Article

Cellular Automata Representation of Submicroscopic Physics

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Abstract

Krasnoholovets theorized that the microworld is constituted as a *tessellation* of primary topological balls. The tessellattice becomes the origin of a submicrospic mechanics in which a quantum system is subdivided to two subsystems: the particle and its inerton cloud, which appears due to the interaction of the moving particle with oncoming cells of the tessellattice. The particle and its inerton cloud periodically change the momentum and hence move like a wave. The new approach allows us to correlate the Klein-Gordon equation with the deformation coat that is formed in the tessellatice around the particle. The submicroscopic approach shows that the source of any type of wave movements including the Klein-Gordon, Schrödinger, and classical wave equations is hidden in the tessellattice and its basic exciations – inertons, carriers of mass and inert properties of matter.

Keywords: Schrödinger, Klein-Gordon, classical wave equation, periodic table, molecule, cellular automata, submicroscopic.

1. Introduction

Elze [1] wrote about possible re-interpretation of quantum mechnics (QM) starting from classical automata principles. This is surely a fresh approach to QM, initiated by some authors including Gerard ‘t Hooft [3]. In the mean time, in a series of papers Shpenkov [2, 3-14] suggested that the spherical solution of Schrödinger’s equation says nothing about the structure of molecules. According to Shpenkov [2, 3-14], the classical wave equation is able to derive a periodic table of elements which is close to Mendeleyev’s periodic table and also other phenomena related to the structure of molecules.

However, the Schrödinger equation is a quantum equation that describes the motion of the appropriate particle-wave since all quantum objects manifest charecteristics of both particles and waves. Considering Shpenkov’s results, one can ask: *why do the particle’s characteristics dissapear and what exactly is the subject of purely wave behaviour in a quantum system?*

Recently, Krasnoholovets has developed a submicroscopic concept in which the motion of a canonical particle occurs in physical space constructed as a cellular structure named the tessellattice(see, e.g. Ref. [15]).

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In this paper, we carry out studies of the Schrödinger equation and classical wave equation and show how they both are related to the idea of tessellatice. The Appendix contains a more detailed proof on how "space" has the form of acoustic/sound wave.

2. Correspondence betwwen Classical Wave & Quantum Mechanics

A connection between classical and quantum mechanics has been studied at least by several researchers (see e.g. Refs. [26-28]). Ward and Volkmer [29] discussed a relation between the classical electromagnetic wave equation and Schrödinger equation. They derived the Schrödinger equation based on the electromagnetic wave equation and Einstein's special theory of relativity. They began with electromagnetic wave equation in one-dimensional case:

$$\frac{\partial^2 E}{\partial x^2}-\frac{1}{c^2}\frac{\partial^2 E}{\partial t^2}=0. \tag{13}$$

This equation is satisfied by plane wave solution:

$$E(x,\ t)=E_0 e^{i(kx-\omega t)}, \tag{14}$$

Where $k=2\pi/\lambda$ and $\omega=2\pi\nu$ are the spatial and temporal frequencies, respectively. Substituting equation (14) into (13), then we obtain

$$\left(\frac{\partial^2}{\partial x^2}-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)E_0 e^{i(kx-\omega t)}=0, \tag{15}$$

or

$$\left(k^2-\frac{\omega^2}{c^2}\right)E_0 e^{i(kx-\omega t)}=0, \tag{16}$$

which arrives us to a dispersion relationship for light in free space: $k=\omega/c$. This is similar to the wave number k in eq. (8).

Then, recalling from Einstein and Compton that the energy of a photon is $\varepsilon=h\nu=\hbar\omega$ and the momentum of a photon is $p=h/\lambda=\hbar k$, which allows us to rewrite eq. (14) using these relations:

$$E(x,\ t)=E_0\, e^{\frac{i}{\hbar}(px-\varepsilon t)}. \tag{17}$$

Substituting expression (17) into eq. (13) we find

$$-\frac{1}{\hbar^2}\left(p^2-\frac{\varepsilon^2}{c^2}\right)E_0 e^{\frac{i}{\hbar}(px-\varepsilon t)}=0, \tag{18}$$

