Essay

A Unit Cell of Quantum Gravity

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Abstract

Using only the most basic elements of Planck scale dynamics, it is possible to derive a large body of knowledge. The key is to discern what comprises a minimal unit cell that incorporates both extent and curvature. This notion arises in the context of Bekenstein-Hawking entropy, where the entropy of a black hole is seen to vary with the surface area in discrete units 4 times the smallest area that is bounded by the Planck length. This paper explains why this fundamental relation for black holes shows us the unit cell for quantum gravity.

Keywords: Bekenstein-Hawking entropy, Planck scale, simplicial fabric, causal structure.

Introduction

Planck's constant represents a minimum scale for quantum transitions, for a wide variety of physical processes. It represents the graininess of physical reality by denoting the smallest increments in which things can vary, or in multiples thereof. Planck's constant appears in quite a few Physics equations, but here the focus is mainly on how areas of space exist only in multiples of the Planck area, and why a surface with exactly four Planck areas holds a special significance in the context of quantum gravity. The reason is astonishingly straightforward; this simple form encodes curvature too! In Einstein's formulation of gravity, the essential element is that the fabric of space or spacetime curves in the presence of mass, where massive objects have trajectories moving along those curves. So a basis for curvature in a quantum mechanical context is a desirable foundation to build a theory of quantum gravity upon. The remainder of this paper discusses this basis and its implications.

A fundamental extent for length and time

When we approach the limits of how small things can get, or seek to examine the smallest steps by which things can vary, a fundamental unit appears known as Planck's constant. If we apply this number to increments of length or time, the size of each step is so small that variations in space and time appear to be absolutely smooth. One might say instead that the graininess of spacetime is very fine indeed, so the fabric can be deformed smoothly to an incredible degree, but not infinitely so. In fact, it appears absolutely smooth, for all practical purposes. The fact that spacetime is grainy at all comes out only in the most extreme settings – like near the event

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*Correspondence: Jonathan J. Dickau, Independent Researcher. E-mail: jonathan@jonathandickau.com Note: This is an Essay written for the 2020 Gravity Research Foundation Awards for essays on gravitation submitted April 1, 2020 horizon of a very small black hole. For the sake of simplicity I'll discuss only those objects that have no charge or spin, which are called Schwarzschild black holes. A spinning black hole can drag the lines of space around it, and this makes for a much messier model that obscures some of the details we would like to highlight here. So a Schwarzschild black hole makes a better example.

$$S_{BH} = \frac{A}{4L_P^2} = \frac{c^3 A}{4G\hbar}$$

Eq. 1. Bekenstein-Hawking equation for Schwarzschild black hole entropy displays granularity

It became apparent to Bekenstein and Hawking [1] that the surface area of a black hole is not smoothly varying, but instead varies in discrete increments, when they tried to work out a fundamental equation for its entropy. So in the equation for Bekenstein-Hawking entropy [2] the smallest increment of entropy corresponds to a black hole growing larger by four Planck areas. This makes some researchers wonder what is so special about having four times the smallest possible area. I think part of the reason is that when the surface of a black hole is tessellated with triangles whose sides are the Planck length, each triangle is a flat surface or facet, but a unit cell comprised of a triangle and the three adjacent also encodes a specific amount of curvature. In order to wrap around the surface of a sphere, the outermost points of the outside triangles (in such an array) must be bent down. If we continue bending all those points in the same direction, the four triangles form a tetrahedron. But in the case where they are part of a simplicial manifold wrapped around a spherical surface, as with the event horizon of a Schwarzschild black hole, we see that adding more triangles allows us to exactly follow a spherical curvature that flattens in small increments, as the radius increases. So as part of a fabric, and as the tessellated sphere grows larger, a unit cell of four triangular Planck areas can represent very tiny changes in curvature.

