, 21] inspiring the hypothesis that effective Planck constant h_{eff} given by $h_{eff}/h_0 = n$, $h = 6h_0$, labels the phases of ordinary matter identified as dark matter. n would correspond to the dimension of extension of rationals.

   The connection between preferred primes and the value of $n = h_{eff}/h_0$ is interesting. One proposal is that preferred primes p in p-adic length scale hypothesis determining the mass scale of particle correspond to so called ramified primes, which characterize the extensions. The p-adic variant of the polynomial defining space-time surfaces in M^8 picture would have vanishing discriminant in order $O(p)$. Since discriminant is proportional to the product of differences of different roots of the polynomial, two roots would be very near to each other p-adically. This would be mathematical correlate for criticality in p-adic sense.

   $M^8 - H$ duality [24, 23] leads to the prediction that the roots r_n of polynomial defining the space-time region in M^8 correspond to preferred time values $t = t_n = \propto r_n$- I have called $t = t_n$ "special moments in the life of self". Since the squares for the differences for the roots are proportional to ramified primes, these time differences would code for ramified primes assignable to the space-time surface. There would be several p-adic time scales involved and they would be coded by $t_{ij} = r_i - r_j$, whose moduli squared are divided by so called ramified primes defining excellent candidates for preferred p-adic primes. p-Adic physics would make itself visible at the level of space-time surface in terms of "special moments in the life of self".

4. p-Adic length scales emerge naturally from $M^8 - H$ duality [24, 23]. Ramified primes would in M^8 picture appear as factors of time differences associated with "special moments in the life of self" associated with CD [23]. One has $|t_i - t_j| \propto \sqrt{p_{ij}}$, p_{ij} ramified prime. It is essential that square root of ramified prime appears here.

   This suggests strongly that p-adic length scale hypothesis is realized at the level of space-time surface and there are several p-adic length scales present coded to the time differences. Knowing of the polynomial would give information about p-adic physics involved. If dark scales correlate with p-adic length scales as proposed, the definition of dark scale should assume the dependence of ramified primes quite generally rather than as a result of number theoretic survival of fittest as one might also think.

The factors $t_i - t_j$ are proportional - not only to the typically very large p-adic prime p_{max} charactering the system - but also smaller primes or their powers. Could the scales in question be of form $l_p = \sqrt{X}\sqrt{p_{max}}L_0$ rather than p-adic length scales $L_{p_{ram}}$ defined by various ramified primes. Here X would be integer consisting of small ramified primes.

p-Adic mass calculations predict in an excellent approximation the mass of the particle is given by $m = (\sqrt{X}/\sqrt{p})m_0$, X small integer and $m_0 = 1/L_0$. Compton length would be given by $L_c(p) = \sqrt{p}/\sqrt{X})L_0$. The identification $l_p = L_c(p)$ would be attractive but is not possible unless one has $X = 1$. In this case one would be considering p-adic length scale L_p. The the interpretation in terms of multi-p-adicity seems to be the realistic option.

3.2 About more detailed form of p-adic length scale hypothesis

More specific form of p-adic length scale hypothesis poses conditions on physically preferred p-adic primes. There are several guesses for preferred primes. They could be primes near to integer powers 2^k, where k could be positive integer, which could satisfy additional conditions such as being odd, prime or be associated with Mersenne prime or Gaussian Mersenne. One can consider also powers of other small primes such as $p = 2, 3, 5$. p-Adic length scale hypothesis in is basic form would generalize the notion of period doubling. For odd values of k one would indeed obtain period doubling, tripling, etc... suggesting strongly chaos theoretic origin.

3.2.1 p-Adic length scale hypothesis in its basic form

Consider first p-adic length scale hypothesis in its basic form.

1. In its basic form states that primes $p \simeq 2^k$ are preferred p-adic primes and correspond by p-adic mass calculations p-adic length scales $L_p \equiv L(k) \propto \sqrt{p} = 2^{k/2}$. Mersenne primes and primes associated with Gaussian Mersennes as especially favored primes and charged leptons ($k \in \{127, 113, 107\}$) and Higgs boson ($k = 89$) correspond to them. Also hadron physics ($k = 107$) and nuclear physics ($k = 113$) correspond to these scales. One can assign also to hadron physics Mersenne prime and the conjecture is that Mersennes and Gaussian Mersennes define scaled variants of hadron physics and electroweak physics. In the length scale between cell membrane thickness fo 10 nm and nuclear size about 2.5 μm there are as many as 4 Gaussian Mersennes corresponding to $k \in \{151, 157, 163, 167\}$.

   Mersenne primes correspond to prime values of k and I have proposed that k is prime for fundamental p-adic length scales quite generally. There are also however also other p-adic length scales - for instance, for quarks k need not be prime - and it has remained unclear what criterion could select the preferred exponents k. One can consider also the option that odd values of k defined fundamental p-adic length scales.