which results in the relativistic total energy of a particle with zero rest mass

$$\varepsilon^2 = p^2 c^2 . \tag{19}$$

Following de Broglie, we may write the total relativistic energy for a particle with non-zero rest mass

$$\varepsilon^2 = p^2 c^2 + m_0^2 c^4 . \tag{20}$$

Inserting expression (20) into eq. (18), it is straightforward from (15) that we get

$$\left(\nabla^2 - \frac{m_0^2 c^2}{\hbar^2} \right) \Psi = \frac{1}{c^2} \frac{\partial^2 \Psi}{\partial t^2} , \tag{21}$$

which is the Klein-Gordon equation [30, 31] for a free particle [29]. Now we want to obtain Schrödinger equation, which is non-relativistic case of eq. (21). The first step is to approximate $\varepsilon^2 = p^2 c^2 + m_0^2 c^4$ as follows

$$\varepsilon = m_0 c^2 \sqrt{1 + \frac{p^2}{m_0^2 c^2}} \approx m_0 c^2 + \frac{p^2}{2 m_0} \approx m_0 c^2 + \Im . \tag{22}$$

After some approximation steps, Ward and Volkmer [29] arrived at the Schrödinger equation

$$-\frac{\hbar^2}{2m} \nabla^2 \phi = i\hbar \frac{\partial \phi}{\partial t} , \tag{23}$$

where the non-relativistic wave function ϕ is also constrained to the condition that it be normalisable to unit probability.

In the meantime, Hilbert and Batelaan [32] explored equivalence between the quantum and acoustic system. A simple physical system was discussed, which mirrorred the quantum mechanical infinite square well with a central delta well potential. They find that the analytic solution to the quantum system exhibits level splitting, as does the acoustic system. They compare the acoustic resonances in a closed tube and the quantum mechanical eigen-frequencies of an infinite square well and showed that the acoustic displacement standing wave is

$$\xi(x) = \xi_{\max} \sin\left(n\pi x / (2a) \right) \tag{24}$$

for the n-th resonance. Eq. (24) has the same shape as the quantum mechanical wave function.
So we can conclude that there exists formal connection between the classical wave equation and Schrödinger equation, but it still requires some assumptions and approximations. Shpenkov's

interpretation of classical wave equation looks as more realistic for atomic and molecular modeling.

3. Cellular Automata Model of Classical Wave Equation

In the previous section, we have argued that Shpenkov's interpretation of classical wave equation looks as more realistic for atomic and molecular modeling. Now we shall outline a cellular automata model of classical wave equation.

But first of all, let us give a few remarks on cellular automata. The term cellular automata cellular automata is plural. Our code examples will simulate just one—a cellular automaton cellular automaton, singular. To simplify our lives, we'll also refer to cellular automata as "CA." Cellular automata make a great first step in building a system of many objects that have varying states over time:

A cellular automaton is a model of a system of "cell" objects with the following characteristics.

- The cells live on a grid grid. (We'll see examples in both one and two dimensions in this chapter, though a cellular automaton can exist in any finite number of dimensions.)
- Each cell has a state. The number of state possibilities is typically finite. The simplest example has the two possibilities of 1 and 0 (otherwise referred to as "on" and "off" or "alive" and "dead").
- Each cell has a neighborhood. This can be defined in any number of ways, but it is typically a list of adjacent cells.[36]

Now consider a set of simple rules that would allow that pattern to create copies of itself on that grid. This is essentially the process of a CA that exhibits behaviour similar to biological reproduction and evolution. (Incidentally, von Neumann's cells had twenty-nine possible states.) Von Neumann's work in self-replication and CA is conceptually similar to what is probably the most famous cellular automaton: the "Game of Life." Perhaps the most significant scientific (and lengthy) work studying cellular automata arrived in 2002: Stephen Wolfram's *A New Kind of Science* (http://www.wolframscience.com/nks/) [36].

