Article

Dark Energy Model in a Five-Dimensional Space-Time with a Massive Scalar Field

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Abstract

The main aim of this investigation is to study the dynamical aspects of a cosmological model with dark energy (DE) and an attractive massive scalar field as source in a five dimensional spherically symmetric space-time. An exact solution of the field equations of general relativity is obtained which represents a massive scalar field DE model in five dimensions. The cosmological parameters of the model are computed and their dynamical behavior is studied. It is noted that our model is a quintessence model which represents accelerated expansion of the universe and is in good agreement with the recent cosmological observation.

Keywords: Massive scalar field, cosmological model, dark energy, five dimensional, space time.

1.Introduction

We live in this universe. Hence it is necessary to know about our universe. There have been several modern cosmological observations to study the nature of the universe. The recent experimental study of type 1a supernova [1-2] and cosmic microwave background data [3] suggests that the universe is spatially flat and accelerating. It is supposed that this is because of the hither to unknown fluid with high negative pressure known as 'dark energy' (DE). Even today, it has been a challenging problem to explain DE which is driving the acceleration of the universe. The simplest candidate for DE is the cosmological constant. But this suffers from coincidence problem. Hence several DE models like quintessence [4] phantom [5] quintom [6], tachyon [7] and holographic models [8] have been considered.

In recent years, scalar field DE models have been attracting several authors, in particular, in the presence of Brans-Dicke [9], Saez-Ballester [10] and Barber's [11] scalar fields. The following are some of the quintessence models obtained in Bianchi type space-times. Rao et al. [12] obtained a Bianchi type-II modified holographic DE model in Barber's second Self-Creation theory of gravity while Reddy [13] discussed Bianchi type-V modified holographic Ricci DE models in Saez-Ballester scalar tensor theory of gravitation. Reddy et al. [14] investigated

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minimally interacting Kaluza-Klein holographic DE model in the presence of Brans-Dicke scalar field while Kiran et al. [15,16] have presented Bianchi type minimally interacting holographic DE models in Brans-Dicke and Saez-Ballester theories of gravitation. Also Rao et al. [17] obtained five dimensional FRW modified holographic Ricci DE model in Saez-Ballester scalar tensor theory. Very recently Aditya and Reddy [18] have investigated FRW type Kaluza-Klein modified holographic DE models in Brans-Dicke theory.

Very recently, investigation of DE models in the presence of scalar-meson fields is attracting many researchers. It is well known that the scalar meson fields are of two categories, namely, zero mass and massive scalar fields. Mass less scalar fields describe long range interactions while massive scalar fields represent short range interactions. Massive and mass scalar fields along with different physical sources have been investigated by various authors [19-23]. Attractive massive scalar fields play a vital role in the discussion of scalar field DE cosmological models. Hence, Reddy [24] obtained Bianchi type-V DE model with zero mass scalar fields. Naidu [25] presented Bianchi type-II modified holographic Ricci DE model in the presence of massive scalar field. Aditya and Reddy [26] have investigated Bianchi type-III massive scalar field DE model while Reddy et al. [27] discussed Kantowski-Sachs DE model in the presence of anisotropic DE fluid. Also, five dimensional Kaluza-Klein DE model with zero mass scalar fields has been presented by Reddy and Ramesh [28]. Reddy and Ramesh [29] discussed inflationary cosmological model with flat potential in the presence of mass less scalar field. Recently, Naidu et al. [30] have obtained a DE model with massive scalar field in Bianchi type-V space-time.

Here our main interest is to explore higher dimensional DE models with massive scalar fields as source. The significance of the higher dimensional space-time in the discussion of early stage of evolution of the universe is well known. This is because of the fact that the cosmos in its early stages might have had higher dimensional era before the universe has undergone compactification transition. Several authors, therefore, have been attracted to this subject and obtained five dimensional cosmological models [31-35]. In particular, Samantha and Dhal [36] have obtained a new class of higher dimensional cosmological models in f(R,T) gravity [37]. Rao and Jayasudha and Raju et al. [38-40] have discussed five dimensional spherically symmetric cosmological models in Brans-Dicke and Saez-Ballester theories of gravitation.

