Exploration

SUSY in TGD Universe

Matti Pitkänen [1]

Abstract

The progress in understanding of $M^8 - H$ duality throws also light to the problem whether SUSY is realized in TGD and what SUSY breaking does mean. It is now rather clear that sparticles are predicted and SUSY remains exact but that p-adic thermodynamics causes thermal massivation: unlike Higgs mechanism, this massivation mechanism is universal and has nothing to do with dynamics. This is due to the fact that zero energy states are superpositions of states with different masses. The selection of p-adic prime characterizing the sparticle causes the mass splitting between members of super-multiplets although the mass formula is same for all of them. The question how to realize super-field formalism at the level of $H = M^4 \times CP_2$ led to a dramatic progress in the identification of elementary particles and SUSY dynamics. The most surprising outcome was the possibility to interpret leptons and corresponding neutrinos as local 3-quark composites with quantum numbers of anti-proton and anti-neutron. Leptons belong to the same super-multiplet as quarks and are antiparticles of neutron and proton as far quantum numbers are consided. One implication is the understanding of matter-antimatter asymmetry. Also bosons can be interpreted as local composites of quark and anti-quark.

Hadrons and hadronic gluons would still correspond to the analog of monopole phase in QFTs. Homology charge would appear as space-time correlate for color at space-time level and explain color confinement. Also color octet variants of weak bosons, Higgs, and Higgs like particle and the predicted new pseudo-scalar are predicted. They could explain the successes of conserved vector current hypothesis (CVC) and partially conserved axial current hypothesis (PCAC). One ends up with the precise understanding of quantum criticality and understand the relation between its descriptions at M^8 level and H-level. Polynomials describing a hierarchy of dark matters describe also a hierarchy of criticalities and one can identify inclusion hierarchies as sub-hierarchies formed by functional composition of polynomials. The Wick contractions of quark-antiquark monomials appearing in the expansion of super-coordinate of H could define the analog of radiative corrections in discrete approach. $M^8 - H$ duality and number theoretic vision require that the number of non-vanishing Wick contractions is finite. The number of contractions is indeed bounded by the finite number of points in cognitive representation and increases with the degree of the octonionic polynomial and gives rise to a discrete coupling constant evolution parameterized by the extensions of rationals. Quark oscillator operators in cognitive representation correspond to quark field q. Only terms with quark number 1 appear in q and leptons emerge in Kähler action as local 3-quark composites. Internal consistency requires that q must be the super-spinor field satisfying super Dirac equation. This leads to a self-referential condition $q_s = q$ identifying q and its super-counterpart q_s. Also super-coordinate h_s must satisfy analogous condition $(h_s)_s = h_s$, where $h_s \to (h_s)_s$ means replacement of h in the argument of h_s with h_s. The conditions have an interpretation in terms of a fixed point of iteration and expression of quantum criticality. The coefficients of various terms in q_s and h_s are analogous to coupling constants can be fixed from this condition so that one obtains discrete number theoretical coupling constant evolution. The basic equations are quantum criticality condition $h_s = (h_s)_s$, $q = q_s$, $D_{\alpha,s}\Gamma_s^\alpha = 0$ coming from Kähler action, and the super-Dirac equation $D_s q = 0$.

One also ends up to the first completely concrete proposal for how to construct S-matrix directly from the solutions of super-Dirac equations and super-field equations for space-time super-surfaces. The idea inspired by WKB approximation is that the exponent of the super variant of Kähler function including also super-variant of Dirac action defines S-matrix elements as its matrix elements between the positive and negative energy parts of the zero energy states formed from the corresponding vacua at the two boundaries of CD annihilated by annihilation operators and *resp.* creation operators.

[1] Correspondence: Matti Pitkänen `http://tgdtheory.fi/`. Address: Rinnekatu 2-4 8A, 03620, Karkkila, Finland. Email: matpitka6@gmail.com.

The states would be created by the monomials appearing in the super-coordinates and super-spinor. Super-Dirac action vanishes on-mass-shell. The proposed construction relying on ZEO allows however to get scattering amplitudes between all possible states using the exponential of super-Kähler action. Super-Dirac equation is however needed and makes possible to express the derivatives of the quark oscillator operators (values of quark field at points of cognitive representation) so that one can use only the points of cognitive representation without introducing lattice discretization. Discrete coupling constant evolution conforms with the fact that the contractions of oscillator operators occur at the boundary of CD and their number is limited by the finite number of points of cognitive representation.

Keywords: SUSY, zero energy, hadron, causal diamond, p-adic, TGD framework.

1 Introduction

What SUSY is in TGD framework is a longstanding question, which found a rather convincing answer rather recently. In twistor Grassmannian approach to $\mathcal{N} = 4$ SYM [5, 2, 3, 4, 7, 6, 1] twistors are replaced with supertwistors and the extreme elegance of the description of various helicity states using twistor space wave functions suggests that super-twistors are realized both at the level of M^8 geometry and momentum space.

In TGD framework $M^8 - H$ duality allows to geometrize the notion of super-twistor in the sense that at the level of M^8 different components of super-field correspond to components of super-octonion each of which corresponds to a space-time surfaces satisfying minimal surface equations with string world sheets as singularities - this is geometric counterpart for masslessness.

1.1 New view about SUSY

The progress in understanding of $M^8 - H$ duality [32] throws also light to the problem whether SUSY is realized in TGD [31] and what SUSY breaking cold mean. It is now rather clear that sparticles are predicted and SUSY remains exact but that p-adic thermodynamics causes thermal massivation: unlike Higgs mechanism, this massivation mechanism is universal and has nothing to do with dynamics. This is due to the fact that zero energy states are superpositions of states with different masses. The selection of p-adic prime characterizing the sparticle causes the mass splitting between members of super-multiplets although the mass formula is same for all of them. Super-octonion components of polynomials have different orders so that also the extension of rational assignable to them is different and therefore also the ramified primes so that p-adic prime as one them can be different for the members of SUSY multiplet and mass splitting is obtained.

The question how to realize super-field formalism at the level of $H = M^4 \times CP_2$ led to a dramatic progress in the identification of elementary particles and SUSY dynamics. The most surprising outcome was the possibility to interpret leptons and corresponding neutrinos as local 3-quark composites with quantum numbers of anti-proton and anti-neutron. Leptons belong to the same super-multiplet as quarks and are antiparticles of neutron and proton as far quantum numbers are consided. One implication is the understanding of matter-antimatter asymmetry. Also bosons can be interpreted as local composites of quark and anti-quark.