A plausible method to describe cellular automata model of wave equation was described for instance by Yang and Young [33]. For the 1D linear wave equation, where c is the wave speed they presented a scheme:

$$\frac{u_i^{n+1} - 2u_i^n + u_i^{n-1}}{(\Delta t)^2} = c^2 \frac{u_{i+1}^n - 2u_i^n + u_{i-1}^n}{(\Delta x)^2}. \quad (25)$$

After some steps eq. (25) can be rewritten in a generic form (by choosing $\Delta t = \Delta x = 1,\ t = n$) as follows $u_i^{t+1} + u_i^{t-1} = g(u^t)$, which is reversible under certain conditions. This property comes from the reversibility of the wave equation because it is invariant under the transformation: $t \rightarrow -t$.

O'Reilly has shown that the coupled Maxwell-Dirac electrodynamic system can be implemented in an analog cellular-automaton operating within a 3D regular face-centered cubic lattice [34]. The result of this approach can be expressed in terms of a second order wave equation, namely: $s_i^{t+1} = s_i^t + \dot{s}_i^{t+1}$. He concludes that the second order wave equation is arguably one of the simplest possible continuous-valued cellular automata update equations that do anything physically interesting, though all of electrodynamics can be built of elaborations of this one fundamental interaction.

Thus, cellular approach allows one to construct equations that describe physical systems without using second order equations.

Correspondence with Konrad Zuse's work: from static space to calculating space

To trace the development of physical thoughts in this field, we would like to mention two books. In the late 1970s, Konrad Zuse conceived an essay entitled *Calculating Space*, in which he advocated that physical laws are discrete by nature and that the entire history of our universe is just the output of a giant deterministic CA.[37]

It shall be clear, that we should let go our assumption of static space (Newtonian), nor dynamical space (Einsteinian), toward calculating space (Zusian). In this new model, space itself has a kind of computing capability, hence intelligence, albeit perhaps not the same kind of human intelligence. If such a new proposition can be proved true, then it may open up an array of explanations on many puzzling cosmology questions, such as: why galaxies apparently grow and then move to other directions (for instance, it is known that our Milky Way is moving toward *The Great Attractor*). Such an observed dynamics is very difficult to comprehend in terms of classical picture based on static space (differential equations).

In closing, we would like to quote Zuse's perceptive predictions made forty years ago: "*Incorporation of the concepts of information and the automaton theory in physical observations will become even more critical, as even more use is made of whole numbers, discrete states and the like.*"[37]

Nonetheless, we shall also keep in mind Zuse's question: "Is nature digital, analog or hybrid?" It is clear that classical physics is built in analogue way, but it does not mean that Nature is perfectly working in accordance with that model. That question needs to be investigated in more precise manner. [37, p. 22]

One way to investigate such a discrete model of space is by assuming a tessellattice model of space, as will be discussed in the next section.

4. The Tessellattice as the Source for the Formalism of Conventional Quantum Mechanics

A detailed theory of real physical space was developed by Bounias and Krasnoholovets starting from pure mathematical principles (see e.g. Ref. [35]). A submicroscopic theory of physical processes occurring in real physical space was elaborated by Krasnoholovets in a series of works (see e.g., monograph [15]). Those studies show that our ordinary space is constructed as a mathematical lattice of primary topological balls, which was named a tessellattice.

In the tesselllattice, primary topological balls play the role of cells. This is a physical vacuum, or aether. Matter emerges at local deformations of the tessellattice when a cell (or some cells) changes its volume following a fractal law of transformations. Such a deformation in the tessellattice can be associated with the physical notion of mass.

The motion of a fractal-deformed cell, i.e. a mass particle, is occurred with the fractal decomposition of its mass owing to its interaction with ongoing cells of the tessellattice. This is a further development of Zuse's idea about calculating space because cells can exchange by fractals, which locally change properties of space.

The interaction of matter with space generates a cloud of a new kind of spatial excitations named 'inertons'. This means that "hidden variables" introduced in the past by Louis de Broglie, David Bohm and Jean-Pierre Vigier have acquired a sense of real quasiparticles of space.

Thus in monograph [15] it has been shown that inertons are carriers of a new physical field (the inerton field), which appears as a basic field of the universe. Inertons as quasi-particles of the inerton field are responsible for quantum mechanical, nuclear and gravitational interactions of matter. Inertons carry mass and also fractal properties of space, i.e. they are real carries of information.

