Perspective

Towards Gross-Pitaevskiian Description of Solar System & Galaxies

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Abstract

In this paper, we argue that Gross-Pitaevskii model can be a more complete description of both solar system and spiral galaxies, especially taking into account the nature of chirality and vortices in galaxies. We also hope to bring out some correspondence among existing models, e.g., the topological vortex approach, Burgers equation in the light of KAM theory, and the Cantorian Navier-Stokes approach. We hope further investigation can be done around this line of approach.

Keywords: Solar system, galaxy, Gross-Pitaevskii, Burgers equation, Navier-Stokes equation.

1. Introduction

From time to time, astronomy and astrophysics discoveries have opened our eyes that the Universe is much more complicated than what it seemed in 100-200 years ago. And despite all pervading popularity of General Relativistic extension to Cosmology, it seems still worthy to remind us to old concepts of Cosmos, for instance the *Hydor theory* of Thales ("that water is the essential element in the Cosmos")[1], and also Heracleitus ("ta *panta rhei kai ouden menei*").[2] Therefore, we can ask: does it mean that the Ultimate theory that we try to find should correspond to hydrodynamics or some kind of turbulence theory?

An indicator of complex turbulence phenomena in Our Universe is the Web like structure. The *Cosmic Web* is the fundamental spatial organization of matter on scales of a few up to a hundred Megaparsec. Galaxies and intergalactic gas matter exist in a wispy web-like arrangement of dense compact clusters, elongated filaments, and sheet-like walls, amidst large near-empty void regions. The filaments are the transport channels along which matter and galaxies flow into massive high-density cluster located at the nodes of the web. The web-like network is shaped by the tidal force field accompanying the inhomogeneous matter distribution.

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[1] https://www.philosophy.gr/presocratics/thales.htm
[2] https://carinawestling.wordpress.com/2010/12/03/τὰ-πάντα-ῥεῖ-καὶ-οὐδὲν-μένει-ta-panta-rhei-kai-ouden-menei/

May be part of that reason that in recent years, there is growing interest to describe the Universe we live in from the perspective of scale-invariant turbulence approach. Such an approach is not limited to hydrodynamics Universe model a la Gibson & Schild, but also from Kolmogorov turbulence approach as well as from String theory approach (some researchers began to explore String-Turbulence).

Recently, Pitkanen describes a solar system model inspired by spiral galaxies [1-2]. While we appreciate his new approach, we find it lacks discussion on the nature of vortices and chirality in galaxy.

In this article, we show some correspondences among existing models, so we discuss shortly, the topological vortice approach, Burgers equation in the light of KAM theory and Golden Mean, and the Cantorian Navier-Stokes approach. We will point out how vortices, turbulence and chirality nature of galaxies seem to suggest a quantised vortex approach, which in turn it corresponds to Gross-Pitaevskiian description.

2. Quantised vortices approach (see also ref. [42-43])

Here we present Bohr-Sommerfeld quantization rules for planetary orbit distances, which results in a good quantitative description of planetary orbit distance in the solar system [6][6b][7]. Then we find an expression which relates the torsion vector and quantized vortices from the viewpoint of Bohr-Sommerfeld quantization rules [3]. Further observation of the proposed quantized vortices of superfluid helium in astro-physical objects is recommended.

<u>Bohr-Sommerfeld quantization rules and quantized vortices</u>

Sonin's book [42] can be paraphrased as follows:

> The movement of vortices has been a region of study for over a century. During the old style time of vortex elements, from the late 1800s, many fascinating properties of vortices were found, starting with the outstanding Kelvin waves engendering along a disconnected vortex line (Thompson, 1880). The primary object of hypothetical investigations around then was a dissipationless immaculate fluid (Lamb, 1997). It was difficult for the hypothesis to find a shared opinion with try since any old style fluid shows gooey impacts. The circumstance changed after crafted by Onsager (1949) and Feynman (1955) who uncovered that turning superfluids are strung by a variety of vortex lines with quantised dissemination. With this revelation, the quantum time of vortex elements started.