Hadrons and perhaps also hadronic gluons would still correspond to the analog of monopole phase in QFTs. Homology charge could appear as a space-time correlate for color at space-time level and explain color confinement. Also color octet variants of weak bosons, Higgs, and Higgs like particle and the predicted new pseudo-scalar are predicted. They could explain the successes of conserved vector current hypothesis (CVC) and partially conserved axial current hypothesis (PCAC).

One ends up with an improved understanding of quantum criticality and the relation between its descriptions at M^8 level and H-level. Polynomials describing a hierarchy of dark matters describe also a hierarchy of criticalities and one can identify inclusion hierarchies as sub-hierarchies formed by functional composition of polynomials: the criticality is criticality for the polynomials interpreted as p-adic polynomials in $O(p) = 0$ approximation meaning the presence of multiple roots in this approximation.

1.2 Connection of SUSY and second quantization

The linear combinations monomials of theta parameters appearing in super-fields are replaced in case of hermitian H super coordinates consisting of combinations of monomials with vanishing quark number. For super-spinors of H the monomials carry same quantum numbers as quarks. Monomials of theta parameters are replaced by local monomials of quark oscillator operators labelled besides spin and weak isospin also by points of cognitive representation with imbedding space coordinates in an extension of rationals defining the adele. Discretization allows anti-commutators which are Kronecker deltas rather than delta functions. If continuum limit makes sense, normal ordering must be assumed to avoid delta functions at zero coming from the contractions.

The monomials (not only the coefficients appearing in them) are solved from generalized classical field equations and are linearly related to the monomials at boundary of CD playing the role of quantum fields and classical field equations determine the analogs of propagators.

The Wick contractions of quark-antiquark monomials appearing in the expansion of super-coordinate of H could define the analog of radiative corrections in discrete approach. $M^8 - H$ duality and number theoretic vision require that the number of non-vanishing Wick contractions is finite. The number of contractions is bounded by the finite number of points in cognitive representation and increases with the degree of the octonionic polynomial and gives rise to a discrete coupling constant evolution parameterized by the extensions of rationals. The polynomial composition hierarchies correspond to inclusion hierarchies for isomorphic sub-algebras of super-symplectic algebra having interpretation in terms of inclusions of hyper-finite factors of type II_1.

Quark oscillator operators in cognitive representation correspond to quark field q. Only terms with quark number 1 appear in q and leptons emerge in Kähler action as local 3-quark composites. Internal consistency requires that q must be the super-spinor field satisfying super Dirac equation. This leads to a self-referential condition $q_s = q$ identifying q and its super-counterpart q_s. Also super-coordinate h_s must satisfy analogous condition $(h_s)_s = h_s$, where $h_s \rightarrow (h_s)_s$ means replacement of h in the argument of h_s with h_s.

The conditions have an interpretation in terms of a fixed point of iteration and expression of quantum criticality. The coefficients of various terms in q_s and h_s are analogous to coupling constants can be fixed from this condition so that one obtains discrete number theoretical coupling constant evolution. The basic equations are quantum criticality condition $h_s = (h_s)_s$, $q = q_s$, $D_{\alpha,s}\Gamma^\alpha_s = 0$ coming from Kähler action, and the super-Dirac equation $D_s q = 0$.

1.3 Proposal for S-matrix

One also ends up to the first completely concrete proposal for how to construct S-matrix directly from the solutions of super-Dirac equations and super-field equations for space-time super-surfaces.

1. The idea inspired by WKB approximation is that the exponent of the super variant of Kähler function including also super-variant of Dirac action defines S-matrix elements as its matrix elements between the positive and negative energy parts of the zero energy states formed from the corresponding vacua at the two boundaries of CD annihilated by annihilation operators and *resp.* creation operators. The states would be created by the monomials appearing in the super-coordinates and super-spinor.

2. Super-Dirac equation implies that super-Dirac action vanishes on-mass-shell. The proposed construction however allows to get also scattering amplitudes between all possible states using the exponential of super-Kähler action. Super-Dirac equation however makes possible to express derivatives of the quark oscillator operators (values of quark field at points of cognitive representation) so that one can use only the points of cognitive representation without introducing lattice discretization.

Discrete coupling constant evolution follows from the fact that the contractions of oscillator operators occur at the boundary of CD and their number is limited by the finite number of points of cognitive representation.

3. S-matrix is trivial unless CD contains the images of 6-D analogs of branes as universal special solutions of the algebraic equations determining space-time surfaces at the level of M^8. 4-D space-time surfaces representing particle orbits meet at the partonic 2-surfaces associated with the 3-D surfaces at $t = r_n$ hyper-surfaces of M^4. The values of $t = r_n$ correspond to the roots of the real polynomial with rational coefficients determining the space-time surface. These transitions are analogs of weak measurements, and in TGD theory of consciousness they give rise to the experience flow of time and can be said to represent "very special moments" in the life of self [30].

4. The creation and annihilation operators at vertices associated with the monomials would be connected to the points assignable to cognitive representations at opposite boundaries of CD and also to partonic 2-surfaces in the interior of CD possibly accompanied by sub-CDs. This would give analogs of twistor Grassmannian diagrams containing finite number of partonic 2-surfaces as topological vertices containing in turn finite number ordinary vertices defined by the monomials. The diagrams would be completely classical objects in accordance with the fact that quantum TGD is completely classical theory apart from state function reduction.

5. This view allows also a formulation of continuum theory since the monomials appearing in the action density in the interior of CD are linear superposition of the monomials at the points of boundary of CD involving 3-D integral so that contractions of oscillator operators only reduce one integration without introducing divergence. One can also normal order the monomials at boundary of CD serving as initial values. If preferred extremals are analogs of Bohr orbits, one can express extremals using either boundary as the seat of initial data.

2 How to formulate SUSY at the level of $H = M^4 \times CP_2$?

In the following I will represent the recent trial for constructing SUSY at the level of $H = M^4 \times CP_2$. The first trial replaced theta parameters of SUSY with quark oscillator operators labelled by spin and isospin and had rather obvious shortcomings: in particular, one did not obtain many-quark states with large quark numbers. The second trial allows quark oscillator operators to have as labels also the points of space-time surface in cognitive representation and thus having coordinates of H belonging to an extension of rationals defining the adele [25].

