Article

Magnetized Locally-rotationally Symmetric Bianchi Type-iii Cosmological Model With Wet Dark Fluid in General Relativity

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Abstract

In this paper, we have investigated the effect of magnetic field and Wet dark fluid on locally-rotationally symmetric Bianchi type-III cosmological which is non-shearing. A new equation of state for the dark energy component of the universe has been used. It is modeled on the equation of state $p = \gamma(\rho - \rho_*)$ which can describe a liquid, for example water. The exact solutions to the corresponding field equations are obtained in quadrature form. The obtained solution have been studied in detailed for both power-law and exponential form. In addition to this we have discuss the geometrical and physical properties of said model.

Keywords: Bianchi type-III, spacetime, wet dark fluid, magnetic flux, cosmological parameters.

I. Introduction

To explain the formation of large scale structure of Universe is one of the basic problems of cosmology even today. The presence of primordial magnetic fields in the early stages of the evolution of the universe has been discussed by several authors (Misner, Thorne and Wheeler [1]; Asseo and Sol [2]; Pudritz and Silk [3]; Kim, Tribble, and Kronberg [4]; Perley and Taylor [5]; Kronberg, Perry and Zukowski [6]; Wolfe, Lanzetta and Oren [7]; Kulsrud, Cen, Ostriker and Ryu [8]; Barrow [9]). Melvin [10], Patel and Maharaj [11] investigated stationary rotating world model with magnetic field.

The occurrence of magnetic fields on galactic scale is well established fact today and their importance for a variety of astrophysical phenomenon is generally acknowledge which was pointed out by Zel'dovich [12], Also, Harrison [13] has suggested that magnetic field could have a cosmological origin. As a natural consequence we should include magnetic field in energy momentum tensor of early universe.

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The formation of cosmic structure are affected in various ways by the interface of dark matter and dark energy, due to this interface, the rate of the background expansion of universe will change, which in turn affect the structure formation in the universe. Secondly, the interface may enhance the effective mass of dark matter particle. Thirdly, this phenomenological aspect of interaction within dark sector may be treated as fifth force, which might support a more effective structure formation. We have motivated of above situation and think what happens, when we use wet dark fluid and electromagnetic field. In this paper, we have discussed the nature of non-shearing LRS Bianchi type-III string cosmological model in presence of wet dark fluid and magnetic flux. The physical and geometrical behaviors of model are discussed. In this paper, we have introduced the magnetic flux in the system of wet dark fluid. This model is in the spirit of generalized Chaplygin gas (GCG) by Gorini et al. [14]. The equation of state for WDF is in the form,

$$p_{WDF} = \gamma(\rho_{WDF} - \rho_*) \tag{1}$$

It is motivated by the fact that, a good approximation for many fluids, including water, in which the internal attraction of the molecules makes negative pressures. One of the virtues of this model is that the square of the sound speed c_s^2, which depends on $\frac{\partial p}{\partial \rho}$ can be positive, even while giving rise to cosmic acceleration in the current epoch. The parameters γ and ρ_* are taken to be positive and we restrict ourselves to $0 \leq \gamma \leq 1$. Note that, if c_s denotes the adiabatic sound speed in WDF, then $\gamma = c_s^2$ (Babichev *et al.* [15]). To find the WDF energy density, we use the energy conservation equation as,

$$\dot{\rho_{WDF}} + 3H(p_{WDF} + \rho_{WDF}) = 0 \tag{2}$$

where H is Hubble parameter.