The quantization of circulation for nonrelativistic superfluid is given by [3]:

$$\oint v dr = N \frac{\hbar}{m_s} \tag{1}$$

where $N, \hbar, m_s$ represents winding number, reduced Planck constant, and superfluid particle's mass, respectively [3]. And the total number of vortices is given by [44]:

$$N = \frac{\omega.2\pi r^2 m}{\hbar} \tag{2}$$

And based on the above equation (2), Sivaram & Arun [44] are able to give an estimate of the number of galaxies in the universe, along with an estimate of the number stars in a galaxy.

However, they do not give explanation between the quantization of circulation (3) and the quantization of angular momentum. According to Fischer [3], the quantization of angular momentum is a relativistic extension of quantization of circulation, and therefore it yields Bohr-Sommerfeld quantization rules.

Furthermore, it was suggested in [6] and [7] that Bohr-Sommerfeld quantization rules can yield an explanation of planetary orbit distances of the solar system and exoplanets. Here, we begin with Bohr-Sommerfeld's conjecture of quantization of angular momentum. As we know, for the wavefunction to be well defined and unique, the momenta must satisfy Bohr-Sommerfeld's quantization condition:

$$\oint_{\Gamma} p.dx = 2\pi.n\hbar, \tag{3}$$

for any closed classical orbit Γ. For the free particle of unit mass on the unit sphere the left-hand side is:

$$\int_0^T v^2.d\tau = \omega^2.T = 2\pi.\omega, \tag{4}$$

where $T = \frac{2\pi}{\omega}$ is the period of the orbit. Hence the quantization rule amounts to quantization of the rotation frequency (the angular momentum): $\omega = n\hbar.$ Then we can write the force balance relation of Newton's equation of motion:

$$\frac{GMm}{r^2} = \frac{mv^2}{r}. \tag{5}$$

Using Bohr-Sommerfeld's hypothesis of quantization of angular momentum (4), a new constant g was introduced:

$$mvr = \frac{ng}{2\pi}. \tag{6}$$

Just like in the elementary Bohr theory (just before Schrodinger), this pair of equations yields a known simple solution for the orbit radius for any quantum number of the form:

$$r = \frac{n^2.g^2}{4\pi^2.GMm^2}, \tag{7}$$

or

$$r = \frac{n^2.GM}{v_o^2}, \tag{8}$$

where r, n, G, M, v_o represents orbit radii (semimajor axes), quantum number (n=1,2,3,…), Newton gravitation constant, and mass of the nucleus of orbit, and specific velocity, respectively. In equation (10), we denote:

$$v_0 = \frac{2\pi}{g} GMm. \tag{9}$$

The value of m and g in equation (9) are adjustable parameters.

Interestingly, we can remark here that equation (8) is exactly the same with what is obtained by Nottale using his Schrödinger-Newton formula [8]. Therefore here we can verify that the result is the same, either one uses Bohr-Sommerfeld quantization rules or Schrödinger-Newton equation. The applicability of equation (8) includes that one can predict new *exoplanets* (i.e., extrasolar planets) with remarkable result.

Therefore, one can find a neat correspondence between Bohr-Sommerfeld quantization rules and motion of quantized vortices in condensed-matter systems, especially in superfluid helium [3]. Here we propose a conjecture that superfluid vortices quantization rules also provide a good description for the motion of galaxies, especially with respect to their chirality nature, as will be discussed later.

3. Golden ratio is directly related to KAM turbulence via Burgers equation

The Cosmic Web is the fundamental spatial organization of matter on scales of a few up to a hundred Megaparsec. Galaxies and intergalactic gas matter exist in a wispy weblike arrangement of dense compact clusters, elongated filaments, and sheetlike walls, amidst large near-empty void regions. The filaments are the transport channels along which matter and galaxies flow into massive high-density cluster located at the nodes of the web. The weblike network is shaped by the tidal force field accompanying the inhomogeneous matter distribution.[15]

Structure in the Universe has risen out of tiny primordial (Gaussian) density and velocity perturbations by means of gravitational instability. The large-scale anisotropic force field induces anisotropic gravitational collapse, resulting in the emergence of elongated or flattened matter configurations. The simplest model that describes the emergence of structure and complex patterns in the Universe is the Zeldovich Approximation (ZA).[15]
It is our hope that the new approach of CA Adhesion model of the Universe can be verified either with lab experiments, computer simulation, or by large-scale astronomy observation data.