A particle moving in the tessellattice is surrounded with its inerton cloud. The particle actualizes the real motion between ongoing cells, though its inertons emitted when the particle rubs again the tessellattice's cells, migrate as excitations hopping from cell to cell. Such sophisticated motion in which the particle is surrounded with its inerton cloud can easily be compared with the formalism of quantum mechanics because the particle wrapped with its inertons can be projected to the particle's wave ψ-function determined in an abstract phase space. In such a pattern, the overlapping of wave ψ-functions of nearest particles means that the particles' inerton clouds overlap and thus we obtain real carriers of the quantum mechanical interaction, which provide a short-range action between the particles studied.

The particle's de Broglie wavelength λ plays the role of a section in which the moving particle emits its inerton cloud (an odd section) and in the next even section λ these inertons come back to the particle passing the momentum on to it. Inertons emitted by the freely moving particle come back to the particle owing to the elasticity of the tessellattice as such.

How can we write the interaction of a moving particle with its inerton cloud? The interaction can be written between the particle and an ensemble of inertons, which accompany the particle. The ensemble is presented as one integral object, an inerton cloud. The speed υ_0 of the particle the particle satisfies the inequality $\upsilon_0 << c$. At such presentation, our study is significantly simplified and is reduced to the consideration of a system of two objects: the particle and its cloud of inertons, which the particle periodically emits and adsorbs when moving along its path. In this case the Lagrangian (2.1) is transformed to the following one written in two-dimensional Euclidean space

$$L = \tfrac{1}{2} m_0 \dot{x}^2 + \tfrac{1}{2}\mu_0 \cdot \left[(\dot{\chi}^{\parallel})^2 + (\dot{\chi}^{\perp})^2 \right] - \frac{2\pi}{T}\sqrt{m_0 \mu_0}\, x \dot{\chi}^{\perp}. \tag{26}$$

In the Lagrangian (26)(2.49) the first term describes the kinetic energy of the particle with the mass *m* and the velocity $\dot{x}$, which moves along the axis *X*; the second term depicts the kinetic energy of the whole inerton cloud whose mass is μ_0 and its center-of-mass has the coordinate $\chi^{\parallel}$ along the particle's path and $\chi^{\perp}$ is the transverse coordinate; the third term is the interaction energy between the particle and the inerton cloud where $1/T$ is the frequency of their collisions.

By using the substitution

$$\dot{x}^{\perp} = \dot{\tilde{\chi}} + 2\pi\sqrt{m_0 / \mu_0}\; x / T, \tag{27}$$

we carry out a kind of a canonical transformation that leads to the following Lagrangian

$$\tilde{L} = \tfrac{1}{2} m_0 \dot{x}^2 - \tfrac{1}{2}(2\pi / T)^2 m_0 x^2 + \tfrac{1}{2}\mu_0 \cdot \left(\dot{\tilde{\chi}}^2 + (\dot{\chi}^{\parallel})^2 \right). \tag{28}$$

We can see from the effective Lagrangian (28)(2.51) that in such a presentation the particle's behavior is described as a classical harmonic oscillator and the accompanying inerton cloud moves by its own hidden principle (though it does not disturb the particle).

The Hamiltonian function according to the definition

$$H = \sum_i \dot{Q}_i \partial L / \partial \dot{Q}_i - L.$$

In our case the Hamiltonian is

$$H = \dot{x}\,\partial L / \partial \dot{x} + \dot{\tilde{\chi}}\,\partial L / \partial \dot{\tilde{\chi}} - \tilde{L}. \tag{29}$$

The effective Hamiltonian based on the Lagrangian (28)(2.51) of the oscillating particle in the system of the center-of-mass of the particle and its inerton cloud in the explicit form becomes

$$H = p^2 / (2m_0) + m_0 (2\pi / T)^2 x^2 / 2. \tag{30}$$

Solutions of the equations of motion given by the Hamiltonian (30) are well known for different presentations. In particular, the function (30) allows one to derive the Hamilton-Jacobi equation

$$(\partial S_1 / \partial x)^2 / (2m_0) + m_0 (2\pi / T)^2 x^2 / 2 = E \tag{31}$$

from which we obtain the equation for a shortened action

$$S_1 = \int_{x_0}^{x} p\,dx = \int_{x_0}^{x} \sqrt{2m_0 [E - (2\pi / T)^2 x^2 / 2]}\, dx. \tag{32}$$