From the equation of state (1) and using equation $3H = \frac{\dot{V}}{V}$ in the equation (2) and on solving we obtained,

$$\rho_{WDF} = \frac{\gamma}{1+\gamma}\rho_* + \frac{C}{V^{1+\gamma}} \tag{3}$$

where C is the constant of integration and V is the volume expansion. Wet dark fluid (WDF) naturally includes two components: a piece that behaves as a cosmological constant as well as a standard fluid with an equation of state $p = \gamma\rho$, if we take $C > 0$, it confirm that, the fluid will not violate the strong energy condition,

$$p_{WDF} + \rho_{WDF} = (1+\gamma)\rho_{WDF} - \gamma\rho_*$$
$$= (1+\gamma)\frac{C}{V^{1+\gamma}} \geq 0 \tag{4}$$

According Holman and Naidu [16], the wet dark fluid has been used as dark energy in the homogeneous, isotropic FRW case and the early stage of expansion of the universe exhibits substantially non-Friedmannian behavior given by Zel'dovich [17]. The author Singh et al. [18] has studied Bianchi type-I universe with wet dark fluid. Recently, Patil et al. [19] have studied Bianchi type-I cosmological model in presence of magnetic flux with viscous fluid, along with this, Patil et al. [20, 21] have studied non shearing LRS Bianchi type-III and Bianchi type-IX string cosmological model in presence of magnetic flux with bulk viscosity as well as Patil et al. [22, 23, 24] have studied LRS Bianchi type-V cosmological model in presence of perfect fluid and magnetic flux with variable magnetic permeability and Bianchi type-IX cosmological model as well as Bianchi type-V cosmological model with two fluids in presence of magnetic flux also Patil et al. [25] have studied Bianchi type-I cosmological model with wet dark fluid in presence of magnetic flux and recently Patil [26] et al. have studied Five dimensional Bianchi Type-I cosmological model in presence of Perfect fluid with Magnetic flux in General Relativity.

In this paper, we have studied the LRS Bianchi type-III cosmological model with matter term and dark energy treated as a dark fluid satisfying the equation of state (1) in presence of magnetic flux. The solution has been obtained in the quadrature form.

II. Field Equations & Solutions

We consider locally-rotationally symmetric Bianchi type-III space time:

$$ds^2 = -dt^2 + A^2dx^2 + B^2(e^{2x}dy^2 + dz^2) \tag{5}$$

in which $A(t)$, $B(t)$ are cosmic scale functions of time t.

The Einstein field equations is,

$$R_j^i - \frac{1}{2}Rg_j^i = T_j^i \tag{6}$$

where T_j^i is energy momentum tensor of Wet dark fluid with electromagnetic field tensor such as,

$$T_j^i = T_j^{i'} + E_j^i \tag{7}$$

Here, we consider $T_j^{i\prime}$ as a source of energy, where,

$$T_j^{i\prime} = (\rho_{WDF} + p_{WDF})u_j u^i - p_{WDF}\,\delta_j^i \tag{8}$$

and the electromagnetic field tensor E_j^i is,

$$E_j^i = \bar{\mu}\left[|h|^2\left(u_j u^i + \frac{1}{2}\delta_j^i\right) - h_j h^i\right] \tag{9}$$

in which h_j is magnetic flux vector defined by,

$$h_j = \frac{1}{\bar{\mu}} * F_{ij} u^i \tag{10}$$

Here $*F_{ij}$ is dual electromagnetic field tensor defined as,

$$*F_{ij} = \frac{\sqrt{-g}}{2}\epsilon_{ij\alpha\beta}F^{\alpha\beta} \text{ and } \epsilon_{0123} = 1 \tag{11}$$

Here $F^{\alpha\beta}$ is electromagnetic field tensor and $\epsilon_{ij\alpha\beta}$ is the totally antisymmetric Levi-Civita tensor, where $\bar{\mu}$ is a constant characteristic of the medium and called the magnetic permeability, typically $\bar{\mu}$ differs from unity only by a few parts in 10^5 ($\bar{\mu} > 1$ for paramagnetic substance and $\bar{\mu} < 1$ for diamagnetic substance),

The co-moving co-ordinates are taken as, $u^0 = 1, u^1 = u^2 = u^3 = 0$ and choose the incident magnetic field in the direction of x-axis, so that the magnetic flux vector has only one nontrivial component of F_{ij} i.e. F_{23}, then the set of Maxwell equations, $F_{\mu\nu;\beta} + F_{\nu\beta;\mu} + F_{\beta\mu;\nu} = 0$, one finds,

