Review Article

Detecting Gravitons on Black Hole Coalesence

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Abstract

This paper is a short review of an article on how quantum hair on black holes may produce measurable signatures in gravitational radiation. This may be a significant step in the understanding of quantum gravitation and some experimental support for theories thereof.

Keywords: Black Hole, coalesence, graviton, detection, quantum hair, gravitational radiation.

1 Introduction

It is increasingly common to hear that quantum gravitation, in particular superstring theory, has gone off into mathematical abstractions. While this is an issue with the complexity of theory, the real problem is one of scale. To produce an exciton of the quantum gravitational field, such as a quantum of black hole, requires transverse momenta or energy 16 orders of magnitude larger than the LHC. To produce a graviton, which is very weakly interacting, that can be at all detectable would require comparable energy. It is not hard to calculate that an LHC type of machine would have to encompass the Milky Way galaxy to achieve Planck energy scale interactions. It is then apparent there will be no Planck energy particle physics experiments.

However, nature may provide events and processes of sufficient power. The coalescence of black holes produces an enormous amount of gravitational radiation. Gravitational waves should be ultimately quantum mechanical, and the production of gravitational waves should have signatures of quantum mechanics[1]. Gravitational radiation is produced when the mass of two black holes is converted into gravitational radiation that carries off information. The theoretical limit for gravitational wave production is where the area of the two equal mass black holes equals the area of the merged black hole. This means $S_{tot} = 8m^2 = 4M^2$. The mass of the merged black hole is then $\sqrt{2}m$ and $(2 - \sqrt{2})m$ of mass energy is produced as gravitational radiation. This $\frac{\sqrt{2}}{2}\%$ of mass energy is a theoretical lower limit for no entropy change. The upper limit is for a final mass $M = 2m$ and maximal entropy increase, where no gravitational radiation is produced. The coalescence of extremal black holes would have this result. LIGO measurements indicate a usual 5% of total mass released as gravitational radiation. This massive amount of gravitational radiation is ultimately quantum mechanical and composed of gravitons. Signatures of these gravitons should then exist in this radiation.

Entropy increase is a measure of hair on the event horizon. For extremal black holes with $r = a$ or Q the horizon has maximal hair and gravitational radiation is not produced. However, even for Schwarzschild black holes of equal mass that have no hair in the form of charge or angular momentum a coalescence results in a black hole with a mass greater than $\sqrt{2}m$. Entropy will in general increase. This is ultimately due to the fact the stretched horizon of a black hole has quantum hair. This is holographic quantum information of what composes the black hole. This quantum hair will then result in a signature in gravitational radiation as gravitational memory[2].

This signature in gravitational waves results from an event horizon induced from of the Casimir effect. In the last 10^{-24} seconds before the horizons actually merge holographic information on the stretched horizon interact and contribute to the entropy of the coalescing black holes. This might appear to be a

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tiny UV physics, but with the tortoise coordinate stretching of wavelengths $r^* = r - ln|r - 2m|$ for the radius $r = 2m + \lambda$ this quantum hair can be expanded into the .1 to 10^2 Hz range[1]. This is within the detector sensitivity of LIGO at the low end and the future eLISA space-based interferometer. This information will imprint itself as gravitational memory. This is where test masses are not restored to their initial configurations after the passage of a gravitational wave.

The production of gravitons in this manner is related to Hawking radiation as a form of exciton generation. Hawking radiation could be thought of as the production of a quantum black hole that pinches off the horizon of a black hole. By the time this quantum makes its way out of the gravitational well it is IR long wavelength radiation. This is a related process, but where the coalescence of black holes induces the rapid production of gravitons through the excitation of quantum hair on horizons. However, this is an explicit generation of gravitons and more closely related to quantum gravitation.

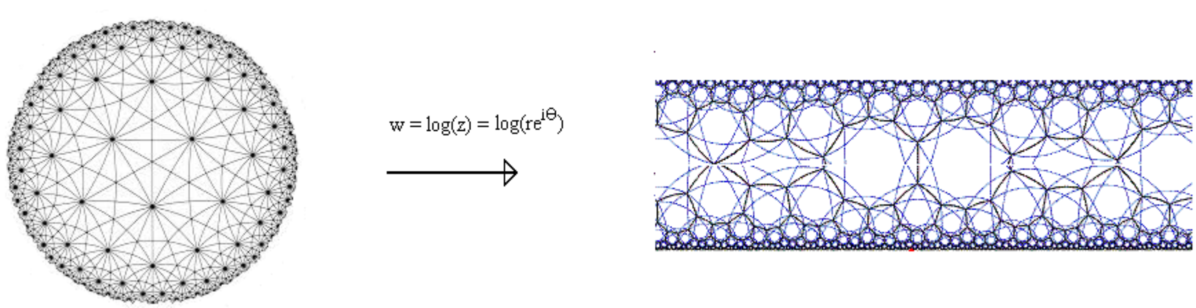
2 Quantum mechanics in a spacetime sandwich between horizons