Inspired by the above investigations and the discussion, we propose to explore five dimensional anisotropic spherically symmetric cosmological model in the presence of DE fluid and massive scalar field. Following is the scheme of this paper: Section 2 deals with the metric and corresponding field equations. In sec.3 we obtain the solution of the field equations and present the cosmological DE model in five dimensions. Sec.4 is devoted to the discussion of cosmological parameters. The last section contains conclusions.

2. Metric and Field Equations

Five-dimensional spherically symmetric anisotropic space-time is described by the metric

$$ds^2 = dt^2 - e^{\alpha}(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2) - e^{\beta} d\psi^2 \tag{1}$$

Here α *and* β are functions of time t and the fifth coordinate ψ is considered space-like [41]
The non-vanishing components of Einstein tensor for the metric (1) are

$$\begin{aligned} G_0^0 &= -\frac{3}{4}\left(\dot{\alpha} + \dot{\alpha}\dot{\beta}\right) \\ G_1^1 = G_2^2 = G_3^3 &= -\left(\ddot{\alpha} + \frac{3}{4}\dot{\alpha}^2 + \frac{1}{2}\ddot{\beta} + \frac{1}{4}\dot{\beta}^2 + \frac{1}{2}\dot{\alpha}\dot{\beta}\right) \\ G_4^4 &= -\frac{3}{2}\left(\ddot{\alpha} + \dot{\alpha}^2\right) \end{aligned} \tag{2}$$

where an overhead dot indicates differentiation with respect to t.
Einstein's field equations in the presence of DE fluid and massive scalar field are written as

$$R_{ij} - \frac{1}{2} g_{ij} R = -(T_{ij}^{(de)} + T_{ij}^{(s)}) \tag{3}$$

Here $T_{ij}^{(de)}$ is the energy momentum tensor of DE given by

$$T_{ij}^{(de)} = (\rho_\wedge + p_\wedge) u_i u_j - p_\wedge g_{ij} \tag{4}$$

and

$$T_{ij}^{(s)} = \varphi_{,i}\varphi_{,j} - \frac{1}{2}(\varphi_{,k}\varphi^{,k} - M^2\varphi^2) \tag{5}$$

where φ is the massive scalar field which satisfies the Klein-Gordon equation.

$$g^{ij}\varphi_{;ij} + M^2\varphi = 0 \tag{6}$$

M is the mass of the scalar field φ, $p_\wedge$ and $\rho_\wedge$ are DE pressure and DE density respectively. Also, comma and semicolon denote ordinary and covariant differentiation respectively. (Gravitational units are used so that $8\pi G = c = 1$).
The energy momentum tensor of DE given by eq. (4) can be parameterized as

$$T_{ij}^{(de)} = \mathrm{diag}\left[1, -\omega_\wedge, -(\omega_\wedge + \gamma), -(\omega_\wedge + \delta) - (\omega_{de} + \xi)\right]\rho_\wedge \tag{7}$$

where

$$\omega_{\wedge} = \frac{p_{\wedge}}{\rho_{\wedge}} \tag{8}$$

is the equation of state (EoS) parameter of DE and γ, δ, ξ are the skewness parameters which are the deviations from $\omega_{\wedge}$ along y, z and φ axes respectively.