2.1 First trial

If SUSY is realized at the level of M^8, it should have a formulation also at the level of H. The basic elements of the first trial form part of also second trial. The basic modification made in the second trial is that finite number of theta parameters replaced with the fermionic oscillator operators labelled by the points of cognitive representations so that they are analogous to fermion fields in lattice, and only local composites of the oscillator operators appear in the super coordinates and super-spinors. This means that SUSY is essential element of the second quantization of fermions in TGD.

1. $M^8 - H$ duality is non-local and means that the dynamics at the level of H is not strictly local but dictated by partial differential equations for super-fields having interpretation as describing purely local many-fermion states made of fundamental fermions with quantum numbers of leptons and quarks (quarks do not possess color as spin like quantum number) ad their antiparticles.

2. Classical field equations and modified Dirac equation must result from this picture. Induction procedure for the spinors of H must generalize so that spinors are replaced by super-spinors Ψ_s

having multi-spinors as components multiplying monomials of theta parameters θ. The determinant of metric and modified gamma matrices depend on imbedding space coordinates h replaced with super coordinates h_s so that monomials of θ appear in two different manners. Hermiticity requires that sums of monomial and its hermitian conjugate appear in h_s. Monomials must also have vanishing fermion numbers. Otherwise one can obtain fermionic states propagating like bosons. For Dirac action one must assume that Ψ_s involves only odd monomials of θ with quark number 1 involving monomials appearing in h_s to get only states with fermion number 1 and correct kind of propagators.

3. One Taylor expands both bosonic action density (6-D Kähler action dimensionally reducing to 4-D Kähler action plus volume term) and Super-Dirac action with respect to the super-coordinates h_s. In Super-Dirac action one has also the expansion of super-spinor in odd monomials with quark number one. The coefficients of the monomials of θ:s are obtained are partial derivatives of the action. Since the number of θ parameters is finite and corresponds to the number of spin-weak-isopin states of quarks and leptons, the number of terms is finite if the θ parameters anti-commute to zero. If not, one can get an infinite number of terms from the Taylor series for the action to the coefficient given monomial. Number theoretical considerations do not favor this and there should exist a cancellation mechanism for the radiative corrections coming from fermionic Wick contractions if thetas correspond to fermionic oscillator operators as it seems to be.

4. One can interpret the superspace as the exterior algebra of the spinors of H. This reminds of the result that the sections of the exterior algebra of Riemann manifold codes for the Riemann geometry (see `http://tinyurl.com/yxrcr8xv`). This generalizes the observation that one can hear the shape of a drum since the sound spectrum is determined by its frequency spectrum defined by Laplacian.

   Super-fields define a Clifford algebra generated by θ parameters as a kind of square root of exterior algebra which corresponds to the Clifford algebra of gamma matrices. Maybe this algebra could code also for the spinor structure of imbedding space or even that of space-time surface so that the super-fields could be seen as carriers of geometric information about space-time surface as a preferred extremal. In 8-D case there is also $SO(1,8)$ triality suggesting that corresponding three Clifford algebras correspond to exterior algebra fermionic and anti-fermionic algebras.

What about the situation at the level of M^8?

1. At M^8 level the components of super-octonion correspond to various derivatives of the basic polynomial $P(t)$ so that space-time geometry correlates with the quantum numbers assignable to super-octonion components - this is in accordance with QCC (quantum-classical correspondence). This is highly desirable at the level of H too.

   Could the space-time surface in M^8 be same for super-field components with degree $d < d_{max}$ in some special cases? The polynomial associated with super octonion components are determined by the derivatives of the basic polynomial $P(t)$ with order determined by the degree of the super-monomial. If they have decomposition $P(t) = P_1^k(t)$, the monomials with degree $d < k$ the roots corresponding to the roots $P_1(t)$ co-incide. Besides this there are additional roots of $d^r P_1/dt^r$ for super-octonion component with r θ parameters.

   A possible interpretation could be as quantum criticality in which there is no SUSY breaking for components having $d < k$ (masses in p-adic thermodynamics could be the same since the extension defined by P_1 and corresponding ramified primes would be same). This would conform with the general vision about quantum criticality.

2. Usual super-field formalism involves Grassmann integration over θ parameters to give the action. M^8 formalism does not involve the θ integral at all. Should this be the case also at the level of H? This would guarantee that different components of H- coordinates as super-field would give

rise to different space-time surface and QCC would be realized. θ integration produces SUSY invariants naturally involved with the definition of vertices involving components of super-fields. Also vertices involving fermionic and bosonic states emerge since bosonic super-field components appear in super-coordinates in super-Dirac action.

This approach does not say anything about second quantization. There is a strong temptation to replace the theta parameters with fermionic oscillator operators. One cannot however obtain second quantization of fermions in this manner since the maximal quark number (and lepton number if leptons are present as fundamental fermions) of the states is 4. To achieve second quantization, one must replace the theta parameters with fermionic oscillator operators labelled besides spin and weak isospin by the coordinates of points of 3-surface, most naturally the points belonging to a cognitive representation characterizing space-time surface for given extension of rationals.

2.2 Second trial

I have already earlier considered a proposal for how SUSY could be realized in TGD framework. As it often happens, the original proposal was not quite correct. The following discussion gives a formulation solving the problems of the first proposal and suggests a concrete formulas for the scattering amplitudes in ZEO based on super-counterparts of preferred extremals. In the sequel I will talk about super Kähler function as functional of 3-surfaces and - super Kähler function action. By holography allowing to identify 3-surfaces with corresponding space-time surfaces as analogs of Bohr orbits, these notions have the same meaning.

2.2.1 Could the exponent of super-Kähler function as vacuum functional define S-matrix as its matrix elements

Consider first the key ideas - some of them new - formulated as questions.

1. Could one see SUSY in TGD sense as a counterpart for the quantization in the sense of QFT so that oscillator operators replace theta parameters and would become fermionic oscillator operators labelled by spin and electroweak spin - as proposed originally - and by selected points of 3-surface of light-cone boundary with imbedding space coordinates in extension of rationals? One would have analog of fermion field in lattice identified as a number theoretic cognitive representation for given extension of rationals. The new thing would be allowance of local composites of oscillator operators having interpretation in terms of analogs for the components of super-field.