The function (32)(2.55) enables the solution *x* as a function of *t* in the form

$$x = \frac{\sqrt{2E / m_0}}{2\pi / T} \sin(2\pi t / T). \tag{33}$$

Now we can calculate the increment ΔS_1 of the action (32)(2.55) of the particle during the period *T*; in terms of the action-angle variables

$$\begin{aligned} \Delta S_1 &= \oint p\,dx = \oint \sqrt{2m_0 \left(E - m_0 (2\pi / T)^2 x^2 \right)}\, dx \\ &= \oint \sqrt{2m_0 \left(E - E \sin^2 (2\pi t / T) \right)} \sqrt{2E / m_0} \cos(2\pi t / T)\, dt \\ &= 2E \int_0^T \cos^2 (2\pi t / T)\, dt = 2E \left(\frac{t}{2} + \frac{\sin(4\pi t / T)}{4(2\pi / T)} \right) \Bigg|_{t=0}^{t=T} = ET. \end{aligned} \tag{34}$$

The final result (34) can be rewritten as follows

$$\Delta S_1 = E \cdot T = E / \nu \tag{35}$$

where the notation $\nu = 1/T$ is entered.

Since the constant E is the initial energy of the particle, i.e., $E = \frac{1}{2} m_0 \upsilon_0^2$, the increment of action (35) can also be presented in the form

$$\Delta S_1 = \tfrac{1}{2} m_0 \upsilon_0^2 \cdot T = m_0 \upsilon_0 \cdot \tfrac{1}{2} \upsilon_0 T = m_0 \upsilon_0 \cdot \lambda \tag{36}$$

where the parameter λ is the spatial amplitude of oscillations of the particle along its path.

If we equate the increment of the action ΔS_1 to the Planck constant h, we immediately arrive at the two major relationships of quantum mechanics introduced by de Broglie for a particle:

$$E = h\nu, \qquad \lambda = h / (m_0\, \upsilon_0). \tag{37}$$

Thus the amplitude of special oscillation of a particle is exactly the particle's de Broglie wavelength.

Having obtained the relationships (37), we can present the complete action for a particle

$$S = S_1 - Et = \int^x p\,dx - Et \tag{38}$$

in two equivalent forms:

$$S = m_0 \upsilon_0 x - Et \tag{39}$$

and

$$S = h \cdot (x/\lambda - \nu t). \tag{40}$$

The relationships (39), (40) and (37) allow the derivation of the Schrödinger equation. If in a conventional wave equation

$$\Delta\psi - \frac{1}{(\upsilon_0/2)^2}\frac{\partial^2\psi}{\partial t^2} = 0 \tag{41}$$

(where $\frac{1}{2}\upsilon_0$ is the average velocity of the particle in the spatial period λ) we insert a wave function, whose phase is based on the action (40),

$$\psi = a\exp\left\{i2\pi[x/\lambda - \nu t]\right\}, \tag{42}$$

and set $\upsilon_0 = \lambda \cdot 2\nu$, we get the wave equation in the following presentation:

$$\Delta\psi + \left(2\pi/\lambda\right)^2 \psi = 0. \tag{43}$$

Then putting $\lambda = h/p$ and extracting the momentum p from the function (32) (i.e., $p^2 = 2mE$) we finally obtain a conventional time-independent Schrödinger equation

$$\Delta\psi + \frac{2m_0 E}{\hbar^2}\psi = 0. \tag{44}$$

Thus, we can see that the moving system of a particle and its inerton cloud obeys the Schrödinger equation.

5. The Deformation Coat of Particle & the Klein-Gordon Equation

As we discussed above, Ward and Volkmer [29] demonstrated the derivation of the Klein-Gordon equation (21) for a mass particle starting from its total relativistic energy $\varepsilon^2 = p^2c^2 + m_0^2c^4$ (20). They also showed that a non-relativistic approximation of the same energy (20) results in the Schrödinger time-dependent equation (23).