2. What makes p-adic length scale hypothesis powerful is that masses of say scaled up variant of hadron physics can be estimated by simple scaling arguments. It is convenient to use electron's p-adic length scale and calculate other p-adic length scales by scaling $L(k) = 2^{(k-127)/2}L(127)$.

Here one must make clear that there has been a confusion in the definitions, which was originally due to a calculational error.

1. I identified the p-adic length scale $L(151)$ mistakenly as $L(151) = 2^{(k-127)/2}L_e(127)$ by using instead of $L(127)$ electron Compton length $L_e \simeq L(127/\sqrt{5}$. The notation for these scales would be therefore $L_e(k)$ identified as $L_e(k) = 2^{(k-127)/2}L_e(127)$ and I have tried to use it systematically but failed to use the wrong notation in informal discussions.

2. This mistake might reflect highly non-trivial physics. It is scaled up variants of L_e which seem to appear in physics. For instance, $L_e(151) \simeq 10$ nm corresponds to basic scale in living matter. Why the biological important scales should correspond to scaled up Compton lengths for electron? Could dark electrons with scaled up Compton scales equal to $L_e(k)$ be important in these scales? And what about the real p-adic length scales relate to these scales by a scaling factor $\sqrt{5} \simeq 2.23$?

3.2.2 Possible modifications of the p-adic length scale hypothesis

One can consider also possible modifications of the the p-adic length scale hypothesis. In an attempt to understand the scales associated with INW structures in terms of p-adic length scale hypothesis it occurred to me that the scales which do not correspond to Mersenne primes or Gaussian Mersennes might be generated somehow from the these scales.

1. Geometric mean $L = \sqrt{L(k_1)L(k_2)}$ would length scale which would correspond to L_p with $p \simeq 2^{(k_1+k_2)/2}$. This is of the required form only if $k = k_1 + k_2$ is even so that k_1 and k_2 are both even or odd. If one starts from Mersennes and Gaussian Mersennes the condition is satisfied. The value of $k = (k_1 + k_2)/2$ can be also even.

   Remark: The geometric mean $(127 + 107)/2 = 117$ of electronic and hadronic Mersennes corresponding to mass 16 MeV rather near to the mass of so called X boson [15] (`https://tinyurl.com/ya3yuzeb`).

2. One can also consider the formula $L = (L(k_1)L(k_2)..L(k_n))^{1/n}$ but in this case the scale would correspond to prime $p \simeq 2^{k_1+...k_n)/n}$. Since $(k_1 + ..k_n)/n$ is integer only if $k_1 + ...k_n$ is proportional to n.

What about the allowed values of fundamental integers k? It seems that one must allow all odd integers.

1. If only prime values of k are allowed, one can obtain obtain for twin prime pair $(k-1, k+1)$ even integer k as geometric mean $\sqrt{k}$ if k is square. If prime k is not a member of this kind of pair, it is not possible to get integers $k-1$ and $k+1$. If only prime values of k are fundamental, one could assign to $k = 89$ characterizing Higgs boson weak bosons $k = 90$ possibly characterizing weak bosons. Therefore it seems that one must allow all odd integers with the additional condition already explained.

2. Just for fun one can check whether $k = 161$ forced by the argument related to electroweak scale and h_{eff} corresponds to a geometric mean of two Gaussian Mersennes. One has $k(k_1, k_2) = (k_2 + k_2)/2$ giving the list $k(151, 157) = 154$), $k(151, 163) = 157$ Gaussian Mersenne itself, $k(151, 167) = 159$, $k(157, 163) = 160$, $k(157, 167) = 162$, $k(163, 167) = 165$. Unfortunately, $k = 161$ does not belong to this set. If one allows all odd values of k as fundamental, the problem disappears.

One can also consider refinements of p-adic length scale hypothesis in its basic form.

1. One can consider also a generalization of p-adic length scale hypothesis to allow length scales coming as powers of small primes. The small primes $p = 2, 3, 5$ assignable to Platonic solids would be especially interesting. $p = 2, 3, 5$ and also Fermat primes and Mersenne primes are maximally near to powers of two and their powers would define secondary and higher p-adic length scales. In this sense the extension would not actually bring anything new.

   There is evidence for the occurrence of long p-adic time scales coming as powers of 3 [3, 4] (`http://tinyurl.com/ycesc5mq`) and [8] (`https://tinyurl.com/y8camqlt`. Furthermore, prime 5 and Golden Mean are related closely to DNA helical structure. Portion of DNA with L(151) contains 10 DNA codons and is the minimal length containing an integer number of codons.

2. The presence of length scales associated with 1 nm and 2 nm thick structures encourage to consider the possibility of p-adic primes near integers $2^k3^l5^m$ defining generators of multiplicative ideals of integers. They do not satisfy the maximal nearness criterion anymore but would be near to integers representable as products of powers of primes maximally near to powers of two.