Now using co moving coordinates and equations (2),(3),(5) and (7) the Einstein field equations for the metric (1) can be written as

$$\frac{3}{4}\left(\dot{\alpha}^2 + \dot{\alpha}\dot{\beta}\right) = -\rho_{\wedge} + \frac{\dot{\varphi}^2}{2} + \frac{M^2\varphi^2}{2} \tag{9}$$

$$\ddot{\alpha} + \frac{3\dot{\alpha}^2}{4} + \frac{\ddot{\beta}}{2} + \frac{\dot{\beta}^2}{4} + \frac{\dot{\alpha}\dot{\beta}}{2} = -\omega_{\wedge}\rho_{\wedge} - \frac{\dot{\varphi}^2}{2} + \frac{M^2\varphi^2}{2} \tag{10}$$

$$\ddot{\alpha} + \frac{3\dot{\alpha}^2}{4} + \frac{\ddot{\beta}}{2} + \frac{\dot{\beta}^2}{4} + \frac{\dot{\alpha}\dot{\beta}}{2} = -(\omega_{\wedge} + \gamma)\rho_{\wedge} - \frac{\dot{\varphi}^2}{2} + \frac{M^2\varphi^2}{2} \tag{11}$$

$$\ddot{\alpha} + \frac{3\dot{\alpha}^2}{4} + \frac{\ddot{\beta}}{2} + \frac{\dot{\beta}^2}{4} + \frac{\dot{\alpha}\dot{\beta}}{2} = -(\omega_{\wedge} + \delta)\rho_{\wedge} - \frac{\dot{\varphi}^2}{2} + \frac{M^2\varphi^2}{2} \tag{12}$$

$$\frac{3}{2}\left(\ddot{\alpha} + \dot{\alpha}^2\right) = -(\omega_{\wedge} + \xi)\rho_{\wedge} - \frac{\dot{\varphi}^2}{2} + \frac{M^2\varphi^2}{2} \tag{13}$$

The energy conservation equation, $T^{ij}_{;j} = 0$ gives

$$\dot{\rho}_{\wedge} + \rho_{\wedge}\left(1 + \omega_{\wedge}\right)\left(\frac{3\dot{\alpha} + \dot{\beta}}{2}\right) = 0 \tag{14}$$

The Klein-Gordon equation (6) becomes

$$\ddot{\varphi} + \dot{\varphi}\left(\frac{3\dot{\alpha} + \dot{\beta}}{2}\right) + M^2\varphi = 0 \tag{15}$$

where an overhead dot denotes differentiation with respect to time t

For the space-time given by Eq. (1), we define the following cosmological parameters which will be useful in solving the field equations (9) - (15)

The average scale factor $a(t)$ and the spatial volume V are given by

$$V = a^4(t) = e^{\frac{3\dot{\alpha}+\dot{\beta}}{2}} \tag{16}$$

Generalized Hubble parameter is

$$H = \frac{1}{4}\left(\frac{3}{2}\dot{\alpha} + \frac{1}{2}\dot{\beta}\right) \tag{17}$$

The scalar expansion θ, shear scalar σ^2 and the average anisotropy parameter are

$$\theta = u^i_{;i} = 4H = \frac{3}{2}\dot{\alpha} + \frac{1}{2}\dot{\beta} \tag{18}$$

$$\sigma^2 = \frac{1}{2}\sigma^{ij}\sigma_{ij} = \frac{3}{8}\dot{\alpha}^2 + \frac{1}{8}\dot{\beta}^2 - \frac{1}{2}\dot{\alpha}^2 - \frac{1}{6}\dot{\beta} + \frac{2}{9} \tag{19}$$

$$A_h = \frac{1}{4}\sum_{i=1}^{4}\left(\frac{H_i - H}{H}\right)^2 \tag{20}$$

where $H_i (i = 1, 2, 3, 4)$ represents directional Hubble parameters.