   SUSY in TGD sense would be realized by allowing local composites of oscillator operators containing 4+4 quark oscillator operators at most. At continuum limit normal ordering would produce delta functions at origin unless one assumes normal ordering from beginning. For cognitive representations one would have only Kronecker deltas and one can consider the possibility that normal ordering is not present. The vanishing of normal ordering terms above some number of them suggested to be the dimension for the extension of rationals would give rise to a discrete coupling constant evolution due to the contractions of fermionic oscillator operators.

2. What is dynamical in the superpositions of oscillator operator monomials? Are the coefficients dynamical? Or are the oscillator operators themselves dynamical - this would mean a QFT type reduction to single particle level? The latter option seems to be correct. Oscillator operators are labelled by points of cognitive representation and in continuum case define an analog of quantum spinor field, call it q. This suggests that this field satisfies the super counter part of modified Dirac equation and must involve also super part formed from the monomials of q and $\overline{q}$. This however requires the replacement of q with q_s in super-Dirac operator and super-coordinates h_s and one ends up with an iteration $q \rightarrow q_s \rightarrow ...$

The only solution to the paradoxical situation is that one has self-referential equation $q = q_s$ having interpretation in terms of quantum criticality fixing the coefficients of terms in $q = q_s$. Analogous condition $h_s = (h_s)_s$ must be satisfied by h_s under substitution $h_s \rightarrow (h_s)_s$. These conditions fix coefficients of terms in H super-coordinate h_s and q_s interpreted as coupling constants so that quantum criticality implying a discrete coupling constant evolution as function of extension of rationals follows. Also super-Dirac equation $D_s q_s = 0$ and field equations $D_{s,\alpha}\Gamma^{\alpha,s} = 0$ for Kähler action guaranteeing hermiticity are satisfied.

3. Could one interpret the time reversal operation taking creation- and annihilation operators to each other as time reflection permuting the points at the opposite boundaries of CD? The positive *resp.* negative energy parts of zero energy states would be created by creation *resp.* annihilation operators from respective vacuums assigned to the opposite boundaries of CD.

4. Could one regard preferred extremal regarded as 4-surface in super imbedding space parameterized by the hermitian imbedding coordinates plus the coefficients of the monomials of quarks and antiquarks with vanishing quark number, whose time evolution follows from dimensionally reduced 6-D super-Kähler action? Could one assume similar interpretation for super spinors consisting of monomials with quark number 1 and appearing in super-Dirac action?

5. In WKB approximation the exponent of action defines wave function. In QFTs path integral is defined by an exponent of action and scattering operator can be formally defined as action exponential. Could the matrix elements for the exponent of the super counterpart of Kähler function plus super Dirac action between states at opposite boundaries of CD between positive and negative energy parts of zero energy states define S-matrix? Could the positive and negative energy parts of zero energy states be identified as many particles states formed from the monomials associated with imbedding space super-coordinates and super-spinors?

6. Could the construction of S-matrix elements as matrix elements of super-action exponential reduce to classical theory? Super-field monomials in the interior of CD would be linear superpositions of super-field monomials at boundary of CD. Note that oscillator operator monomials rather than their coefficients would be the basic entities and the dynamics would reduce to that for oscillator operators as in QFTs. The analogs of propagators would relate the monomials to those at boundary ly to the monomials at the boundary of CD, and would be determined by classical field equations so that in this sense everything would be classical. Note however that the fixed point condition $q = q_s$ and super counterpart of modified Dirac equation are non-linear.

   Vertices would be defined by monomials appearing in super-coordinate and super-spinor field appearing in terms of those at boundary of CD. If two vertices at interior points x and y of CD are connected there is line leading from x to a point z at boundary of CD and back to y and one would have sum over points z in cognitive representation. This applies also to self energy corrections with $x = y$. At the possibly existing continuum limit integral would smoothen the delta function singularities and in presence of normal ordering at continuum would eliminate them.

   In the expressions for the elements of S-matrix annihilation operators appearing in the monomials would be connected to the passive boundary P of CD and creation operators to the active boundary. If no partonic 2-surfaces appear as topological vertices in the interior of CD, this would give trivial S-matrix!

   $M^8 - H$ duality however predicts the existence of brane like entities as universal 6-D surfaces as solutions of equations determining space-time surfaces. Their M^4 projection is $t = r_n$ hyperplane, where r_n corresponds to a root of a real polynomial with algebraic coefficients giving rise to octonion polynomial, and is mapped to similar surface in H. 4-D space-time surfaces representing incoming and outgoing lines would meet along their ends at these partonic 2-surfaces.

Partonic 2-surfaces at these hyper-surfaces would contain ordinary vertices as points in cognitive representation. Given vertex would have at most 4+4 incoming and outgoing lines assignable to the monomial defining the vertex. This picture resembles strongly the picture suggested by twistor Grassmannian approach. In particular the number of vertices is finite and their seems to be no superposition over different diagrams. In this proposal, the lines connecting vertices would correspond to 1-D singularities of the space-time surfaces as minimal surfaces in H. Also stringy singularities can be considered but also these should be discretized.

By fixing the set of monomials possibly defining orthonormal state basis at both boundaries one would obtain given S-matrix element. S-matrix elements would be matrix elements of the super-action exponential between states formed by monomials of quark oscillator operators. Also entanglement between the monomials defining initial and finals states can be allowed. Note that this in principle allows also initial and final states not expressible using monomials but that monomials are natural building bricks as analogs of field operators in QFTs.

7. The monomials of imbedding space coordinates are imbedding space vectors constructible from Dirac currents (left- or right-handed) with oscillator operators replacing the induced spinor field and its conjugate. The proposed rules for constructing S-matrix would give also scattering amplitudes with odd quark number at boundaries of CD. Could the super counterpart of the bosonic action (super Kähler function) be all that is needed to construct the S-matrix?

   In fact, classically Dirac action vanishes on mass shell: if this is true also for super-Dirac action then the addition of Dirac action would not be needed. The super-Taylor expansion of super-Kähler action gives rise to the analogs of perturbation theoretic interaction terms so that one has perturbation theory without perturbation theory as Wheeler might state it. The detailed study of the structure of the monomials appearing in the super-Kähler action shows that they have interpretation as currents assignable to gauge bosons and scalar and pseudo-scalar Higgs.