What could be the interpretation of the integer k appearing in $p \simeq 2^k$? Elementary particle quantum numbers would be associated with wormhole contacts with size scale of CP_2 whereas elementary particles correspond to p-adic size scale about Compton length. What could determine the size scale of wormhole contact? I have proposed that to p-adic length scale there is associated a scale characterizing wormhole contact and depending logarithmically on it and corresponds to $L_k = (1/2)log(p)L_0 = (k/2)log(2)L_0$. The generalization of this hypothesis to the case of $p \simeq 2^k3^l5^m...$ be straightforward and be $L_{k,l,m} = (1/2)(klog(2) + llog(3) + mlog(5) + ..)$.

3.3 Dark scales and scales of CDs and their relation to p-adic length scale hierarchy

There are two length scale hierarchies. p-Adic length scale hierarchy assignable to space-time surfaces and the dark hierarchy assignable to CDs. One should find an identification of dark scales and understand their relationship to p-adic length scales.

3.3.1 Identification of dark scales

The dimension n of the extension provides the roughest measure for its complexity via the formula $h_{eff}/h_0 = n$. The basic - rather ad hoc - assumption has been that n as dimension of extension defines not only h_{eff} but also the size scale of CD via $L = nL_0$.

This assumption need not be true generally and already the attempt to understand gravitational constant [28] as a prediction of TGD led to the proposal that gravitational Planck constant $h_{gr} = n_{gr}h_0 = GMm/v_0$ [2] could be coded by the data relating to a normal subgroup of Galois group appearing as a factor of n.

The most general option is that dark scale is coded by a data related to extension of its sub-extension and this data involves ramified primes. Ramified primes depend on the polynomial defining the extension and there is large number polynomials defining the same extension. Therefore ramified ramifies code information also about polynomial and dynamics of space-time surface.

First some observations.

1. For Galois extension the order n has a natural decomposition to a product of orders n_i of its normal subgroups serving also as dimensions of corresponding extensions: $n = \prod_i n_i$. This implies a decomposition of the group algebra of Galois group to a tensor product of state spaces with dimensions n_i [30].

2. Could one actually identify several dark scales as the proposed identifications of gravitational, electromagnetic, etc variants of h_{eff} suggest? The hierarchy of normal subgroups of Galois group of rationals corresponds to sub-groups with orders given by $N(i,1) = n_in_{i-1}...n_{i-1}$ of n define orders for the normal subgroups of Galois group. For extensions of $k-1$:th extension of rationals one has $N(i,k) = n_in_{i-1}...n_{i-k}$. The most general option is that these normal subgroups provide only the data allowing to associate dark scales to each of them. The spectrum of h_{eff} could correspond to the $\{N_{i,k}\}$ or at least the set $\{N_{i,1}\}$.

3. The extensions with prime dimension $n = p$ have no non-trivial normal subgroups and $n = p$ would hold for them. For these extensions the state space of group algebra is prime as Hilbert space and does not decompose to tensor product so that it would represent fundamental system. Could these extensions be of special interest physically? SSFRs would naturally involve state function reduction

cascades proceeding downwards along hierarchy of normal subgroups and would represent cognitive measurements [30].

The original guess was that dark scale $L_D = nL_p$, where n is the order n for the extensions and p is a ramified prime for the extension. A generalized form would allow $L_D = N(i,1)L_{p_k}$ for the sub-extension such that p_k is ramified prime for the sub-extension.

3.3.2 Can one identify the size scale of CD as dark scale?

It would be natural if the scale of CD would be determined by the extension of rationals. Or more generally, the scales of CD and hierarchy of sub-CDs associated with the extension would be determined by the inclusion hierarchy of extensions and thus correspond to the hierarchy of normal sub-groups of Galois group.

The simplest option would be $L_{CD} = L_D$ so that the size scales of sub-CD would correspond dark scales for sub-extension given by $L_{CD,i} = N(i,1)L_{p_k}$, p_k ramified prime of sub-extension.

1. The differences $|r_i - r_j|$ would correspond to differences for Minkowski time of CD. CD need not contain all values of hyperplanes $t = r_i$ and the evolution by SSFR would gradually bring in daylight all roots r_n of the polynomial P defining space-time surface as "special moments in the life of self". If the size scale of CD is so large that also the largest value of $|r_i|$ is inside the upper or lower half of CD, the size scale of CD would correspond roughly to the largest p-adic length scale.

   CD contains sub-CDs and these could correspond to normal subgroups of Galois extension as extension of extension of ....

2. One can ask what happens when all special moments $t = r_n$ have been experienced? Does BSFR meaning death of conscious entity take place or is there some other option? In [29] I considered a proposal for how chaos could emerge via iterations of P during the sequence of SSFRs.

   One could argue that when CD has reached by SSFRs following unitary evolutions a size for which all roots r_n have become visible, the evolution could continues by the replacement of P with $P \circ P$, and so on. This would give rise to iteration and space-time analog for the approach to chaos.

3. Eventually the evolution by SSFRs must stop. Biological arguments suggests that metabolic limitations cause the death of self since the metabolic energy feed is not enough to preserve the distribution of values of h_{eff} (energies increase with $h_{eff} \propto Nn$, for N:th iteration and h_{eff} is reduced spontaneously) [31].

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