$F_{23} = I, \quad I = constant.$

Using equations (10), (11) in equation (9), which takes the following form,

$$E_0^0 = E_1^1 = -E_2^2 = -E_3^3 = -\frac{I^2}{2B^4 e^{2x}\bar{\mu}}$$

For simplification we take $e^{-2x} = \bar{\mu}$

Therefore above equations becomes,

$$E_0^0 = E_1^1 = -E_2^2 = -E_3^3 = -\frac{I^2}{2B^4} \tag{12}$$

for $i = j = 0, 1, 2, 3$ equation (8) becomes,

$$T_0^{0'} = \rho_{WDF} \text{ and } T_1^{1'} = T_2^{2'} = T_3^{3'} = -p_{WDF} \tag{13}$$

Using equations (12), (13) in equation (6), it takes the following forms,

where the dots over the letters denote the ordinary derivative with respective to cosmic time t:

$$2\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{B}^2}{B^2} - \frac{1}{A^2} = \left(\rho_{WDF} - \frac{I^2}{2B^4}\right) \tag{14}$$

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} = \left(-p_{WDF} - \frac{I^2}{2B^4}\right) \tag{15}$$

$$\frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\ddot{A}}{A} = \left(-p_{WDF} + \frac{I^2}{2B^4}\right) \tag{16}$$

$$\frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\ddot{A}}{A} - \frac{1}{A^2} = \left(-p_{WDF} + \frac{I^2}{2B^4}\right) \tag{17}$$

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = 0 \implies \frac{\dot{A}}{A} = \frac{\dot{B}}{B} \tag{18}$$

From equation (18), we have

$$A = mB \tag{19}$$

where m is constant of integration.

From equation (19), we have

$$\frac{\ddot{A}}{A} = \frac{\ddot{B}}{B} \tag{20}$$

The volume scale factor V is defined as,

$$V = AB^2e^x \tag{21}$$

Using equations (21), (19) and (18), we have

$$\frac{\dot{V}}{V} = 3\frac{\dot{B}}{B} \tag{22}$$

As the Hubble parameter H is defined as,

$$H = \frac{1}{3}\frac{\dot{V}}{V}$$

Using equation (22) in above equation, we have

$$H = \frac{\dot{B}}{B} \tag{23}$$

The scalar expansion θ is defined as,

$$\theta = u^i_{;i} = \left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right) = \frac{3\dot{B}}{B} \tag{24}$$

and shear scalar σ is defined as,

$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij}$ which on solving , we get

$\sigma^2 = 0$

$$\Longrightarrow \sigma = 0 \tag{25}$$

Using the Equation (23) in equation (2), we have

$$\dot{\rho_{WDF}} + 3\frac{\dot{B}}{B}(p_{WDF} + \rho_{WDF}) = 0$$

Using equation (1) in above equation, we have

$$\dot{\rho_{WDF}} + 3\frac{\dot{B}}{B}(\gamma\rho_{WDF} - \gamma\rho_* + \rho_{WDF}) = 0$$

$$\Longrightarrow \frac{\dot{\rho}_{WDF}}{\left(\rho_{WDF} - \frac{\gamma\rho_*}{(1+\gamma)}\right)} = -3(1+\gamma)\frac{\dot{B}}{B} ,$$

On integrating above equation and solving, we have

$$\rho_{WDF} = \frac{C}{B^{3(1+\gamma)}} + \frac{\gamma\rho_*}{(1+\gamma)} \tag{26}$$

Using above value in equation (1), we have

$$p_{WDF} = \frac{\gamma C}{B^{3(1+\gamma)}} - \frac{\gamma\rho_*}{(1+\gamma)} \tag{27}$$

From equations (26) and (27), we have

$$\rho_{WDF} + p_{WDF} = \frac{(1+\gamma)C}{B^{3(1+\gamma)}} \tag{28}$$

Using equations (18), (19) and (20) in equations (14) and (16) and Subtracting equation (16) from Equation (14), we have

$$2\frac{\dot{B}^2}{B^2} - 2\frac{\ddot{B}}{B} - \frac{1}{m^2B^2} = (\rho_{WDF} + p_{WDF}) - \frac{2I^2}{2B^4}$$