From Zeldovich Approximation to Burgers' equation to Cellular Automaton model

In this section, we will outline a route from ZA to Burgers' equation and then to CA model. The simplest model that describes the emergence of structure and complex patterns in the Universe is the Zeldovich Approximation (ZA). In essence, it describes a ballistic flow, driven by a constant (gravitational) potential. The resulting Eulerian position x(t) at some cosmic epoch t is specified by the expression[15]:

$$x(t) = q + D(t)u_o(q), \tag{10}$$

where q is the initial "Lagrangian" position of a particle, D(t) the time-dependent structure growth factor and

$$u_0 = -\nabla_q \Phi_0 \tag{11}$$

its velocity. The nature of this approximation may be appreciated by the corresponding source-free equation of motion,

$$\frac{\partial u}{\partial D} + (u \cdot \nabla_x)u = 0. \tag{12}$$

The use of ZA is ubiquitous in cosmology. One major application is its key role in setting up initial conditions in cosmological N-body simulations. Of importance here is its nonlinear extension in terms of *Adhesion Model* [15]:

The ZA breaks down as soon as self-gravity of the forming structures becomes important. To 'simulate' the effects of self-gravity, Gurbatov *et al*. included an artificial viscosity. This results in the Burgers' equation as follows [15]:

$$\frac{\partial u}{\partial D} + (u \cdot \nabla_x)u = \nu.\nabla_x^2 u, \tag{13}$$

a well known PDE from fluid mechanics. This equation has an exact analytical solution, which in the limit of $\nu \to 0$, the solution is [15]:

$$\phi(x, D) = \max_q \left[\Phi_0(q) - \frac{(x-q)^2}{2D} \right]. \tag{14}$$

This leads to a geometric interpretation of the Adhesion Model. The solution follows from the evaluation of the convex hull of the velocity potential modified by a quadratic term. We found that the solution can also be found by computing the weighted Voronoi diagram of a mesh weighted with the velocity potential. For more detailed discussion on Adhesion Model of the Universe, see for example [18].

Now, let us consider another routes to solve Burgers equation: (a) by numerical computation with *Mathematica*, see [17]; and (b) by virtue of CA approach. Let us skip route (a), and discuss less known approach of cellular automata.

We start with the Burgers' equation with Gaussian white noise which can be rewritten as follows [16]:

$$\frac{\partial u}{\partial t} + \xi = 2u\frac{\partial u}{\partial x} + \frac{\partial^2 u}{\partial x^2} + \eta. \tag{15}$$

By introducing new variables and after straightforward calculations, we have the automata rule [16]:

$$\begin{aligned} \phi_i^{t+1} &= \phi_{i-1}^t + \max[0, \phi_i^t - A, \phi_i^t + \phi_{i+1}^t - B, \Psi_i^t - \phi_{i-1}^t] \\ &- \max[0, \phi_{i-1}^t - A, \phi_{i-1}^t + \phi_i^t - B, \Phi_i^t + \phi_{i-1}^t] \end{aligned} \tag{16}$$

In other words, in this section we give an outline of a plausible route from ZA to Burgers' equation then to CA model, which suggests that it appears possible –at least in theory- to consider a nonlinear cosmology based on CA Adhesion model.

From KAM theory to Golden section

Another possible way to describe the complex structure of Universe, is the Kolmogorov-Arnold-Moser (KAM) theorem, which states that if the system is subjected to a weak nonlinear perturbation, some of the invariant tori are deformed and survive, while others are destroyed. The ones that survive are those that have "sufficiently irrational frequencies" (the non-resonance condition, so they do not interfere with one another). The golden ratio being the most irrational number is often evident in such systems of oscillators. It is also physically significant in that circles with golden mean frequencies are the last to break up in a perturbed dynamical system, so the motion continues to be quasi-periodic, i.e., recurrent but not strictly periodic or predictable.

