Article

On a Natural Solution for the Hierarchy Problem Using Dimensional Regularization

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Abstract

This brief report suggests a straightforward solution for the hierarchy problem of the Standard Model using dimensional regularization of quantum field theory (QFT). Our viewpoint breaks away from traditional approaches to the hierarchy problem based on supersymmetry (SUSY), Technicolor, extra-dimensions, anthropic arguments, fine-tuning or gauge unification near the Planck scale.

Key words: hierarchy problem, dimensional regularization, natural solution.

1. Introduction

It is well known that renormalization of perturbative QFT is conceived as a two-step program: regularization and subtraction. One first controls the divergence present in momentum integrals by inserting a suitable "regulator", and then brings in a set of "counter-terms" to cancel out the divergence. Momentum integrals have the generic form [1]

$$I = \int_0^\infty d^4 q F(q) \tag{1}$$

Two regularization techniques are frequently employed to manage (1), namely "momentum cutoff" and "dimensional regularization". When the first scheme is applied for regularization in the **ultraviolet** (UV) limit, the infinite bound of (1) is replaced by a finite upper scale M,

$$I \to I_M = \int_0^M d^4 q F(q) \tag{2}$$

Explicit calculation of the convergent integral (2) amounts to a generic sum of three polynomial terms

$$I_M = A(M) + B + C(1/M) \tag{3}$$

Dimensional regularization proceeds instead by shifting the momentum integral (1) from a four-dimensional space to a continuous D - dimensional space

$$I \to I_D = \int_0^\infty d^D q F(q) \tag{4}$$

Introducing the parameter $\varepsilon = 4 - D$ leads to

$$I_D \to I_\varepsilon = A'(\varepsilon) + B' + C'(1/\varepsilon) \tag{5}$$

It follows that M and ε are not independent regulators and relate to each other via [5]

$$\textbf{UV:} \quad \varepsilon = 4 - D = \frac{1}{\log(M/M_0)} \tag{6}$$

where M_0 stands for an arbitrary and finite reference scale with $M_0 < M$.

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A similar technique can be used to regularize field theory in the **infrared** (IR) limit whereby M is taken to represent the lowest bound scale. A strictly positive ε on less than four dimensions ($D<4$) requires taking the reciprocal of the logarithm appearing in (6) to comply with $M_0 > M$. The infrared version of (6) accordingly reads:

$$\textbf{IR}: \quad \varepsilon' = 4 - D = \frac{1}{\log(M_0/M)} \tag{7}$$

2. Assumptions

Our model rests on the following assumptions:
P1) Electroweak symmetry breaking (EWSB) is interpreted as a second-order phase transition which turns massless photons into massive gauge bosons [4].
P2) The deep IR cutoff of field theory is set by the cosmological constant scale [2]

$$M_{IR} = (\Lambda_{cc})^{1/4} \tag{8}$$

where Λ_{cc} represents the cosmological constant.
P3) The deep UV cutoff of field theory is set by the Planck scale:

$$M_{UV} = M_{Pl} \tag{9}$$

3. Solving the hierarchy problem

Making use of P1), P2) and P3) leads to the conclusion that, as the electroweak scale (M_{EW}) is approached from above or below, (6) and (7) naturally converge to each other. Taking $M_0 = M_{EW}$ and replacing (8) and (9) in (7) and (6), respectively, yields

$$\boxed{\frac{M_{EW}}{M_{cc}} = \frac{M_{Pl}}{M_{EW}} \rightarrow (\Lambda_{cc})^{1/4} = \frac{M_{EW}^2}{M_{Pl}}} \tag{10}$$

4. Conclusions

1) Asymptotic approach to four-dimensional space-time explains the existence of the deep IR cutoff (Λ_{cc}) and deep UV cutoff (M_{Pl}). Stated differently, fractal space-time description supplied by the condition $\varepsilon > 0$ and $\varepsilon' > 0$ appears to be linked to these natural bounds [6].
2) Fixing two out of the three scales involved in (10) automatically determines the third one.
3) The gauge hierarchy problem, cosmological constant problem and the existence of the electroweak phase transition appear to be deeply interconnected.
4) Our simple derivation stands in sharp contrast with sophisticated approaches to the hierarchy problem based on supersymmetry (SUSY), Technicolor, extra-dimensions, anthropic arguments, fine-tuning or gauge unification near the Planck scale [3].

References

[1] Weinberg S., The Quantum Theory of Fields; Cambridge University Press: Cambridge, UK, 1995; Vol.1.
[2] Arkani-Hamed, N., http://arxiv.org/PS_cache/hep-th/pdf/0501/0501082v1.pdf

[3] Ross Graham G., http://rsta.royalsocietypublishing.org/content/359/1779/405.full.pdf
[4] Goldfain E., http://www.worldacademicunion.com/journal/1749-3889-3897IJNS/IJNSVol3No3Paper02.pdf
[5] van Baal, P., http://www.lorentz.leidenuniv.nl/~vanbaal/FT/extract.pdf
[6] Goldfain E., http://www.vixra.org/abs/1011.0061