   Super Dirac action is however needed. Super-Dirac equation for q and $D_{\alpha,s}\Gamma^{\alpha}_s = 0$ allow to reduce ordinary divergences $\partial_\alpha j^\alpha$ of fermionic currents appearing in super-Kähler action to commutators $[A_{\alpha,s}j^\alpha]$. Therefore no information about q at nearby points is needed and one avoids lattice discretization: cognitive representation is enough.

8. Topological vertices represent discontinuities of the space-time surface bringing strongly in mind the non-determinism of quantum measurement, and one can ask whether the 3-branes and associated partonic 2-surfaces. Could the state function reductions analogous to weak measurements correspond to these discontinuities? Ordinary state function reductions would change the arrow of time and the roles of active and passive boundaries of CD [28]. In TGD inspired theory of consciousness these time values would correspond to "very special moments" in the life of self [30].

9. The unitarity of S-matrix can be understood from the structure of the exponent of Kähler action. The exponent decomposes to a sum of real and purely imaginary parts. The exponent of the hermitian imaginary part is a unitary operator for a given space-time surface. Real exponent containing also radiative corrections from the normal ordering gives exponent of Kähler function as vacuum functional in WCW (sum in the case of cognitive representations) and by choosing the normalization factor of the state appropriately one obtains unitary S-matrix.

2.3 More explicit picture

The following sketch tries to make the picture of the second trial more explicit.

1. The construction of S-matrix should reduce to super-geometry coded by super Kähler function determined by the 6-D Kähler action for twistor lift by dimensional reduction. This might be

possible since zero energy states have vanishing total conserved charges and exponent of super-Kähler function has matrix elements only between states at opposite boundaries of CD having same total charges.

2. Construction should reduce to preferred extremals and their super-deformations determined by variational principle with boundary conditions. The boundary values of super-deformations at either boundary could be also interpreted as initial values for preferred extremals analogous to Bohr orbits. The expectations for the super action with fixed initial values between positive and negative energy parts would give the scattering amplitudes assignable to a given space-time surface. There would be functional integral over space-time surfaces using exponent of Kähler function as weight. In number theoretic vision this would reduce to sum over preferred extremals labelled by cognitive representations serving as WCW coordinates.

3. Number theoretic vision suggests a discretization in terms of cognitive representation consisting of points with coordinates in extension of rationals defining the adele. This representation could be associated with the boundaries of CD and possibly with M^4 time=constant hyperplanes assignable with the universal special solutions in M^8. At the partonic 2-surfaces associated with these hyper-planes 4-D extremals would meet along their ends: topological particle vertices would be in question. Is string world sheets and partonic 2-surfaces correspond to singularities, the boundaries of strings world sheets as intersections of the string world sheets and orbits of partonic 2-surfaces should represent fermion lines.

4. Creation operators would be assigned with the passive boundary of CD - call it P - and annihilation operators as their conjugates would act as creation operators at the opposite boundary, active boundary - call it A. Time reversal symmetry of CD suggests that annihilation operator as conjugate of creation operator labelled by the a point of boundary of CD corresponds to the same point in common coordinates for light-cone boundary. This would conform also with the basic character of the half-algebras associated with super-symplectic symmetries.

   The original proposal was that oscillator operators have only spin and electroweak spin as indices but the standard view about spin and statistics requires that also the points of the 3-surface must label them. Also the fact that the total quark number can be larger than 4 of course requires this too. Algebraically the only difference with respect to this proposal is that one allows also the points of 3-surface at the boundary of CD as labels.

5. Number theoretical vision requires that only points of 3-surface having imbedding space coordinates in the extension of rationals defining the adelic physics are allowed. In the generic case the number of points in the cognitive representation would be finite and would increase with the dimension of extension so that at the limit of algebraic numbers they form a dense set of 3-surface.

   Since action has infinite expansion in powers of super coordinates the contractions of oscillator operators would give rise to a renormalization of the coefficients of the monomials and of classical action. For cognitive representations one would avoid normal ordering problems sine the number of contractions is limited by the number of points in cognitive representation. This would give rise to discrete coupling constant evolution as function of the extension of rationals.

6. In continuum theory all points of 3-D boundary would label quark oscillator operators and one must normal order the oscillator operators in given local monomial. Also now the idea about connecting creation and annihilation operators to opposite boundaries of CD would allow to get rid of infinities due to contractions.

The action exponential would lead to a rather concrete proposal for the coefficients of the monomials appearing in super-fields.

1. The deformations of imbedding space coordinates would be expressible as WCW-local superpositions of isometry generators or as WCW-global superpositions of Hamiltonian currents contracted with the coordinate deformations. The latter would conform with super-symplectic symmetries of WCW. CP_2 Hamiltonian currents would give color quantum numbers. S^2 Hamiltonian currents would be also present. One could see space-time local Kac-Moody symmetries assignable to light-like partonic orbits and string world sheets as a dual representations at space-time level of symplectic symmetries at imbedding space level.

2. Spinor modes would be expressible as superpositions of imbedding space spinor modes having expansion as super-Taylor series at the boundaries of CD. This would give spin and electroweak quantum numbers.

Does one really obtain description of gauge bosons and gravitons by using the exponent?

1. Could the coefficients of super-monomials at boundary of CD allow interpretation in terms of gauge bosons? These entities could have well-defined quantum numbers so that this might be possible. Quark spin and isospin would represent additional spin degrees of freedom. The Hamiltonians of H of CP_2 expressible for given 3-surface as local superpositions of SU(3) Killing vector fields would represent color degrees of freedom.

   For string world sheets one would naturally have transversal M^4 super-coordinates and CP_2 super-coordinates as analogs of fields. Could this allow to get gauge bosons as excitations of strings as in string theories.

2. Gauge bosons could be also bi-local composites of fermion and anti-fermion at opposite boundaries of wormhole contact or at opposite wormhole contacts of wormhole flux tube. Gravitons could be 4-local composites. Baryons and mesons could be this kind of non-local composites. One can consider also the analog of monopole phase of QFTs in which particles would be multilocal composites.

3. The bosonic action is for induced metric and and induced Kähler form. QFT wisdom would suggest that their super-analogs could correspond to external particles. One could indeed take the induced gauge potentials or -fields at boundary and form their contractions with Killing vectors of isometries to obtain general coordinate invariant quantities and form their super-analogs as normal ordered local composites. One can consider the same idea for induced gravitational field or its deviation from Minkowski metric.