An important consequence of the KAM theorem is that for a large set of initial conditions, the motion remains perpetually quasi-periodic, and hence stable. KAM theory has been extended to non-Hamiltonian systems and to systems with fast and slow frequencies.

Those KAM tori that are not destroyed by perturbation become invariant Cantor sets, or "Cantori". The frequencies of the invariant Cantori approximate the golden ratio. The golden ratio effectively enables multiple oscillators within a complex system to co-exist without blowing up the system. But it also leaves the oscillators within the system free to interact globally (by resonance), as observed in the coherence potentials that turn up frequently when the brain is processing information.

Obviously, this can be tied in to the creation of subatomic particles such as electrons and positrons. At a certain scale of smallness, the media in the local volume becomes isotropic, while larger volumes exhibit occupation by ever-larger turbulence formations and exhibit extremes of **an**isotropy in the media.

The Kolmogorov Limit is $10e^{-58}$ m, which is the smallest vortex that can exist in the aether media. Entities smaller than this, down to the SubQuantum infinitesimals (Bhutatmas) (vortex lines) are the primary cause of gravitation (a "sink" model of gravitation caused by superluminal infinitesimals).[3]

Figure 1. Turbulent flow generated by the tip vortex of the aeroplane wing shown up by red agricultural dye. (after Mae Wan-Ho [38]).

Shadow gravity is valid in the situation of gravitational interaction between two discrete masses that divert the ambient gravitational flux-density away from each other. This happens due to absorption (rare), scattering (more common), and refraction (most of the time) of gravitational infinitesimals.

Gravitational flux density is a variable depending on stellar, interstellar, and intergalactic events. A simplified model of vorticity fields in large scale structures of the Universe is depicted below:

What is more interesting here, is that it can be shown that there is correspondence between Golden section and in coupled oscillators and KAM Theorem, but also between Golden section and Burgers equation. [35]. For more discussion, on Golden Mean and its ramifications, see for instance [39[40][41].

[3] Thanks to discussions with Robert Neil Boyd, PhD.

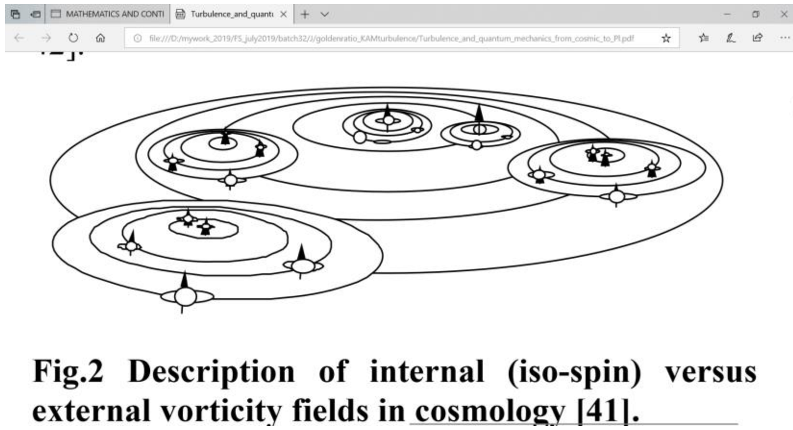

Figure 2. Vorticity fields in cosmology (after Siavash Sohrab [34]).

4. Cantorian Navier-Stokes approach

<u>Vorticity as the driver of Accelerated Expansion</u>

According to Ildus Nurgaliev [26], velocity vector V_α of the material point is projected onto coordinate space by the tensor of the second rank $H_{\alpha\beta}$:

$$V_\alpha = H_{\alpha\beta} R^\beta \tag{17}$$

where the Hubble matrix can be defined as follows for a homogeneous and isotropic universe:

$$H_{\alpha\beta} = \begin{pmatrix} H & \pm\omega & \pm\omega \\ \mp\omega & H & \pm\omega \\ \mp\omega & \mp\omega & H \end{pmatrix} \tag{18}$$

where the global average vorticity may be zero, though not necessarily [7]. Here the Hubble law is extended to 3x3 matrix.

