Article

Bimetric Convergence

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Abstract

If the early universe obeys constructivist principles, there is a dual or multiple definition of dimensionality – in the first epoch – giving it an upper and lower bound. Various theories of gravity have been suggested which are bimetric, in having a dual definition in their formulation. In this brief letter, we explore a scenario where *D* is delimited initially, on both the upper and lower end of the spectrum, and later converges toward familiar values. Spacetime in the local universe appears to be 4-d at this time, but some theories suggest it is lower-dimensional at the outset or in the microscale, and some say it may be higher-dimensional at vast distances or in an earlier epoch. Both options might be true simultaneously. So if we admit the possibility that spacetime is bimetric and study what happens if we leave *D* open-ended at both extremes initially, in the Planck epoch, we can better understand why reality has the nature it does today.

Keywords: Planck scale, bimetric gravity, constructivist geometry, higher dimensions.

Introduction

In the following analysis, the constructivist ideal is assumed. We avoid *ad hoc* assumptions about the nature of reality, because we aim for an *ab initio* formulation – working from first principles. Things only exist which can be determined in a process that is both a measurement or observation and a creative act or construction. In this setting, dimensionality is indeterminate or undefined, in a cosmos before form, or where nothing is known through determination. This is connected with the process of geometrogenesis, by which spacetime emerges. Going back to the Planck epoch, we arrive at a state where familiar particles cannot form yet and spacetime as we know it is unformed too. So the construction of objects and spaces defines what is possible during this first cosmological era or epoch. If we assume an infinite dimensional upper limit and a lower limit of 0-d or a point-like condition, in the earliest phase of cosmological evolution, it allows formulation of a basis for reality in higher and lower dimensions simultaneously, which converges toward a congruent single definition for *D* at any given level of scale – in the current era. We can see this as a specific example of a larger pattern where bottom-up and top-down processes happen simultaneously, a view now championed by cosmologist George Ellis [1]. Since the normal view is that reality proceeds only from the simplest forms to more complex ones, we examine an alternative here.

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Constructing reality from the top-down

While we often assume reality proceeds from the bottom up, with simple forms and spaces appearing first, it is wise to consider what if the most complex forms possible shaped the earliest phases of reality's unfoldment instead, or as well. Mathematical objects of high dimension like the Monster group or E_8 could have an impact on Physics in the familiar world of 3-d space or 4-d spacetime if or because the early universe is higher-dimensional. The idea of Monstrous Moonshine has gained considerable traction [2], showing the Monster group has an influence on real-world Physics, and the exploratory work of Garrett Lisi [3] and of F. D. 'Tony' Smith [4] strongly suggests that the footprint of Lie group E_8 on Modern Physics is greater than most people are aware of.

I favor a view where all mathematical objects, both discovered and undiscovered, are influencing Physics now [5] – whether we realize it or not. This view implies that mathematical invariances present in the structure of, or required by the construction of higher-dimensional spaces, are automatically relevant to Physics. This is an extension of ideas going back at least to Plato [6], where universal ideals and archetypal forms outside of space and time give rise to familiar conditions in the real world. It was beyond the tools Plato had available to treat higher-d objects and spaces, and it remains unfamiliar territory for a large segment of the Physics and Math community. So we examine here what is encountered if we go from a very high dimension downward.

One of the first things to realize is that familiar concepts fail us in higher-d spaces, because they do not accurately represent how form is structured. While we live in a world populated by objects with well-defined surfaces, which move relative to each other, this state might be the product of earlier conditions where such objects could not exist, and where movement through space over time does not conform to conventional definitions of size and distance. Perhaps the familiar constancy of interiority/exteriority, and size/distance, is emergent [7] from an earlier higher-dimensional state where the background space is both non-associative and non-commutative. This would greatly influence geometric properties for both objects and spaces, in that earlier epoch.

But various complications arise in high dimensions. Reality can be like a Möbius strip, where the inside and outside of things flow into each other. It can be discontinuous, so it is only well-defined in ranges like the panels of a box kite. And higher-d reality can be purposefully featureless, except for congruent forms that exist in a range of dimensions, or in one specific dimensionality. These are the kinds of complications that arise, examining cosmological origins in the context of higher-dimensional Mathematics. But while most mathematical physicists assume a specific background state, and examine the dynamics within that condition, here we attempt to examine a background-absent condition through top-end delimitation.