Using equation (28) in above equation, and simplifying above equation, we have,

$$\frac{dB}{dt} = \sqrt{\frac{B}{m^2} - \frac{I^2}{4B^2} - \frac{1}{3B^{(1+3\gamma)}}} \;\rightarrow\; dt = \frac{dB}{\sqrt{\frac{B}{m^2} - \frac{I^2}{4B^2} - \frac{1}{3B^{(1+3\gamma)}}}} \tag{29}$$

On integrating above equation, we have

$$t = \int \frac{dB}{\sqrt{\frac{B}{m^2} - \frac{I^2}{4B^2} - \frac{1}{3B^{(1+3\gamma)}}}} \tag{30}$$

Hence the geometry of the space-time in the presence of electromagnetic field by using equations (19) and (29) in equation (5), we have

$$ds^2 = -dB^2\left(\frac{B}{m^2} - \frac{I^2}{4B^2} - \frac{1}{3B^{(1+3\gamma)}}\right)^{-1} + m^2B^2dx^2 + B^2(e^{2x}dy^2 + dz^2) \tag{31}$$

Here we have expressed all term in the form of metric potential B

III. Some Special Cases: as $0 \leq \gamma \leq 1$,

Case-I: for $\gamma = 0$,

Equations (26) and (27) become,

$\rho_{WDF} = \frac{C}{B^3}$

$p_{WDF} = 0$

Graphical Representation:- For $\gamma = 0$

$\rho_{WDF} = \frac{C}{B^3}$, $p_{WDF} = 0$

For $C = 1$,

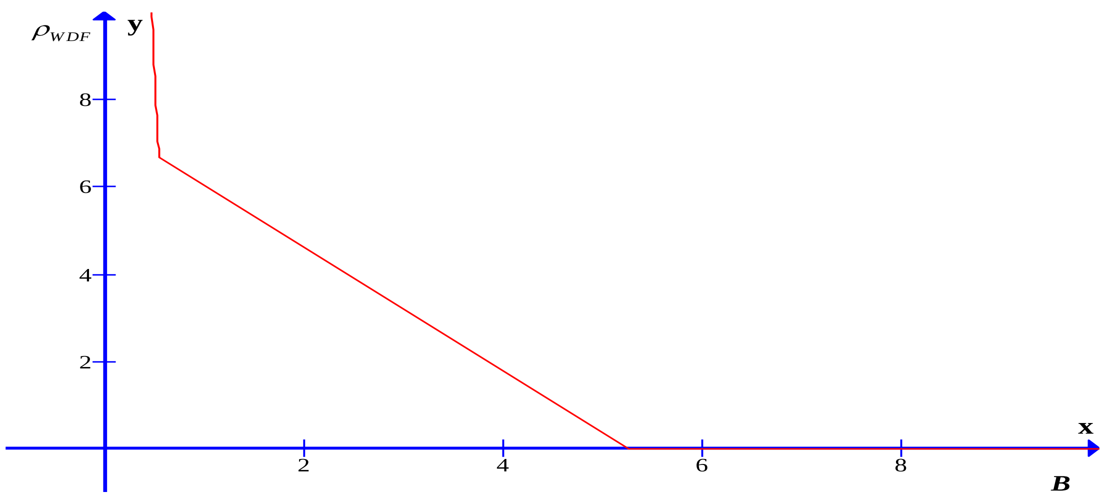

Fig. 1

From fig.1, as metric potential increases density decrease gradually and it coincide with the x-axis.