   Formally this would correspond to an addition to the action exponential of perturbative terms of type jA appearing in QFTs representing coupling to external currents and take the limit $j \to 0$. In QFT picture this works since various gauge fields are functionally independent but in TGD framework this is not the case. Second problem is to to construct a complete orthonormalized set of states in this manner. Therefore it seems this description can make sense only at QFT limit of TGD.

2.3.1 Dimensionally reduced 6-D Kähler action as an analog of SYM action

The 6-D dimensionally reduced Kähler action reduces to a sum of 4-D Kähler action and volume term and will be simply referred to as Kähler action. The super variant of this action is obtained by replacing imbedding space coordinates with their super counterparts. Super-Kähler action is analogous to pure SYM action.

1. Space-time would be super-surface in super counterpart of $H = M^4 \times CP_2$ with coordinates h^k having super components proportional to multi-spinors multiplying the monomials of oscillator operators. The ocillator operator monomials rather than only the multi-spinor coefficients of the oscillator monomials transforming like vectors of H are regarded as analogs of quantum fields

expressible by classical field equations as linear superpositions of their values at the boundary of CD for preferred extremals. The dynamics of monomials would reduce to that for oscillator operators labelled by points of cognitive representation and having interpretation as restriction of quantized quark field satisfying super-Dirac equation and the quantum criticality condition $q = q_s$.

For M^4 this is expected to work but in the case of CP_2 this approach need not be so straightforward. The symmetries and projective space property allowing to use projective coordinates might help to overcome the possible technical problems.

2. Fermionic creation operators and annihilation operators labelled not only by spin and weak isospin as in the original proposal but also by the finite number of points of the cognitive representation. Therefore oscillator operators are analogous to the values of fermion field in discretization obeying super variant of modified Dirac equation. Both leptonic and quark like oscillator operators corresponding to two different H-chiralities and having different couplings to Kähler gauge potential could be present but octonionic triality allows only quarks. The vacuum expectation value of the action action exponentials contains only monomials with vanishing B (and L if leptons are present as fundamental fields). The matrix elements between positive and negative energy parts of zero energy states gives S-matrix.

   Real super-coordinates can be assumed to be hermitian and thus contain only sums of monomials and their conjugates having vanishing fermion numbers. This guarantees super-symmetrization respecting bosonic statistics at the level of propagators since all kinetic terms involve two covariant derivatives - one can indeed transform ordinary derivatives of monomials coming from the Taylor expansion to covariant derivatives involving also the coupling to Kähler form since the total Kähler charge of terms vanishes.

The lack of anti-commutativity of fermionic oscillator operators implies the presence of terms resulting in contractions.

1. The super-Taylors series would involve a finite number of partial derivatives of action. Wick contractions of oscillator operators would give rise to an infinite number of terms in continuum case. The appearance of infinite Taylor series defining the coefficients of super-polynomial is however troublesome from the point of view of number theoretic vision since there is no guarantee that the coefficients are rational functions. The finite number of points in the cognitive representation implying finite number of oscillator operators however allows only finite number of terms in the super-Taylor expansion.

   The monomials appearing in action in the interior of CD can be expressed as linear superpositions of those at boundary also in continuum case. Therefore each monomial is 3-D integral over the monomials at the boundary of CD. As a consequence, the contractions giving delta functions only eliminate one integration but do not give rise to infinities. A general solution to the divergence problems emerges.

   This is actually nothing new: one of the key ideas behind the notion of WCW is that path integral over space-time surfaces is replaced by a functional integral over 3-surfaces in WCW holographically equivalent with preferred extremals as analogs of Bohr orbits. The non-locality of the theory due to the replacement of point-like particles with 3-surfaces would solve the divergence problems.

An interesting possibility in line with the speculations of Nima-Arkani Hamed and others is that the action defining space-time as a 4-surface of imbedding space could emerge from the anti-commutators of the oscillator operator monomials as radiative corrections so that the bosonic action would vanish when the super-part of h_s vanishes.

2.3.2 Super-Dirac action

Before doing anything one can recall what happens in the case of modified Dirac action.

1. One has separate modified Dirac actions $\overline{\Psi}D\Psi$, $D = \Gamma^\alpha D_\alpha$ for quarks and leptons (later it will be found that modified Dirac action for quarks might be enough) and the covariant derivatives differ since there is a coupling to n-ple of included Kähler potential. For leptons one has $n = -3$ and for quarks $n = 1$. This guarantees that em charges come out correctly. This coupling appears in the covariant derivative D_α of fermionic super field.

2. One obtains modified Dirac equations for quarks and leptons by variation with respect to spinors. The variation with respect to the imbedding space coordinates gives quantized versions of classical conservation laws with respect to isometries. One also obtains and infinite number of super-currents as contractions of modes of the modified Dirac operator with Ψ.

3. Classical field equations for the space-time surface emerge as a consistency condition guaranteeing that the modified Dirac operator is hermitian: canonical momentum currents of classical action must be conserved and define conserved quantum when contracted with Killing vectors of isometries. Quantum-classical correspondence (QQC) requires than for Cartan algebra of symmetry algebra the classical Noether charges are same as the fermionic Noether charges.

   It turns out that the super-symmetrization of modified Dirac equation gives only fermions and they fermionic superpartners in this manner if one requires that propagators are consistent with statistics.

Consider first the situation without the quantum criticality condition $q = q_s = \Psi_s$. H coordinates are super-symmetrized and induced spinor field becomes a super-spinor $\Psi_s = \Psi^N O_N(q, \overline{q})$ with Ψ_N depending on h_s (summation over N is understood).

1. As in the case of bosonic action the vacuum expectation value gives modified Dirac action conserving fermion numbers but one could assume that the monomials in the leptonic (quark) modified Dirac action have either non-vanishing L (B) and vanishing B (L). It seems that the lepton (baryon -) number of monomials can vary from 1 to maximum value. A more restrictive condition would be that the value is 1 for all terms.

2. Super-Dirac spinor is expanded in monomials $O_N(q, \overline{q})$ of q and its conjugate $\overline{q}$, whose anti-commutator is non-trivial. One can equally well talk about quark like oscillator operators. The sum $\Psi = \Psi^N O_N$ defining super-spinor field. The multi-spinors Ψ_N are functions of space-time coordinates, which are ordinary numbers. Quark oscillator operators are same as appearing in the imbedding space super-coordinates. Only monomials O_N having total quark number 1 are allowed. Super-spinor field however contains terms involving quark pairs giving rise to spartners of multi-quark states with fixed quark number 1. The conjugate of super-spinor is defined in an obvious manner.