The Reisner-Nordstrom metric for a charged black hole has a near horizon condition that is equivalent to $AdS_2 \times \mathbb{S}^2$[1]

$$ds^2 = \left(\frac{r}{m}\right)^2 dt^2 - \left(\frac{m}{r}\right)^2 dr^2 - m^2 d\Omega^2.$$

This is also the condition that Carroll, Johnson and Randall found for the extremal spacelike region in the Kerr metric[3]. This suggests a connection to anti-de Sitter geometry, and this is argued to be more completely the case for the sandwiched region between two black holes within 10^{-25} seconds of merging. This however must be argued without reference to an exact metric or solution.

The sandwich region is argued to be locally AdS_4 from two points[1]. The first is to look at two black holes that are far from a merger and approximately calculate the curvatures between them. The second argument is to look at the above metric but where the near horizon condition expands the area of the region with $m^2 \rightarrow r^2 + \rho^2$. This can be seen physically reasonable as the spherical nature of the metric is clearly perturbed and expanded. Also, the first approach indicates clearly there is a spatially hyperbolicity. It is then evident this sandwiched region is a deformed variant of AdS_4. To model this deformation the spatial hyperbolic space $H^3 \subset AdS_4$ in this region is mapped into a strip. The same could be seen geometrically if the antipodal points of a Poincar disk are sent to infinity. This is diagrammatically represented below



This map sends the Poincar disk to a strip, that in three dimensions is the hyperbolic sandwich. In a classical setting a particle trajectory is given by these arcs. This hyperbolic path is also the type of trajectory a particle from Unruh radiation follows. This spatial sandwich is then similar to a Rindler wedge and the quantum hair on the event horizon generates quanta in this region. The most elementary quantum equations to analyze are the Dirac and Klein-Gordon equations. The analysis performed reduced the dimension by one to consider AdS_3 in 2 plus 1 spacetime. The near horizon condition for a near extremal black hole in 4 dimensions is considered for the BTZ black hole. This AdS_3 spacetime is then a foliations f hyperbolic spatial surfaces H^2 in time. These surfaces under conformal mapping are a Poincare disk. The motion of a particle on this disk are arcs that reach the conformal boundary as $t \rightarrow \infty$. This is then the spatial region we consider the dynamics of a quantum particle. This particle we start out treating as a Dirac particle, but the spinor field we then largely ignore by taking the square of the Dirac equation to get a Klein-Gordon wave[1].

Define the z and $\bar{z}$ of the Poincare disk with the metric

$$ds^2_{p-disk} \;=\; R^2 g_{z\bar{z}} dz d\bar{z} \;=\; R^2 \frac{dz d\bar{z}}{1 \;-\; z\bar{z}}$$

with constant negative Gaussian curvature $\mathcal{R} \;=\; -4/R^2$. This metric $g_{z\bar{x}} \;=\; R^2/(1 \;-\; \bar{z}z)$ is invariant under the $SL(2,\ \mathbb{R}) \ \sim SU(1,\ 1)$ group action, which, for $g \;\in\; SU(1,\ 1)$, takes the form

$$z \;\rightarrow\; gz \;=\; \frac{az \;+\; b}{\bar{b}z \;+\; \bar{a}},\ g \;=\; \begin{pmatrix} a & b \\ \bar{b} & \bar{a} \end{pmatrix}. \tag{2.1}$$