Case-II: for $\gamma = 1$,

Equations (26) and (27) obtain as,

$\rho_{WDF} = \frac{C}{B^6} + \frac{\rho_*}{2}$

$p_{WDF} = \frac{C}{B^6} - \frac{\rho_*}{2}$

Graphical Representation:-For $\gamma = 1$

$\rho_{WDF} = \frac{C}{B^6} + \frac{\rho_*}{2}$

For $C = 1, \rho_* = 1$

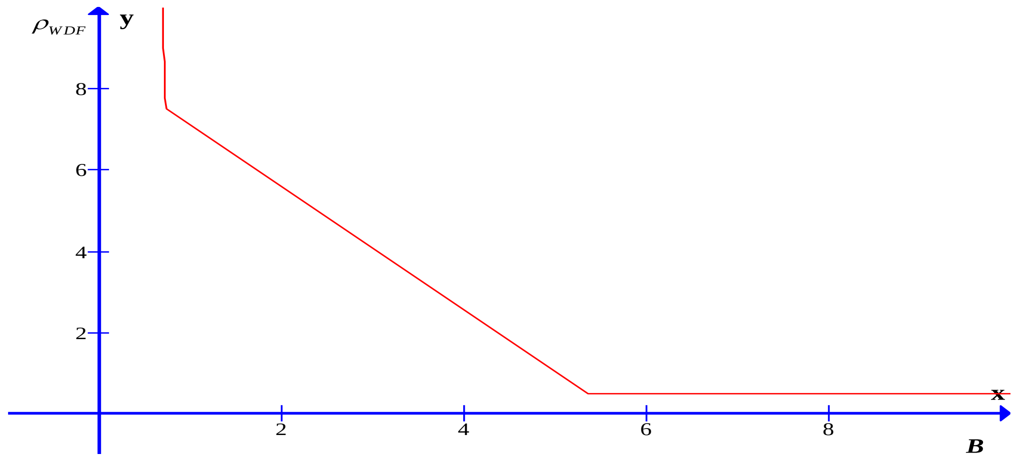

Fig. 2

From fig.-2, it clears that as metric potential increases, density decreases and after a stage, it is stable.

$p_{WDF} = \frac{C}{B^6} - \frac{\rho_*}{2}$

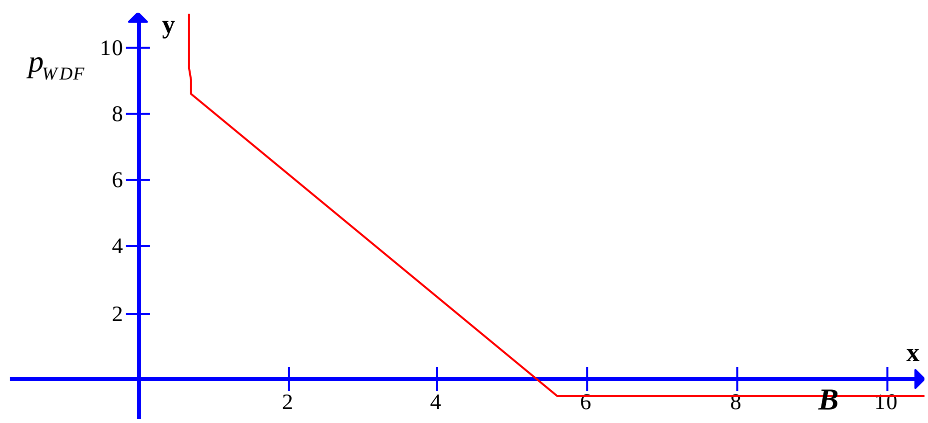

Fig. 3

From fig.-3, it shows that as metric potential increase, pressure decreases and it goes to negative.

Case-III: We take exponential form of metric potential, Such as $B = \alpha e^{\beta t}$

Equations (26) and (27) become,

$$\rho_{WDF} = \frac{C}{\alpha e^{3\beta t(1+\gamma)}} + \frac{\gamma \rho_*}{(1+\gamma)}$$

$$p_{WDF} = \frac{\gamma C}{\alpha e^{3\beta t(1+\gamma)}} - \frac{\gamma \rho_*}{(1+\gamma)}$$

Graphical Representation for $B = \alpha e^{\beta t}$,

$$\rho_{WDF} = \frac{C}{\alpha e^{3\beta t(1+\gamma)}} + \frac{\gamma \rho_*}{(1+\gamma)}$$