There might be a universal tendency for dimensions to reduce from higher to lower, allowing specialized forms to emerge. And there may be important mathematical objects yet to discover and explore, which could shed light on some of what we already know. But it appears that forms we *have* discerned serve a coordinating function, in terms of relating the infinite back to the finite and allowing discrete material forms to have existence. The Monster Group was shown by Robert Griess in 1982 [8] constructible as a group of rotations in 196883-dimensional space, corresponding to a 196884-d algebra, giving a formal definition to an object already known to exist. When a connection was seen between the Monster group and the modular functions, particularly with the j-invariant, this became known as Monstrous Moonshine [9]. This bridge between two distinct areas of Math allows Physics researchers to construct conformal field theories over its structure, which suggests that this object of very high dimension shapes Physics in familiar spaces as well.

We may assume that this object is among the first encountered moving downward from infinite dimensions toward the finite, but we may find that congruent forms do exist in yet higher dimensions, which also have an impact on our reality. Or there might be forms as yet undiscovered, in an intermediate range of dimensions which is already partly familiar but not fully explored. Just as nature is full of surprises for those who study Physics, reality is equally difficult to predict in some areas of Mathematics. There is a clear sense that there is something there to discover which is waiting to be revealed, however, rather than a blank slate that mathematicians create upon.

This means that if the most congruent forms appear only in specific dimensions, this fact itself should be relevant or meaningful to the structure of Physics. So it follows that if the universe started without an upper limit to its dimension, nature encountered things like the Monster Group on the way to formulating the reality we know as the cosmos of today. And this structure served as a filter or sieve for what came after. So what we see in the cosmos today must incorporate that structure in some way, if dimensionality is at first unconstrained and constraints appear from the top down, as well as from the bottom up. If we thus assume reality was maximally bimetric at the outset, there are some very interesting consequences indeed. Most notably, this makes it explicit why time is clearly directional and irreversible in familiar domains.

I will move next to E_8, which I briefly mentioned earlier. It is a beautiful, jewel-like structure of incredible complexity and elegance that occupies an important place in the theory of Planck scale physics, because it is a storehouse of symmetries. It is perhaps the most deeply symmetrical structure it is possible to construct. It is the largest of the exceptional Lie groups, and it is constructible as the product of the octonions with the octonions, giving it 248 dimensions. The E_8 Lattice, or Gosset lattice, is the most compact structure possible in 8-d [10], in terms of the close-packing of spheres. I will talk more about the extent of the spheres in a bit.

But first, I want to mention that the remaining exceptional groups, E_7, E_6, F_4, and G_2 also derive from the octonions, while the more familiar Lie groups SO(n), SU(n), and Sp(n), derive from the reals, the complex numbers, and the quaternions respectively [11]. The highest dimension in which the remaining exceptional Lie groups reside decreases in descending order after E_8, with E_7 having 133 dimensions, E_6 78, F_4 52 dimensions, and G_2 having 14, but with one of its fundamental representations in 7-d. So from 248-d space down to 7 dimensions, the exceptional Lie groups provide a wealth of specific structures we have only recently had the technology to map [12]. However, it appears this structure has already shaped the reality we live in from the early universe forward.

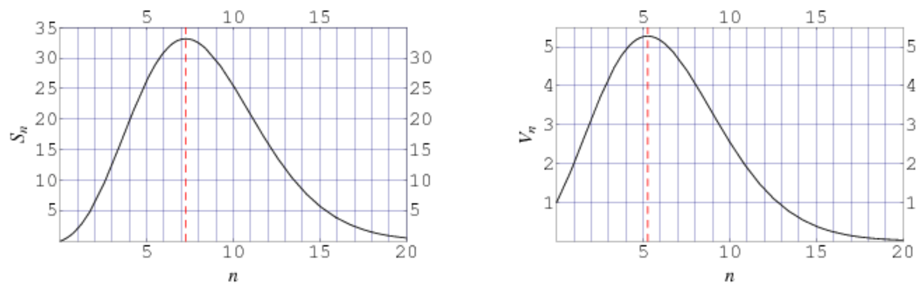

Fig. 1. The (hyper-) surface area of an n-sphere (on left) has a peak near 7 (an 8-d 7-sphere) and drops off afterward while (hyper-) volume peaks for a ball (space contained by a sphere) in 5 dimensions, and the unequal maxima appear to drive geometrogenesis and inflation to create a 5-d volume.