Now we will use Newtonian equations to emphasize that cosmological singularity is consequence of the too simple model of the flow, and has nothing to do with special or general relativity as a cause [26]. Standard equations of Newtonian hydrodynamics in standard notations read:

$$\frac{d\vec{\upsilon}}{dt} = \frac{\partial\vec{\upsilon}}{\partial t} + \vec{\upsilon}\nabla\vec{\upsilon} = -\nabla\varphi + \frac{1}{\rho}\nabla\rho + \frac{\mu}{\rho}\Delta\vec{\upsilon} + ..., \tag{19}$$

$$\frac{\partial \rho}{\partial t} + \nabla \rho \vec{\upsilon} = 0, \tag{20}$$

$$\Delta \varphi = 4\pi G \rho \tag{21}$$

Procedure of separating of diagonal H, trace-free symmetrical σ, and anti-symmetrical ω elements of velocity gradient was used by Indian theoretician Amal Kumar Raychaudhury (1923-2005). The equation for expansion θ, sum of the diagonal elements of [7]:

$$\dot{\theta} + \frac{1}{3}\theta^2 + \sigma^2 - \omega^2 = -4\pi G \rho + div(\frac{1}{\rho}\sum f) \tag{22}$$

is most instrumental in the analysis of singularity and bears the name of its author. [26]

System of (25)-(27) gets simplified up to two equations [26]:

$$\dot{\theta} + \frac{1}{3}\theta^2 - \omega^2 = 0, \tag{23}$$

$$\dot{\omega} + \frac{2}{3}\theta\omega = 0. \tag{24}$$

Recalling $\theta = 3H$, the integral of (30) takes the form [26]:

$$H^2 = H_\infty^2 - \frac{3\omega_0^2 R_0^4}{R^4}. \tag{25}$$

<u>How to write down Navier-Stokes equations on Cantor Sets</u>

Now we can extend further the Navier-Stokes equations to Cantor Sets, by keeping in mind their possible applications in cosmology.

By defining some operators as follows:

1. In Cantor coordinates [28]:

$$\nabla^\alpha \cdot u = div^\alpha u = \frac{\partial^\alpha u_1}{\partial x_1^\alpha} + \frac{\partial^\alpha u_2}{\partial x_2^\alpha} + \frac{\partial^\alpha u_3}{\partial x_3^\alpha}, \tag{26}$$

$$\nabla^\alpha \times u = curl^\alpha u = \left(\frac{\partial^\alpha u_3}{\partial x_2^\alpha} - \frac{\partial^\alpha u_2}{\partial x_3^\alpha}\right)e_1^\alpha + \left(\frac{\partial^\alpha u_1}{\partial x_3^\alpha} - \frac{\partial^\alpha u_3}{\partial x_1^\alpha}\right)e_2^\alpha + \left(\frac{\partial^\alpha u_2}{\partial x_1^\alpha} - \frac{\partial^\alpha u_1}{\partial x_2^\alpha}\right)e_3^\alpha . \tag{27}$$

2. In Cantor-type cylindrical coordinates [29, p.4]:

$$\nabla^\alpha \cdot r = \frac{\partial^\alpha r_R}{\partial R^\alpha} + \frac{1}{R^\alpha}\frac{\partial^\alpha r_\theta}{\partial \theta^\alpha} + \frac{r_R}{R^\alpha} + \frac{\partial^\alpha r_z}{\partial z^\alpha}, \tag{28}$$

$$\nabla^{\alpha} \times r = \left(\frac{1}{R^{\alpha}} \frac{\partial^{\alpha} r_{\theta}}{\partial \theta^{\alpha}} - \frac{\partial^{\alpha} r_{\theta}}{\partial z^{\alpha}} \right) e_R^{\alpha} + \left(\frac{\partial^{\alpha} r_R}{\partial z^{\alpha}} - \frac{\partial^{\alpha} r_z}{\partial R^{\alpha}} \right) e_{\theta}^{\alpha} + \left(\frac{\partial^{\alpha} r_{\theta}}{\partial R^{\alpha}} + \frac{r_R}{R^{\alpha}} - \frac{1}{R^{\alpha}} \frac{\partial^{\alpha} r_R}{\partial \theta^{\alpha}} \right) e_z^{\alpha} . \tag{29}$$