3. The metric determinant and modified gamma matrices appearing in the Dirac action are expanded as Taylor series in hermitian super-coordinate $h_s + \overline{h}_s$ with $h = h^N O_N$. This as as in the case of bosonic action.

   There are also couplings to gauge potentials defined by the spinor connection of CP_2 and the expansion of them with respect to the imbedding space coordinates gives at the first step rise covariant derivatives of gauge potentials giving spinor curvature. At next steps one obtains covariant derivatives of spinor curvature, which however vanish so that the number of terms coming from the dependence of spinor connection on CP_2 coordinates is expected to be finite. Constant curvature property of CP_2 is therefore be essential (not that also M^4 would have covariantly constant spinor curvature in twistor lift and give rise to CP breaking).

The super-coordinate expansion of the metric determinant $\sqrt{g}$ and modified gamma matrices Γ^α and covariant derivatives D_α involving dependence on H coordinates give additional monomials of q parameters appear as hermitian monomials. Classical field equations correspond to $D_\alpha \Gamma^\alpha = 0$ guaranteeing the hermiticity of $D = \Gamma^\alpha D_\alpha$.

4. When super-coordinates of H are replaced with ordinary imbedding space coordinates the only Wick contractions are between O^N and $\overline{O}^N$ in the vacuum expectation of Dirac action, and the action reduces to super-Dirac action with components satisfying modified Dirac equation. Propagator is Dirac propagator for all terms and the presence of only quark number 1 odd components in Ψ and even components in h^s guarantees that Fermi statistics is not violated at the level of propagators. The dependence on h_s induces coupling between different components of the super-spinor. The components of super-spinor are interpreted as second quantized objects.

5. The terms in the action would typically involve n-tuples of partial derivatives $L_{k_1\alpha_1 \ldots k_n 1\alpha_n}$ defined earlier for $L = \sqrt{g}$ coming from super-Taylor expansions. Similar derivatives come from the modified gamma matrices Γ^α.

   Also now one obtains loops from the self contractions in the terms coming from the expression of action and gamma matrices. These terms should vanish and as already found this would requires vanishing of currents perhaps identifiable as Noether currents of symmetries. This guarantees that the Taylor expansion contains only finite number of terms as required by number theoretic vision.

The multi-fermion vertices defined by the action would be non-trivial but involve always contraction of all fermion indices between monomials formed from oscillator operators in Ψ and their conjugates in $\overline{\Psi}$ if the loop contractions sum up to zero. One could interpret these supersymmetric vertices as a redistribution of fermions of a local many-fermion state between external local many-fermion states particles represented by the monomials appearing in the vertices. The fermions making the initial state would be same as in final state and all distributions of fermion number between sfermion lines would be allowed. The action obtained by contraction would has SUSY as symmetry but the propagation of different sfermions is fermionic and does not look like that for ordinary spartners.

The quantum criticality condition $q = q_s$ makes the situation non-linear and should fix the coefficients of various terms in super-Taylor expansions as fixed point values of coupling constants.

2.3.3 Could super-Kähler action alone give fermionic scattering amplitudes?

The concrete study of the super-counterpart of Kähler action led to a realization of an astonishing possibility: super-Kähler action alone could give also fermionic scattering amplitudes.

1. In principle this is possible if in S-marix one has contractions of quark creation operator and annihilation operator appearing in quark-antiquark bilinear with different partonic 2-surfaces. This would give fermionic line connecting the points of the cognitive representation at the boundary of CD with points at partonic 2-surfaces in $t = r_n$ hyper-planes in the interior of CD or at the opposite boundary of CD.

   As a matter of fact, this must be the case if the exponent for the sum of super-Kähler and super-Dirac action gives the scattering amplitudes as its matrix elements! The reason is that super-Dirac action vanishes or its solutions.

   The super-Dirac equation must be however present and corresponding variational principle must be satisfied. The hermiticity of the modified Dirac operator requires the vanishing of the covariant derivatives of the modified gamma matrices meaning that bosonic field equations are satisfied. This must be true also for the super variants of the modified gamma matrices.

   If super-Dirac equation is satisfied, the action of modified Dirac operator without connection (ordinary rather than covariant derivative) terms on the discretized quark fields can be expressed in terms of spinor connection as $\Gamma^\alpha - s\partial_\alpha \Psi = \Gamma^\alpha_s A_{\alpha,s}\Psi$ and there is no need for explicit information about the behavior of quark field in the nearby points so that cognitive representation is enough. Otherwise one must have the usual lattice type discretization.

2. The super expansion of super-Kähler action contains only ordinary derivatives of 4-currents defined by quark bi-linears. If the quark field operators with continuous arguments are behind those with discretized arguments and satisfy modified Dirac equation, one can transform the action on quark and antiquark fields to a multiplication with induced gauge potential. This gives nothing but the coupling terms to the gauge potentials in the standard perturbation theory, where one assumes free solutions of Dirac action as approximate solutions. One therefore obtains on mass shell variant of the perturbation theory! Perturbation theory without perturbation theory, might Wheeler say. Or more concretely: the fact that one can treat super-coordinates only perturbatively.

3. The natural guess is that all terms in the expansion of super-Kähler can be transformed to interaction terms and super-Kähler action gives the analog of perturbation theory as a discretized version. The leptonic terms associated with $(3,3)$ term in super-Kähler action should transfrom to the analog of interaction terms for leptonic Dirac action. Whether Kähler gauge potential and and spinor connection are developed in super-Taylor series in ordinary manner or remains an open questions.

2.4 What super-Dirac equation could mean and does one need super-Dirac action at all?

What does super-Dirac equation actually mean? Super Dirac action vanishes on mass shell and super-Kähler action would give all scattering amplitudes. Are super-Dirac action and super-spinor field needed at all? Should one interpret the oscillator operators defining analog of quark field q as the super-Dirac field Ψ_s as conceptual economy suggests. But doesn't this imply $q = q_s$?

One can consider 3 options as an attempt to answer these questions. Options I and II are not promising. Option III leads to very nice concrete realization of quantum criticality.