It is important to note that the Leech lattice in 24-d is the most compact structure possible, where more close-packed spheres will nest together [13] with no frustration and no space left over in between. This runs counter to the conventional notion that as we add more dimensions, things become more spacious. While a unit sphere increases in both volume and surface area as we go from 3 to 5 dimensions, the volume of a hypersphere (its hyper-volume) decreases after we reach 5-d and the (hyper-) surface area decreases after 7-d, so spheres are more and more compact until we reach 24-d. So if familiar forms need space to spread out in, or extend into, they cannot exist in spaces of very high dimension. But in some ways, the compactness offsets the need for space, in this scenario, by creating spaces between spaces. It is as Mr. Feynman said "there is always more room at the bottom."

So very compact arrangements may afford more 'places' to put things, despite the fact there is less actual space to go around. However, the reality of both everyday life and Cosmology takes place in open or extended spaces where things are free both to have size and to move around relative to each other. The familiar reality rests on the property of associativity which helps establish interiority/exteriority and commutativity which makes size/distance determinations into measurable and repeatable quantities. But before the appearance of fermionic particles, we cannot assume these properties. So if the universe was higher-dimensional in the Planck era, that cosmos was very different from the one we inhabit today. However, a cosmos even one dimension higher might be literally inside-out from the one we inhabit.

There are several theories under serious consideration where space has a higher dimension at great distances or in a prior epoch. Since looking farther and farther into space means we are also looking at things which happened further back in time, it is equivalent to say one or the other. But we find that options with additional dimensions allow us to easily address the hierarchy problem, where gravity is the weakest force by far (and seems unnaturally so) and to solve issues relating to the universe expanding in a way that avoids the need for dark matter or dark energy. We see small discrepancies from what Relativity theory tells us we should see, unless we put in DE and DM correcting factors, but changing the way gravity behaves at vast distances could make this kind of adjustment unnecessary. This was the hope for DGP gravity [14], a braneworld formulation with a 5-d parent or precursor. DGP gravity was seen by other researchers [15] to represent a scenario where a black hole in a 5-d parent or precursor becomes a white hole giving birth to our 4-d universe. A similar scenario also occurs in Einstein-Cartan gravity [16]. But in Cascading DGP [17], the 5-d reality had a 6-d parent and then rolled down further to create 4-d spacetime. Models with a higher-d origin continue to be explored.

Simpler Spaces and Earlier Times

It is in many ways easier to create models of the universe that start with very simple structure and build upon that, to arrive at conditions similar to those we observe today. It is evident from several quantum gravity theories spacetime flattens out near the Planck scale, to become 2-dimensional. In CDT theory (Causal Dynamical Triangulations) [18], a simplicial fabric is 2-d near the Planck scale and evolves over time to become a 4-d spacetime resembling our cosmos. The question still arises of whether this rules out the possibility for a higher dimensional origin, or just makes things more interesting. The knowledge of geometry and algebra in higher-d spaces appears necessary to a complete understanding of reality regardless of a lower-d bound, under any number of assumptions, including those favoring String Theory as a TOE.

But we are left with gaps in our knowledge because some of our starting assumptions appear incompatible. It is easier to envision things evolving from the bottom up, with simple forms appearing first and then more complex objects and processes appearing later. This is the picture we arrive at using conventional assumptions, in the Big Bang or Inflationary universe models. However, nature behaves as though the most advanced Maths we can discover have already been put to use in her handiwork since the very beginning. Perhaps nature allows only those forms that adhere to both the laws of higher and lower-dimensional reality at once, or she is showing us that both sets of rules are employed in turn. If things are indeed formed from both above and below, complex forms can emerge more directly, so it is worth examining this possibility.