The Dirac equation $i\gamma^\mu D_\mu \psi \;+\; m\psi \;=\; 0$, for $D_\mu \;=\; \partial_\mu \;+\; iA_\mu$ on the Poincare disk has the Hamiltonian matrix

$$\mathcal{H} \;=\; \begin{pmatrix} m & H_w \\ H^*_w & -m \end{pmatrix} \tag{2.2}$$

for the Weyl Hamiltonians

$$H_w \;=\; \frac{1}{\sqrt{g_{z\bar{z}}}} \alpha_z \left(2D_z \;+\; \frac{1}{2}\partial_z(ln\ g_{z\bar{z}})\right),$$
$$H^*_w \;=\; \frac{1}{\sqrt{g_{z\bar{z}}}} \alpha_{\bar{z}} \left(2D_{\bar{z}} \;+\; \frac{1}{2}\partial_{\bar{z}}(ln\ g_{z\bar{z}})\right),$$

with $D_z \;=\; \partial_z \;+\; iA_z$ and $D_{\bar{z}} \;=\; \partial_{\bar{z}} \;+\; iA_{\bar{z}}$. here α_z and $\bar{\alpha}_z$ are the 2×2 Weyl matrices[1]. These wave equations are then a quantum version of the hyperbolic dynamics of Mirzakhani and Eskin[4].

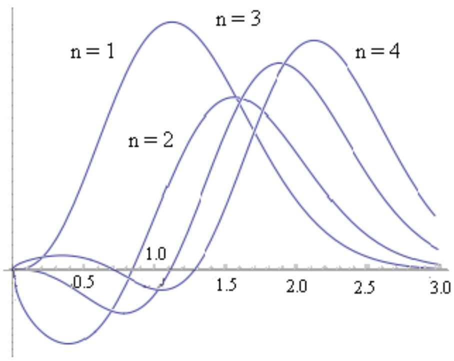
With these two field sources and some approximations, such as considering the fields in the middle of the sandwich, the wave solution is given by a parabolic cylinder function plus a Laguerre polynomial. A numerical output of these is seen below. The parabolic cylinder function has as special cases harmonic oscillator solutions. This connects these analyses with the normal mode theory of Corda. This means that black hole quantum mechanics, at least to some fair degree of approximation, as dynamics that is similar to an atom. The emission and absorption of quanta by black holes shares features similar to an atom[5]. This follows similar analysis for normal or harmonic oscillator modes with black holes[6].The emission of quanta by this process is then a superposition of a hydrogen atom-like system and a harmonic oscillator. Black hole dynamics then connects with aspects of standard quantum physics.

The differential equations are conformal and solutions conformal invariant This is consistent with the AdS/CFT correspondence identified by Maldacena[7]. Further the parabolic cylinder function defines a path integral with the Lagrangian

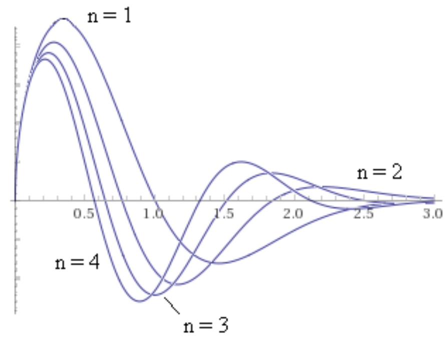
$$\mathcal{L} \;\rightarrow\; \frac{1}{2}\partial_\mu\chi\partial^\mu\chi \;+\; \alpha \;+\; \frac{1}{2}\mu^2\chi^2 \;+\; \frac{1}{4}\lambda\chi^4,$$

where $\frac{2}{3}\alpha \;\rightarrow\; \frac{1}{4}\lambda$. The functional derivatives are then

$$\left((p^2 \;+\; m^2)\frac{\delta}{\delta J} \;+\; \lambda\frac{\delta^3}{\delta J^3}\right) Z \;=\; -i\left\langle\frac{\delta S}{\delta\chi}\right\rangle,$$



Parabolic cylinder function for n = 1, 2, ... 4 represented as a Hermite polynomial solution of the form $x^{1/4} e^{-x^2} H_n(x^2)$.



Laguerre wave function $x^{1/4} e^{-x^2} L^0_n(x^2)$ for hydrogen atomic-like states for n = 1, 2, 3, 4.

These are the wave function components contributed by the parabolic cylinder functions, or Hermite polynomials and the Laguerre polynomials. These depend on $x^2 = k\xi^2$ so the wave function is radial. These are not normalized.

This cubic form has three parabolic cylinder solutions. We may think of this as $ap + bp^3 = J$ and is a cubic equation for the source J that is annulled at three points. The correspond to distinct solutions with distinct paths. These three solutions correspond to three contours and define three distinct vacua. The overall action is a quartic function, which will have three distinct vacua, where one of these is the low energy physical vacua. It is worth noting this transformation of the problem has converted it into a system similar to the Higgs field[1].