For $\gamma = 1, C = 1, \rho_* = 1, \beta = 1$

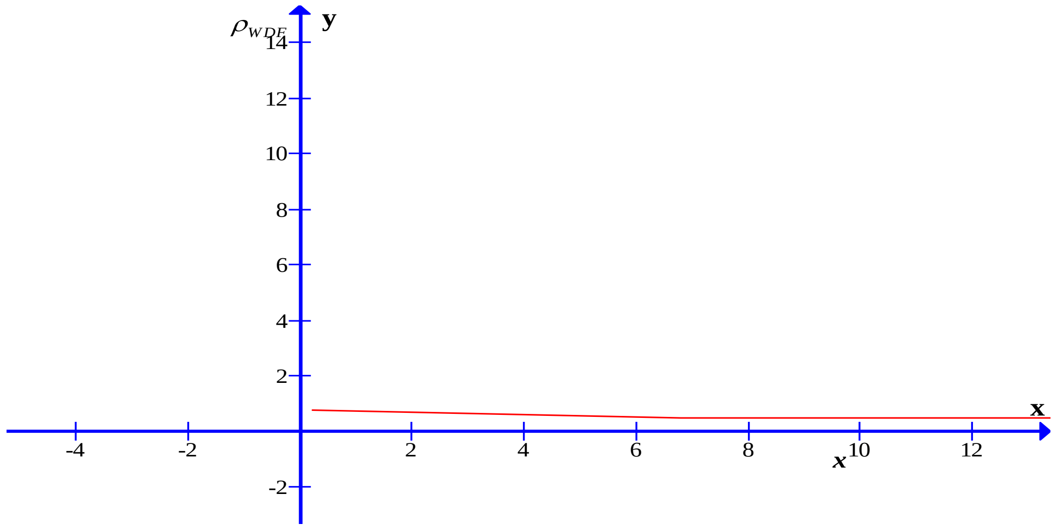

Fig. 4

From fig.-4 it is clear that, the density gradually decreases and after some stage it become steady as metric potential increases.

$$p_{WDF} = \frac{\gamma C}{\alpha e^{3\beta t(1+\gamma)}} - \frac{\gamma \rho_*}{(1+\gamma)}$$

For $\gamma = 1, C = 1, \rho_* = 1, a = 1, b = 1$

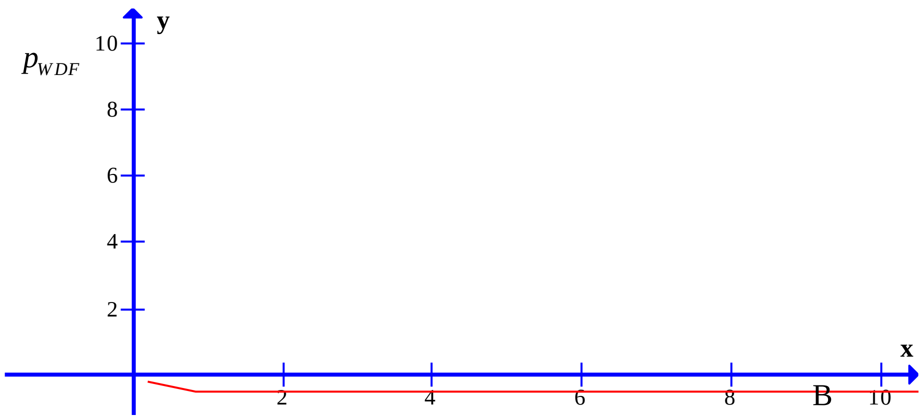

Fig. 5

From fig.-5 it is clear that, the pressure decreases and goes to negative as metric potential increases.