Then Yang, Baleanu and Machado are able to obtain a general form of the Navier-Stokes equations on Cantor Sets as follows [28, p.6]:

$$\rho \frac{D^{\alpha} \upsilon}{Dt^{\alpha}} = -\nabla^{\alpha} \cdot (pI) + \nabla^{\alpha} \left[2\mu \left(\nabla^{\alpha} \cdot \upsilon + \upsilon \cdot \nabla^{\alpha} \right) - \frac{2}{3} \mu \left(\nabla^{\alpha} \cdot \upsilon \right) I \right] + \rho b \tag{30}$$

The next task is how to find observational cosmology and astrophysical implications. This will be the subject of future research.

5. Correspondence with Gross-Pitaevskiian description and a description of chirality nature of galaxies

In this section we will point out how vortices and chirality nature of galaxies seem to suggest a superfluid vortex approach, which in turn it corresponds to Gross-Pitaevskiian description. The nature and origin of chirality in galaxies remain an elusive topic to explain. However we can recall some recent works to suggest an explanation.

First of all, let us quote from abstract of a recent paper [9], where Tapio Simula wrote, which can be rephrased as follows:

> Right now, and electromagnetism have a similar starting point and are new properties of the superfluid universe, which itself rises up out of the hidden aggregate structure of progressively basic particles, for example, atoms. The Bose–Einstein condensate is identified as the tricky dull matter of the superfluid universe with vortices and phonons, separately, comparing to huge charged particles and massless photons.

In lieu of his model of electromagnetic and gravitation fields in terms of superfluid vortices, we can also come up with a model of chirality in cosmology from Proca equations. As Proca equations can be used to describe electromagnetic field of superconductor, we find it as a possible approach too [45].

Now we are going to discuss how it can be used as a model of chirality nature of galaxies. Cappoziello and Lattanzi argue that spiral galaxies are axi-symmetric objects showing 2D-chirality when projected onto a plane [36]. In their enantiomers model, chirality in spiral galaxies and chirality Spiral galaxies are axi-symmetric objects showing 2D-chirality when projected onto a plane, and their progressive loss of chirality surrounding its galaxy center, can point out of vorticity in superfluidity.

See the following figure, which is quite in agreement with Figure 2 by Sohrab:

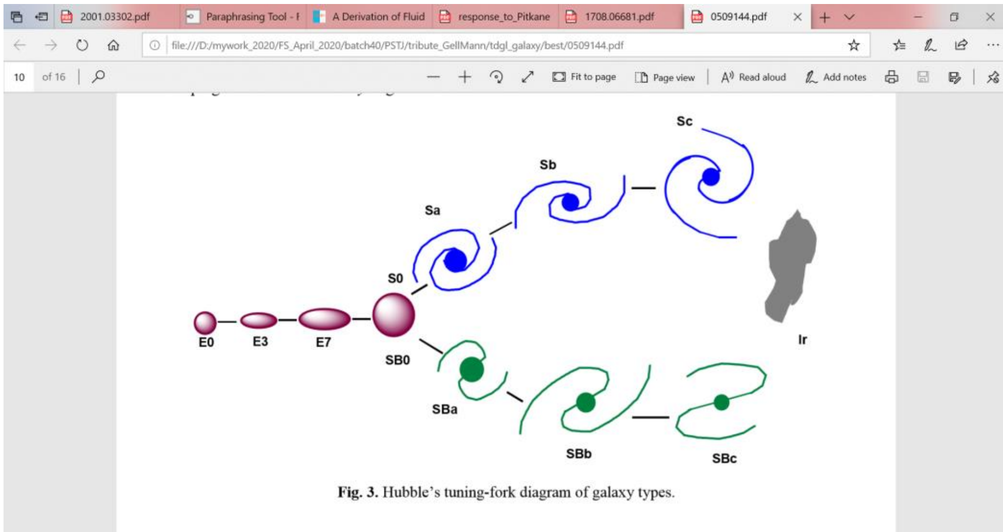

Figure 3 . After Cappoziello & Lattanzi [36].