The primary interest in these analyses is to explore how quantum mechanics of gravitation and in particular gravitons can be detected or inferred in the detection of gravitational radiation. These signatures will be found in BMS translations[2]. These could be detected by the eLISA program. This is a triangle of spacecraft in an orbit chasing Earth. Each spacecraft has a different inclination so the orbital planes of the three spacecraft is inclined relative to the ecliptic by about 0.33 degree. This defines a plane of the triangular spacecraft formation tilted 60 degrees from the plane of the ecliptic. Each spacecraft houses a laser interferometer and keeps it from being accelerated by external forces. The LISA Pathfinder mission to test this result was a success[8].

Quantum hair will generate information that reaches $\mathcal{I}^\infty$ as translations. This is BMS information that will adjust the metric of spacetime and extension the spatial relationship between the three interferometers.

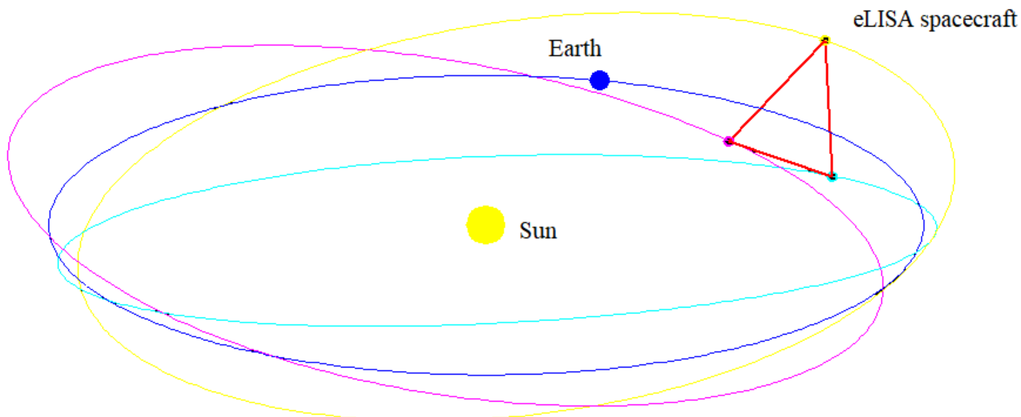
The Bondi metric is [9]

$$ds^2 = du^2 - 2dudr + 2r^2\gamma_{z\bar z} + \frac{2m}{r}du^2 + rC_{zz}dz^2 + rC_{\bar z\bar z}d\bar z^2 + D^zC_{zz}dudz + D^{\bar z}C_{\bar z\bar z}dud\bar z + \dots,$$

for $\gamma_{z\bar z} = 2(1 + z\bar z)^{-2}$ the metric on the unit $\mathbb{S}^2$ sphere. The term m_B is the Bondi mass term, say for a BH and source of mass-energy propagating out to $\mathcal{I}^+$, where the coordinate u is defined. The terms C_{zz} and $C_{\bar z\bar z}$ determine Weyl curvature terms for gravitational wave propagating out. An Einstein field equation is

$$D^2_{\bar z}C_{zz} - D^2_zC_{\bar z\bar z} = 0,$$

which gives the simple solution $C_{zz} = D^2_zC(z,\ \bar z)$. Here $C(z,\ \bar z)$ is a scalar potential. The change in



this potential is a change in Weyl curvature with the passage of a gravitational wave[9]. These define the information generated by quantum hair.

3 Discussion

This development clearly needs refinement. There is a need for explicit calculation of the BMS translations expected. The calculation was also performed with a high degree of approximation. There are of course no exact solutions possible, and further work may require numerical analysis. This may lead to future detection of signatures for gravitons. This with future experiments with quantization on the large and superposed metrics may yield some future experimental understanding of quantum gravitation.

The formalism with the near horizon anti-de Sitter space and conformal symmetry is in line with AdS/CFT correspondence and M-theory. This also leads with the connection to normal modes some general transformation of spacetime solutions via quantum modes. The AdS and dS spacetimes are separated by a light cone in one dimension larger. The de Sitter spacetime on the outside connects with the AdS inside the cone at $\mathcal{I}^\infty$. Thus ultimately the two share the same quantum information. Thus, while string theory has difficulties in de Sitter spacetime with a positive vacuum energy, the field theoretic content of AdS or CFT on the boundary connect to the quantum field theoretic in dS. Further, this all points to methods for understanding how different physical systems in general relativity transform into each other.

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