Case-IV: We take Power Law form, $B = at^b$

Equations (26) and (27) obtain as,

$$\rho_{WDF} = \frac{C}{a^{3(1+\gamma)}t^{3b(1+\gamma)}} + \frac{\gamma\rho_*}{(1+\gamma)}$$

$$p_{WDF} = \frac{\gamma C}{a^{3(1+\gamma)}t^{3b(1+\gamma)}} - \frac{\gamma\rho_*}{(1+\gamma)}$$

Graphical Representation for $B = at^b$

$$\rho_{WDF} = \frac{C}{a^{3(1+\gamma)}t^{3b(1+\gamma)}} + \frac{\gamma\rho_*}{(1+\gamma)}$$

For $\gamma = 1, C = 1, \rho_* = 1, a = 1, b = 1$

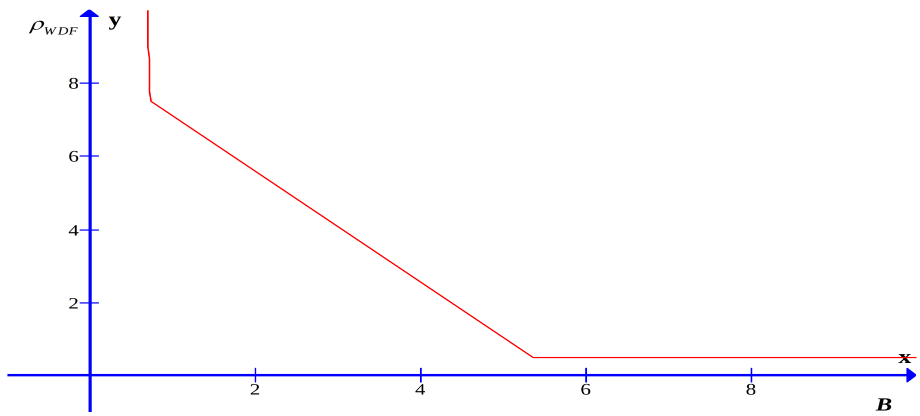

Fig. 6

From fig.-6, it clears that as metric potential increases, density decreases and after a stage, it is stable.

$$p_{WDF} = \frac{\gamma C}{a^{3(1+\gamma)}t^{3b(1+\gamma)}} - \frac{\gamma\rho_*}{(1+\gamma)},$$ For $\gamma = 1, C = 1, \rho_* = 1, a = 1, b = 1$

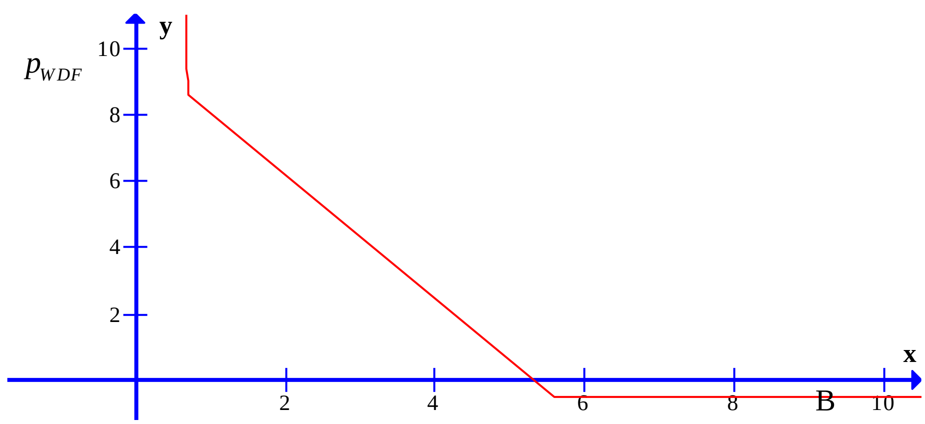

Fig. 7

From Fig.7 it is clear that, the pressure decreases and goes to negative as metric potential increases.

Conclusion

In this paper, we have investigated the nature of LRS Bianchi type-III cosmological model for the dark energy component of the universe. The solution has been obtained in quadrature form. Here we have expressed cosmic time in terms of metric potential B also the model has been expressed in terms of metric potential and have been discussed in details. The model get shrink in presence of magnetic field and expand in its absence respectively. As metric potential B tends to zero, model tends to infinity and as B increases or decreases respectively it shrink or expand. Here we have obtained the graphical representation of density and pressure of wet dark fluid against the metric potential for some special cases and discussed their nature. As the shear scalar $\sigma = 0$, then the ratio $\left(\frac{\sigma}{\theta}\right)^2$ is zero, hence the model is isotropic in nature.

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