In the conventional view, things must always start from a simpler basis, for the materials to be available by which we might construct more complex objects. We think of lines being comprised of points, and flat surfaces like a grid of lines, and so on (despite the insistence of Klein and of Grassmann they should get equal weight). So it is natural to speak of particles arranged into atoms, which form molecules, and continuing from there. We can think of these things as occupying or defining parcels of space, as well. We know too that fermionic particles and constructs made from them obey the Pauli Exclusion Principle, so they cannot occupy the same space under normal conditions (unless they can be cooled to form a BEC).

In addition, we appear to be surrounded by a vast expanse of space, which extends even beyond the observable cosmos. So it is easy to imagine that space is something self-existing, which does not have to be formed or to emerge. However, it is seen by theorists that geometrogenesis must somehow create a fabric of space and time, in order for familiar spaces to exist. It may be true that point-like objects cannot exist in any form, and that a line-like projection can only exist as part of a larger construct. Indeed, if the smallest definable parcel of space must also contain a time-like projection to have duration or persistence, it is unavoidable that it should be 2-d as a lower limit.

This is easy to explain in purely geometric terms. The simplest structures that can contain space are 2-d figures like a triangle, square, or circle. Without a container to hold the space open, there can be no emptiness, and no definable parcel of space! In constructivist geometry, only that which can be constructed truly exists. And this seems the correct rule to apply for the inception of the cosmos. This assumption is in good agreement with a large number of quantum gravity theories [19], which show or predict that spacetime becomes 2-d near the Planck scale. In these theories, the dimensionality of spacetime D is seen to run smoothly from 2-d to 4-d, over the evolution of the cosmos, with a fractional or fractal dimension along the way. Or equivalently, spacetime itself is folded and flattened at the smallest levels of scale, near the Planck scale, so that it becomes 2-d.

Steven Carlip sees this dynamic as a hallmark or generic feature of all quantum gravity theories he calls spontaneous dimensional reduction [20], which may point us toward the correct theory. However, none of this excludes that D might be initially unbounded from the top down as well, because at the outset of cosmic evolution there was nothing present to constrain it. In the matter-free regime, before the appearance of fermionic particles at baryogenesis, D might have a very high upper limit (as it is unconstrained), which would have become bounded by that event. In effect, the binding of energy into particles with mass also fixed the upper bound for D.

Regardless, the quality of extensivity and the possibility for things to have a measurable extent may require the emergence of spacetime geometry, or geometrogenesis, to allow things to have an area or volume. We take it as automatic that area and volume are well-defined, but pregeometry rules the Planck domain [21]. Spaces with appropriate properties needed to evolve first, before conventional forms. Volumetric space was not possible at the outset of cosmic evolution, even with a large number of dimensions available, because what is needed first are parcels of space that persist in time. There need to be space-like extents with a time-like extension, and there need to be open pockets with measurable extent, before space as we know it can truly be said to exist.

Briefly stated, measurability requires the existence of separable objects with well-defined surfaces. A ball in a space devoid of all else has no particular size. But a collection of balls with uniform size can measure a space or other objects. And a ball of yet larger size can serve as a container for all of them. However, this does not preclude higher or lower dimensional objects from simultaneously existing. Ergo, the fabric of spacetime might have a separate evolution from and a different dimension than the higher-dimensional bulk, just as the surface of a sphere is of one lower dimension than the volume it contains. A 3-d sphere has a 2-d surface, a 4-d sphere has a 3-d surface, and a 5-d hypersphere has a (hyper-) surface that is 4-d, for example.