The deceleration parameter is

$$q = -1 + \frac{d}{dt}\left(\frac{1}{H}\right) \tag{21}$$

3. Solution of field equations and DE model

From Eqs.(10), (11) and (12) we, immediately, obtain

$$\gamma = \delta = 0 \tag{22}$$

Using Eq. (22), Eqs. (9) - (15) reduce to the following independent equations:

$$\frac{3}{4}\left(\dot{\alpha}^2+\dot{\alpha}\dot{\beta}\right)=-\rho_{\wedge}+\frac{\dot{\varphi}^2}{2}+\frac{M^2\varphi^2}{2} \tag{23}$$

$$\ddot{\alpha}+\frac{3\dot{\alpha}^2}{4}+\frac{\ddot{\beta}}{2}+\frac{\dot{\beta}^2}{4}+\frac{\dot{\alpha}\dot{\beta}}{2}=-\omega_{\wedge}\rho_{\wedge}-\frac{\dot{\varphi}^2}{2}+\frac{M^2\varphi^2}{2} \tag{24}$$

$$\frac{3}{2}\left(\ddot{\alpha}+\dot{\alpha}^2\right)=-(\omega_{\wedge}+\xi)\rho_{\wedge}-\frac{\dot{\varphi}^2}{2}+\frac{M^2\varphi^2}{2} \tag{25}$$

$$\ddot{\varphi}+\dot{\varphi}\left(\frac{3\dot{\alpha}+\dot{\beta}}{2}\right)+M^2\varphi=0 \tag{26}$$

[Eq. (14) being conservation equation].

Now, the field equations (23) - (26) are a system of four independent equations in five unknowns $\alpha,\beta,\rho_{\wedge},\omega_{\wedge}$ and φ. To obtain a determinate solution we use the relation between metric potentials given by [36,38]

$$\alpha=k\beta \tag{27}$$

where $k\neq 0$ is a constant.

Using Eq. (27) in Eq. (26) we have

$$\ddot{\varphi}+\dot{\varphi}\left(\frac{3k+1}{2}\right)\dot{\beta}+M^2\varphi=0 \tag{28}$$

To reduce the mathematical complexity we have [22,23,29]

$$\left(\frac{3k+1}{2}\right)\dot{\beta}=-\frac{\dot{\varphi}}{\varphi} \tag{29}$$

which amounts to a power law between average scale factor and the scalar field φ.

Now using Eq. (29) in Eq. (28) we get

$$\varphi=\exp\left(\varphi_0 t-\frac{M^2t^2}{2}+\varphi_1\right) \tag{30}$$

From Eqs. (29) and (30) we obtain

$$\beta = \left(\frac{M^2 t^2 - 2\varphi_0 t - 2\varphi_1}{3k+1} \right) \tag{31}$$

From Eqs. (27) and (31) we have

$$\alpha = \left(\frac{k}{3k+1} \right) \left(M^2 t^2 - 2\varphi_0 t - 2\varphi_1 \right) \tag{32}$$

Here φ_0 and φ_1 are constants of integration.

Now using Eqs. (31) and (32),in Eq. (1) the DE modelcan be written as

$$\begin{aligned} ds^2 = dt^2 - \exp\left[\left(\frac{k}{3k+1} \right) \left(M^2 t^2 - 2\varphi_0 t - 2\varphi_1 \right) \right] \left[dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2 \right] \\ - \exp\left[\frac{M^2 t^2 - 2\varphi_0 t - 2\varphi_1}{3k+1} \right] d\psi^2 \end{aligned} \tag{33}$$

and the scalar field in this model is given by Eq. (30).

4. Discussion of Cosmological parameters

In this section we construct the cosmological parameters corresponding to the model (33) and discuss their physical significance.

Spatial volume of the model is

$$V = \exp\left(\frac{M^2 t^2}{2} - \varphi_0 t - \varphi_1 \right) \tag{34}$$

The average Hubble parameter is

$$H = \frac{1}{4} \left(M^2 t - \varphi_0 \right) \tag{35}$$

The scalar expansion $\theta$is

$$\theta = \left(M^2 t - \varphi_0 \right) \tag{36}$$

The shear scalar is

$$\sigma^2 = \left(M^2 t - \varphi_0 \right)^2 \left[\frac{3k^2+1}{2(3k+1)} \right] - \left(M^2 t - \varphi_0 \right) \left[\frac{3k+2}{6(3k+1)} \right] + \frac{2}{9} \tag{37}$$