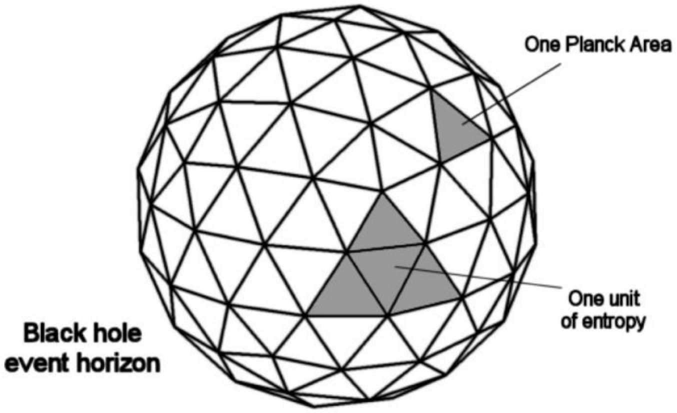

Fig. 1. Tessellated small Black Hole horizon shows Planck scale and entropy relation

We find that a structure consisting of just four triangles has considerable flexibility. The corners can be bent up or down by any amount, until they meet. The figure can flex both ways with one corner bent in the opposite direction from another, or be coplanar. So it can serve as an element in a wave-like propagation across a fabric. These features make this structure a good candidate for a unit cell of quantum gravity. Variations on this theme have already been tried, such as Causal Dynamical Triangulations (or CDT) [3], but there were a few wrinkles to work out to make it a functional theory.

For CDT, it was necessary to stipulate that where edges meet along a time-like boundary, the directions of time must match. Then they allow this structure to be extended from triangles (the 2-simplex) to tetrahedra (or 3-simplices), and then to a 4-d generalization of this structure (the 4-simplex). This produces the well-known feature of dimensional reduction [4] near the Planck scale, which happens automatically, and appears to be a universal feature of all quantum gravity theories [5]. However, this is not the only complication we encounter near the Planck scale. In fact, one of the hallmarks of Planck scale dynamics is that uncertainty reigns, or becomes absolute. One can think of the Planck length as a unit of positional uncertainty, as well as a minimal extent for position to vary, because we can never know the position of anything more precisely. So the Planck length is a unit of length whose uncertainty is as great as its extent. It is like the freedom to vary by a certain amount, before it counts as a change.

In this context, an array of four Planck-sized triangles takes on a different meaning, because its size is just large enough that it is at least a Planck length in all directions, with an extra Planck length's uncertainty added in. This may make a 2 Planck-length triangular figure the smallest parcel of space it is possible to define. So if a single Planck-sized triangle can be viewed as having an uncertain or indefinite extent, the next larger triangular figure surpasses the threshold where it can shrink to Planck-sized – but no further – so it must be an actual extent. That is, a 4 Planck-triangle area is arguably the smallest figure which is verifiably *at least* Planck-sized in extent. This implies a hysteresis effect where once space exceeds a threshold; it must shrink to below where it started to return to its original size. And positional uncertainty can never be smaller than the Planck length, so there is a lower limit to how small a parcel of space can shrink. Therefore, this mechanism is seen to implement a ratcheting effect, such that once spacetime grows larger past a certain point it cannot return to its original size, but must instead grow larger still. This is likely related to BH surfaces growing ever larger over time. But it implies that a similar mechanism is involved in geometrogenesis at the universe's origin. So we will examine this ratcheting effect more closely.

Enough Space to put Things

There is uncomfortable obscurity about anything that occurs or occurred near the Planck scale, either in the microscale or during the universe's first instants. We simply do not know enough about dynamics in that regime to make sense of it. But it appears the existence of a Planck scale implies a profound relationship between minimal extents of space and time which create a spacetime fabric, and corresponding amounts of mass and energy that induce curvature in the fabric. A Planck-sized triangle is the minimal container for a parcel of space. The question is, if the fabric starts out being 2-d at the Planck scale, or if it is otherwise unconstrained and undefined at the outset, how do we evolve spaces dimensionally sufficient to contain the familiar forms like massive particles that have a definite size? This is largely a matter of the formation of persistent bubbles within the energetic fluid of the early universe. But without some process of geometrogenesis this could not occur. The earlier discussion posited that mass-energy engenders curvature in the fabric of spacetime, which in Einstein's Relativity is the source of gravitation. But this leaves it unclear exactly how volumetric spaces emerge and how they came to possess duration or persistence.