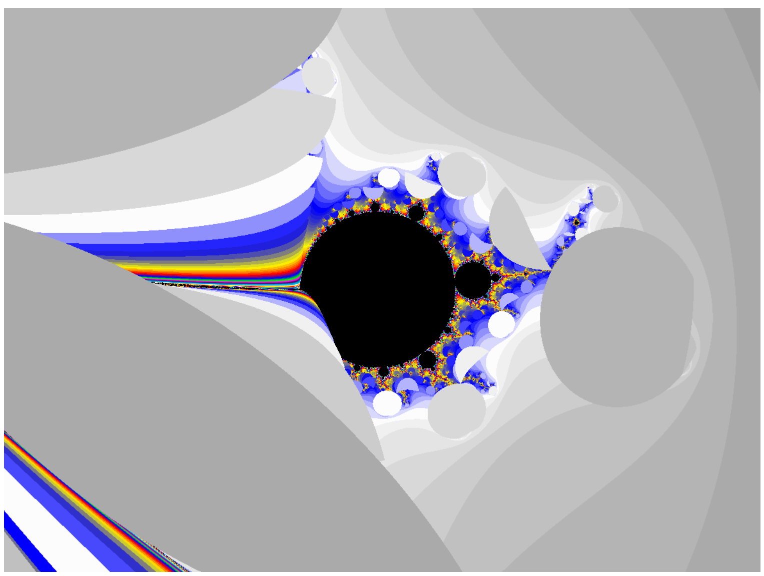

Fig. 2. The region of the Mandelbrot Butterfly figure centered at (–0.125,0.75*i*) shows a persistent bubble of space of comparable size to a proton embedded in the spacetime fabric.

This last case is especially relevant because, in a cosmological setting, it is most natural to think in terms of evolving spheres, and a higher-dimensional sphere or hypersphere can have a boundary that is 3-d or higher. So this leaves open the possibility for familiar particles to exist in spaces of high dimension along the boundaries, or in a boundary region which is a brane with specific properties in a higher-d bulk or embedding space. Ergo, in the early universe, things could have evolved in a way very much like that envisioned in the concordance cosmology, except that the nature and dimensionality of the background space, or of spacetime, was evolving too. Seen this way, it is easier to explain baryogenesis as an outgrowth of geometrogenesis, because the existence of stable volumetric spaces was a necessary precursor but was not automatic. That is, persistent bubbles needed to form, large enough to fit a proton or neutron, before these particles could exist as enduring forms (i.e. – having duration or persistence). This phenomenology appears automatically if one assumes that bimetric convergence is a feature of emergent dimensionality. But it also pops out prominently in cosmologies derived from the Mandelbrot Set [22]. In the region of the Mandelbrot Butterfly centered at (–0.125,±0.75*i*), as seen in Fig. 2 above, we see represented a persistent bubble of space comparable in size to protons or anti-protons in a particle nursery during baryogenesis, allowing particles to be embedded in the spacetime fabric.

Conclusions

The most important point to take away from this analysis is that reality is open-ended at the outset, even though certain properties appear to be fixed or static now, and it behaves as though the attributes of forms and spaces in various dimensions all contribute to our reality. These things would automatically be true, if emergent dimensionality proceeds via bimetric convergence, because reality would have passed through both higher and lower-dimensional phases of evolution on its way to where we are now. Nor is the effect limited to what happened in the early universe. If String Theory is correct, we live side by side with higher dimensions all of the time, or they remain part of everything. There exists a remarkable array of orderly features in higher-dimensional objects and spaces which mysteriously function to inform the laws of nature; despite the fact the universe appears 3-dimensional with a single arrow of time. The image of higher-dimensional structures reflected in lower-d forms is not likely to be purely coincidental or inconsequential however.

The fact there is a semblance of exotic structures in the physical world is evidence that cosmic evolution *did* get shaped by these features, and that they continue to influence physical law. And yet, many people still wonder if it is relevant to think about higher dimensions and to talk about top-down evolution as part of Physics. Why think about higher-dimensional reality or study its properties if we do not live there, and it does not affect everyday physical relations? The prevailing notion is it violates the principle of Occam's razor to invoke higher dimensions, without a specific physical phenomenology which would either generate or require them. This appears reasonable. I feel strongly that the opposite argument is justified, however, and that mathematical structures in all dimensions impinge on our reality. The sequentially evolutive properties [23] of non-associative spaces may be a necessary component of cosmic evolution, which makes higher dimensions essential.

However, there are limits. If the constructivists are correct, only that which can be constructed is possible or real. Furthermore, the invocation of higher-d spaces designed to produce a particular

Physics is perhaps wrong-headed, where instead we should think about how such spaces could evolve in the first place. But in my mind, the structure is there already. We do not need to invent special structures to contain the Physics of the natural world. If we knew the wealth of structure Mathematics has to offer, waiting to be explored or discovered; discovering the key to Physics would be almost automatic.