Usually the Klein-Gordon equation [30, 31] is applied for the description of an abstract relativistic particle that does not possess spin. However, the submicroscopic concept of physics presented in monograph [15] makes *it possible to relate the Klein-Gordon equation to a real object,* namely, a deformation coat that is developed around the mass particle created in the tessellatice.

In fact the creation a particle means the appearance of a local deformation, i.e. a volumetric fractal deformation of the appropriate cell of the tesselllatice. The local deformation must induce a tension state in ambient cells, which may extend only to a definite radius R. So behind the radius R, the tessellattice does not have any distortion, it is found here in a degenerate state.

The study [15] shows that in the microworld such fundamental physical parameters as mass and charge vary at the motion. Namely, in a section (the even section) equal to the particle's de Broglie wavelength λ the mass m is transferred to a tension ξ and the charge e changes to the magnetic monopole g. In the odd section λ the mass and charge are restored. The same happened with cells that form the particle's deformation coat. When the particle is moving, it pulls its deformation coat as well, i.e. ambient cells adjust to state of the particle. In the deformation coat the state of cells oscillates between the tension ξ and mass m. A collective oscillating mode of the deformation coat is specified by the energy [15] $E = \hbar\omega$, which in turn equals the total energy of the particle mc^2.

The discussed oscillations can be described by a plane wave mode $E(x,t) = E_0 e^{i(kx-\varepsilon t)}$ (17). Then following the arguments (17) – (21), we immediately derive the Klein-Gordon equation (21). Note that in our case the particle that obeys the Klein-Gordon equation is the deformation coat that accompanies the moving particle. This deformation coat is specified with the radius equal to the particle's Compton wavelength [15] (see p. 57).

If the speed υ of a particle satisfies the inequality $\upsilon << c$, we following reasoning (22) and (23) will arrive at the Schrödinger equation (23).

6. Conclusion

We have reviewed a plausible cellular automaton molecular model for classical wave equation, as an alternative to Cellular automaton quantum mechanics (by Elze, Gerard 't Hooft etc).

Then we have considered the submicroscopic concept that allows one to easily derive the Schrödinger and Klein-Gordon equations starting from first submicroscopic principles. It is interesting that for the first time we now can identify the Klein-Gordon equation with a real object that is described by this equation – it is the particle's deformation coat that is induced in the tessellattice at around the appropriate created canonical particle.

The submicroscopic concept, which is based on space constituted as the tessellattice of primary topological balls, introduces a new physical field, namely the inerton field, which appears as a fundamental field of the universe. Inertons emerge at any motion of particles; in particular, they arise in atoms and around owing to uninterrupted motion of electrons, nuclei and nucleons.

Thus the motion of a quantum system is characterized by its separation to two joined subsystems: the particle itself and its inerton cloud. Their oscillation dynamics exhibits obvious features of the wave motion. Although the deformation coat that accompanies the moving particle behaves in a special way, it is described by the Klein-Gordon equation, which also manifests the wave properties.

Our analysis shows that oscillations of inertons are present in any movement of a material object. Inertons clearly demonstrate wave behavior. This means that inerton oscillations appear in atoms and molecules. Hence inerton oscillations justify Shpenkov's model [4–14], which applies a classical wave equation of sound to atoms and molecules: the wave function Ψ used by Shpenkov describes oscillations of an inerton field and the location of the corresponding nodes in the oscillating wave studied.

Thus, quantum mechanical models, cellular automata, and a cellular automaton molecular model that uses a wave equation can be covered by studies originated from the tessellattice and the submicroscopic behavior of quantum systems, which involves an inerton field that binds canonical particles with the tessellattice and between themselves.

Nonetheless, there remains many questions to ponder, for example: whether the notion of cellular automata corresponds neatly to Zuse's calculating space hypothesis [37], and whether the latter in turn leads to cellular intelligence (see for instance [37a]). Therefore, further investigations in this direction are recommended, which will shed light on the cornerstones of the microworld.

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