If we assume that this did take place, we can make some observations about how it had to happen. For one thing, energy is a resonant phenomenon whose waves have a characteristic wavelength or frequency, and a tendency to extend or propagate where allowed. But that wavelength varies in discrete multiples of the Planck length, so that it is like an enormous arpeggio or a descending scale starting at the highest note possible and going down from there, all the way to the current background energy. There is a sense that this resonant energy couples directly to the spacetime background in the early universe, at least up to a point where massive particles first appear. So a void of a specific size tends to hold energy of a characteristic wavelength or frequency. In this context, it is plain that a space-occupying form like a proton is possible to create only once we have volumetric bubbles of sufficient size to allow its existence. That is, fermionic particles can appear only once there are bubbles of comparable size or larger, in the early universe. So the challenge in quantum gravity theories is to engineer ways this process could unfold naturally. Ergo, there is an incentive to chart a course from the first instant to that point.

Enough Time for Space and Objects to Form

The other component we must consider in this analysis is time. Having space for things to exist in does not automatically give either space itself or things contained therein persistence or permanence. As noted by Plato [6], the persistence of things in time is the limited expression of eternal or permanent existence. The property of duration gives things a definite lifetime, or a half-life. And even when we are talking about a parcel of space, it must persist long enough to contain or participate in interactions, in order to exist at all. One can easily see this by asking the question, if something persists for zero time, can it actually be said to exist? Only when things persist for a definite time do they have duration, and only then can they be observed. This is the difference between virtual particles and actual particles. Virtual particles pop out of nothing in pairs, and they exist as an extended property of the vacuum, but they are ephemeral having no persistence or duration, so they are not directly observable, and are only weakly detectable through their interactions. Actual particles do participate in interactions with both the background and other particles, and they are for the most part observable, so they are completely physical while virtual particles are not. Therefore the property of persistence in time appears essential to physical existence.

While many take the view that time disappears or is irrelevant at the Planck scale, I think instead that time is more fundamental or primal than space, because it is essential to physical existence. If spacetime was not persistent with properties that support the existence of massive particles, we could not exist, and neither would the cosmos. In my view, it is the progress of processes unfolding that marks the passage of time, so arguments that things can happen outside of time or before the beginning of time are questionable. That is to say, the sense of directionality and of procedural progression is itself a clear marker of time's passage. If the argument against time being fundamental is made on the basis of symmetry in natural law alone, I am especially wary, because I imagine that the cosmos is globally asymmetric, in its progression over the eons, even while symmetries in the local universe may exactly apply out to millions of light years. Any other conclusion appears incompatible with the 2nd law of Thermodynamics. Many cosmologists project the 'heat death' of the universe and a cold dark end to the cosmos, which indisputably reflects this sensibility. So while space and time may be interchangeable in Special Relativity, so that spacetime has meaning apart from its components, we openly acknowledge that time is asymmetric. However a similar global asymmetry also appears to be a generic feature characterizing all theories of entropic or emergent gravity [7].

So there are both (or at least) procedural and thermodynamic arrows of time involved in cosmological evolution. In the current era, it appears that the two arrows face in the same direction for the most part. We do not see a glass that has smashed on the floor reassemble itself spontaneously, although it might be possible theoretically. In the early universe, it may be true that these two arrows of time were independent. However, to fully understand this, we need to look at the prehistory of the cosmos as well. It may rightly be said that geometry pertains mainly to forms that exist in or have extents in space. And what we have been discussing here is how space as we know it comes to be or evolves. We have established already that we cannot assume the existence of volumetric spaces or spacetime is automatic, but instead it requires a process by which appropriately dimensioned spaces are unfolded. Ergo, the earliest phase of cosmological evolution deals with pregeometry [8] as it pertains to the prerequisite items and conditions necessary for geometrogenesis to commence. This is where the language of directed graphs and category theory serves us well, while regular geometry fails to capture important details.