The average anisotropy parameter is

$$A_h = 3 \left(\frac{k-1}{3k+1} \right)^2 \tag{38}$$

Now using Eqs. (30) - (32) in Eq. (23) we obtain DE density as

$$\rho_{\wedge} = \left(M^2 t - \varphi_0\right)^2 \left[\frac{k(k+1)}{(3k+1)^2}\right] - \exp\left(2\varphi_0 t - M^2 t^2 + 2\varphi_1\right)\left[\frac{\left(M^2 t - \varphi_0\right)^2 + M^2}{2}\right] \quad (39)$$

Now using Eqs. (30) - (32) in Eq. (24) we get the EoS parameter of DE as

$$\omega_{\wedge} = -\frac{1}{\rho_{\wedge}}\left[\frac{\left(M^2 t - \varphi_0\right)^2 (3k^2 + 2k + 4)}{(3k+1)^2} + \left(\frac{2k+1}{3k+1}\right)M^2 + \exp\left(2\varphi_0 t - M^2 t^2 + 2\varphi_1\right)\left\{\frac{\left(M^2 t - \varphi_0\right)^2 + M^2}{2}\right\}\right] \quad (40)$$

where $\rho_{\wedge}$ is given by Eq. (39).

From Eq. (24), (25) and (30) - (32) we obtain the skewness parameter ξ as

$$\xi = -\frac{1}{\rho_{\wedge}}\left[\left(\frac{k-1}{3k+1}\right)\left\{\left(M^2 t - \varphi_0\right)^2 + M^2\right\}\right] \quad (41)$$

where $\rho_{\wedge}$ is given by Eq. (39).
The deceleration parameter is

$$q = -\left[1 + \frac{4M^2}{\left(M^2 t - \varphi_0\right)^2}\right] \quad (42)$$

The jerk parameter is given by[42]

$$j(t) = -\frac{\dddot{a}}{aH^2} = 1 + 2q + \frac{\dot{q}}{H} = 1 + \frac{12M^2}{(M^2 t - \varphi_0)^2} + \frac{32M^2}{(M^2 t - \varphi_0)^4} + \frac{16M^2}{(M^2 t - \varphi_0)^5} \quad (43)$$

The model given by Eq. (33) describes five dimensional DE model with massive scalar field with the above cosmological parameters and the massive scalar field given by Eq. (30). It can be observed that the spatial volume of the universe increases exponentially from a finite value so that we have exponential expansion of the universe. The parameters H, θ, σ^2 are finite at $t = 0$ and tend to infinity as $t \to \infty$. Also DE density $\rho_{\wedge}$ is always positive and increases with time. It can be seen that the EoS parameter is function of cosmic time $\omega_{\wedge} > -1$ which implies that the model lies in the quintessence region. The skewness parameter also increases with time. The average anisotropy parameter is constant and is anisotropic throughout the evolution except when $k=1$. It is observed that the deceleration parameter $q=-1$ at late times showing that our universe accelerates. In cosmology jerk parameter plays an important role in describing the models close to ΛCDM and it is given by Eq.(43) It is believed that a transition of the universe from deceleration to acceleration occurs for models with negative values of q and positive values of j. At late times our model has q=-I and j=1. Thus our model represents accelerated expansion of the universe which is in accordance with the modern cosmological observations.

5. Conclusions

Five-dimensional anisotropic DE models are significant in the discussion of early stages of evolution of the universe. Scalar fields are useful in describing DE cosmological model. Hence, in this article, we have discussed an anisotropic DE model in the frame work of five dimensional spherically symmetric space-time. It is observed that the model non-singular and there is an exponential expansion of volume from a finite value. The model represents a quintessence universe. It is also observed that our model represents an accelerating expansion of the universe. Thus all the results obtained are in good agreement with the recent exponential data.

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