Dedication: My departed colleague Ray Munroe explored a geometric road to unification, with the view that reality is formed and informed by both the simplest and the most complex forms in Mathematics – a view which I share. He and I were often inspired by the work of F.D. 'Tony' Smith, where Tony shared with us a wealth of knowledge about the reality of higher-dimensional objects and spaces few have explored, before his demise last year. This paper is dedicated to the research and the memory of these fine gentleman scholars.

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Fig. 1 Weisstein, Eric – graphs from 'Ball' and 'Hypersphere' at Wolfram's Mathworld
Fig. 2 Dickau, Jonathan J. – image created in Chaos Pro 4.0

Appendix

Tony Smith responded to my questions about spheres in various dimensions, in an e-mail several years ago, with a detailed reply which is reproduced below. This highlights the fact there are unexpectedly strange and wonderful things lurking in higher dimensions.

F.D. 'Tony' Smith wrote:

Sphere, torus, Klein bottle, Möbius strip, etc are all basic geometric concepts. The simplest of these is the sphere.

When people tried to use Math to classify spheres of various dimensions, they found out that classification was not at all simple but had lots of subtleties. For example, a thing that looks like a sphere from a combinatorial/piecewise linear point of view can (in some dimensions) have many different smooth/differential structures:

sphere - number of possible smooth/differential structures
S_1 - 1
S_2 - 1
S_3 - 1 = 2x1 / 2 (but S_3 is a subset of any exotic R4# and there are uncountably many exotic R4 spaces)
S_4 - 1
S_5 - 1

S_6 - 1
S_7 - 28 = 8x7 / 2 = 23 x (23 - 1) / 2
S_8 - 2
S_9 - 8
S_{10} - 6
S_{11} - 992 = 32x31 = 25 x (25 - 1)
S_{12} - 1
S_{13} - 3
S_{14} - 2
S_{15} - 16,256 = 128x127 = 27 x (27 - 1)
S_{16} - 2
S_{17} - 16
S_{18} - 16

As John Baez has noted,
there are various distinct questions floating around, including:

A) how many topological manifolds are homotopy-equivalent to the sphere?
B) how many PL (= piecewise-linear = combinatorial) manifolds are homeomorphic to the sphere?
C) how many smooth manifolds are PL equivalent to the sphere?

For dimension 3, question A is the Poincare conjecture.
It was proven by Grisha (Grigori) Perelman.
For dimension 3, questions B and C are solved and the answer is 1.
For dimension 4, question A is solved (in the 1980s, by Freedman) and the answer is 1.
For dimension 4, question C is solved and the answer is 1.
For dimension 4, question B is open (the smooth Poincare conjecture in dimension 4).

To try to make sense of this look at spheres by their Homotopy Groups PI(k)(S_n), which is roughly the number of ways you can wrap a k-sphere around an n-sphere. For example, PI(n)(S_n) is the infinite cyclic group Z, and each element of Z corresponds to a winding number of a wrapping of S_n around S_n. If you want to look at homotopy groups from the point of view of all spheres of all dimensions, and take the orthogonal group O(n) as the group of rotations/reflections of S_n, then you can say that O(∞) is the orthogonal group for infinite-dimensional real space which contains as subgroups all orthogonal groups O(n) for all finite n and is effectively the symmetry of all spheres of whatever dimension.

Then you find that the homotopy relation is periodic with period 8:

Bott periodicity PI(n+8)(O(∞)) = PI(n)(O(∞))

The orthogonal structure is directly related to Clifford algebra and Clifford algebra also has the periodicity structure Cl(8N) = Cl(8) x ...(N times tensor product)... x Cl(8) So, in some sense the geometry of spheres is described by Clifford algebra which is why I use Clifford algebra as the basis for my physics model. Once you describe spheres, you can use that to describe torus - sphere with a hole / Klein bottle - sphere with a twist ... etc ...
So I think that Clifford algebras are a nice Math way to describe spheres which are the basic structures of the universe.

Tony