So if we follow through with this analysis, we find that evolving geometry involves moving from a unified state to one characterized by separated or extended forms. By a unified state, I mean there is an absence of boundaries or surfaces that serve as topological distinctions, and by the evolution of geometry, I refer to the emergence of such forms. This is absolutely necessary for the cosmos to exist.

While we may imagine a geometric point as an abstraction, a purely point-like entity cannot exist in physical reality, because its energy would be boundless or infinite. We know that as we go from visible light to X-rays and Gamma rays, individual quanta or photons become smaller and smaller in size – approaching a point-like condition. But we also know there is a hard limit at the Planck scale, because it is not possible for something to get any smaller, and space itself becomes undefined instead. In my view, this does not cancel out the directionality of time, but instead strengthens the requirement for a time direction. One finds it useless to have an entity that is space-wise point-like, if it is also time-wise point-like, because this makes it fleeting or ephemeral to the extent of being virtual. But in the 0-brane, we have something with a point-like projection in space that brackets or contains an instant of time, and is therefore called an instanton. This appears to imply that the first manifestation at the Planck scale needed to be a minimum time step, and by the previous arguments this would automatically unfold the spacetime fabric to its present dimensions.

Directions for Future Research

The later processes of cosmological evolution we understand well needed to first evolve an arena for that process to take place in – under the above assumptions. The first step in finding the right answers is to understand that space or spacetime doesn't happen automatically. It is only possible for there to be an open or empty space if there is a structure to hold that space open or keep it from contracting or collapsing – i.e. containers. This makes the properties of a triangle as a minimal container that also resists collapse or twisting quite attractive. Space and energy are thus contained in the early cosmos, in a causal structure that unfolds or evolves over time. Conversely, it can be seen that a bubble of high energy in a cooler medium would tend to expand or to create more space – as with steam in water.

So there is a force and a rationale for energy to resist containment, or to exceed its bounds wherever possible. This is the dynamic tension between opposing principles that drives or defines geometrogenesis in the earliest moments of the universe. To restate this, there needed to be a phase of spacetime evolution before massive particles of familiar types could appear. And in that era the role of energy was largely to create more space and time – enough for a large number of particles to reside in. We see that this resulted in a large expanse of space that persists in time with long duration. And we know that once that space grew large enough, and the collection of particles cooled, they formed atoms and molecules to make all of the forms now familiar to us.

The challenge is using what we have learned already to explore other avenues of research and to modify techniques now in use, to allow unexplored areas of the parameter space to be investigated and navigated. In the CDT model, for example, the simplicial fabric can be allowed to extend to higher dimensions (using the 5-simplex…) then used to explore dynamics in cosmological scenarios where the early cosmos evolved in higher-d spacetime. However complications creep in rapidly because geometry becomes first non-commutative then non-associative when adding more dimensions beyond three.

One needs to manage evolutive behaviors by observing rules of order and sequence that become more complicated as you go. At some point one must employ permutahedra and associahedra, in addition to constructions of polyhedral figures, to discern or model which configurations of space-like extents are allowed. Luckily, a few brave souls have explored those reaches ahead of us. Marni Dee Sheppeard provides an excellent overview of how these complications may be resolved [9], as part of her motivic gravity program, which would require of the reader extensive research to compile. It is not for the timid, to explore Planck scale dynamics, pregeometry, geometrogenesis, and the emergence of spacetime – but it is worthwhile to study because so much remains to discover. Hopefully, this short letter supplies sufficient food for thought to those who wish to investigate this regime further.

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Fig. 1 – BHEntropyF1 by Jacob Bekenstein, Scholarpedia 3(10):7375