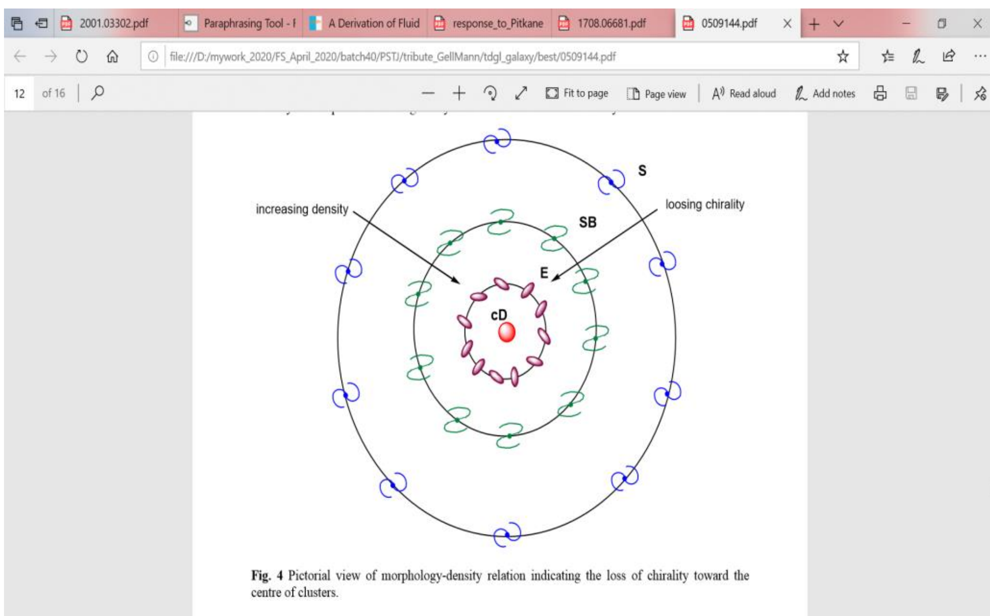

Figure 4. Progressive loss of chirality. After Cappoziello & Lattanzi [36].

With regards to question posed above: what kind of medium of interaction capable of doing such a quantal action? Allow us to quote from Fernandez-Hernandez *et al's* abstract [37], which can be paraphrased as follows:

> The ultra-light scalar fields are likewise called scalar field dull issue model. Right now study turn bends for low surface brilliance winding cosmic systems utilizing two scalar

field models: the Gross-Pitaevskii *Bose-Einstein condensate in the Thomas-Fermi estimation* and a scalar field arrangement of the Klein-Gordon condition. We additionally utilized the zero circle guess universe model where photometric information isn't thought of.

Therefore, we come up with a conclusion that it seems *Gross-Pitaevskiian description of Bose-Einstein condensate is necessary for correct modelling of spiral and non-spiral galaxies*. Interestingly, in two rather old papers both of us (VC & FS) have argued for derivation of Schrödinger equation model of planetary orbits of solar system from TDGL (Gross-Pitaevskii) description [46-47]. Therefore, it seems we can arrive at this conclusion: it is possible to come up with consistent description of both solar system and galaxy dynamics, including its chirality and rotation curves, by virtue of Gross-Pitaevkiian description of BEC/superfluidity.

6. Conclusion

In this paper, we have discussed several approaches in description of planetary systems as well as galaxies. Sections 2-4 have been presented in earlier papers.

Therefore, we come up with a conclusion that there are sufficient grounds to argue in favour of Gross-Pitaevskiian description of Bose-Einstein condensate; i.e. it is necessary for correct modelling of spiral and non-spiral galaxies. Interestingly, in a rather old paper both of us (VC & FS) have argued for derivation of Schrodinger equation model of planetary orbits of solar system from TDGL (Gross-Pitaevskii) description.

Summarizing, it seems we can arrive at this conclusion: it is possible to come up with consistent description of both solar system and galaxy dynamics, *including its chirality and rotation curves, by virtue of Gross-Pitaevkiian description of BEC.*

Of course, this short article is far from being complete. We hope further investigation can be done around this line of approach. The remaining questions include how to find observational cosmology and astrophysical implications. Future research is recommended.

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