2.4.1 Option I: No super-Dirac action and constant oscillator operators

1. If oscillator operators can be regarded as constant, the super Taylor expansion for super Kähler action would give ordinary divergences of the fermionic currents and the action of derivative would be on modified gamma matrices and charge matrix A commutator of $[A_\alpha, \Gamma^\alpha Q]$ and the outcome would be non-vanishing so that one would obtain the coupling terms also now. Could the commutator $[A_\alpha, \Gamma^\alpha]$ be interpreted in terms of gravitational interaction and the commutator $[A_\alpha, Q]$ as electro-weak interaction? In any case, there would be no need for super-Dirac action!

2. There is however an objection. Quark oscillator operators are labelled by the points of cognitive representation and in continuum case they are analogous to the values of quantized spinor field. Should one identify this spinor field with super-spinor field and solve it using a generalization of modified Dirac equation to super-Dirac equation? Can one argue that oscillator operators labelled by points represent superpositions of constant oscillator operators involving integration over 3-D surface at light-cone boundary and are indeed constant?

This option does not look promising.

2.4.2 Option II: q satisfies ordinary Dirac equation

1. Could one assume that the solution q_0 of ordinary Dirac equation defines the solution to be used as q in the super-Kähler action. The coupling terms of super-Kähler action obtained using $D_0 q_0 = 0$ would be proportional to the classical spinor connection. Classical Kähler action does not involve gauge potentials so that internal consistency would not be lost at this level. The super-variant of

Kähler action however involves derivatives of the analogs of fermion currents and there transformation to purely local objects requires the introduction of electroweak gauge potentials so that the symmetry between super-Kähler and super-Dirac would be lost.

2. This would save from developing gauge potentials A_k to super Taylor series - as found this would give only 2 terms by the covariant constancy of spinor curvature. The divergence would reduce to a term involving only a commutator $[A_{alpha}, Q]$, where A_α is purely classical. If Q is Kähler charge, this commutator would vanish, which looks strange since electroweak hyper-charge is proportional to Q_K. This could be seen as a failure. If Kähler gauge potential is replaced with its super-variant $A_\alpha + J_{\alpha l}\delta h_s^l$ the commutator is non-vanishing as it should be.

3. Leptons would not appear in $q = q_0$ but since the exponent of super-Kähler action would define the scattering amplitudes by the vanishing of (super-)Dirac action, one could say that leptons emerge as 3-quark composites. SUSY would be dynamical after all!

Mathematically this option looks awkward and must be dropped from consideration.

2.4.3 Option III: q is a solution of super-Dirac equation

It is best to start from an objection.

1. Assume that q is given Super-Dirac equation

$$D_s(q)q = 0 \ .$$

This non-linear equation involves powers of q and its conjugate. The problem is that super-Dirac equation is non-linear in q and there are actually 7 separate equations for the part of q with quark number one. 7 equations is too much. The only manner to solve the problem is to replace q with q_s to get $D_s q_s = 0$. But this would require replacing q with q_s in $D_s(q)$ and it would seem that one has an infinite recursion.

2. This suggests that q is self-referential in the sense that one has

$$q_s = q \ . \tag{2.1}$$

q would be invariant under iteration $q \rightarrow q_s$. This would give excellent hopes of fixing q uniquely. This allows also physical interpretation. The fixed points of iteration give typically fractals and quantum criticality means indeed fractality. This condition could therefore realize quantum criticality, and would give hopes about unique solution for $q = q_s$ for given extension of rationals.

Also h_s should satisfy similar self-referentiality condition expressing quantum criticality:

$$h_s = (h_s)_s \ . \tag{2.2}$$

The general ansatz for h_s^k involves analogs of electroweak vector currents formed from quark field and lepton field as its local composites. q_s has analogous structure. The currents contracted with the Hamiltonian vector fields of symplectic transformations of light-cone boundary appear in the Minkowski salars and have some coefficients having an interpretation as coupling constants. $q = q_s$ condition defining quantum criticality would fix the values of these coupling parameters for given extension of rationals and would realize discrete coupling constant evolution.

3. Many consciousness theorists love the idea of self-referentiality described by Douglas Hofstadter in fascinating manner in his book "Gödel, Escher, Bach". They might get enthusiastic about the naive identification of q_s and h_s with field of consciousness. In TGD inspired theory of consciousness the self-referentiality of consciousness is understood in different manner but $q = q_s$ and $h_s = (h_s)_s$ as quantum correlated for the self-referentiality is certainly a fascinating possibility.

Consider now a more detailed picture.

1. What does one really mean with q_s? q_s could contain parts with quark number 1 and 3 but a very natural requirement is that it has well-defined fermion number and thus has only a part with quark number 1. The part with quark number 3 is not needed since super-Kähler action would contain it: leptons would emerge as local 3-quark composites from super-Kähler action.

2. Super-Dirac equation would be given by

$$\begin{aligned} D_s(q)q &= 0 \ , \\ D_s(q) &= \Gamma^{\alpha,s}(q)D_{\alpha,s}(q) \ . \end{aligned} \tag{2.3}$$

$D_s(q)$ is super-Dirac operator and

$$\Gamma_s^\alpha = T_s^{\alpha k}\gamma_k \tag{2.4}$$

are super counterparts of the modified gamma matrices $\Gamma^\alpha = T^{\alpha k}\gamma_k$ defined by the contractions of canonical momentum currents of Kähler action with the gamma matrices γ_k of H:

$$T_k^\alpha = \frac{\partial L_K}{\partial(\partial_\alpha h^k)} \ . \tag{2.5}$$

One would have $\gamma_{k,s} = \gamma_k$ by covariant constancy. L_K denotes Kähler action density, which is sum of 4-D Kähler action and volume term. The field equations of super Kähler action give

$$D_{\alpha,s}\Gamma_s^\alpha = 0 \tag{2.6}$$

guaranteeing the hermiticity of the super Dirac operator.

3. The basic equations would thus reduce to

$$\begin{aligned} q = q_s \ , \\ D_{\alpha,s}\Gamma_s^\alpha = 0 \ , \\ D_s(q)q = 0 \ . \end{aligned} \tag{2.7}$$

In the continuum case one could think of solving the field equations iteratively.

1. One would first by solve $q = q_0$ for classical modified Dirac operator $D(h_0)$ defined by the ordinary coordinates h_0 of H. Next one would solve $q_1 = q_0 + \Delta q_1$ for the super version $D_1 = D(q_0)$. This would allow to solve next iterate $h_1 = h_0 + \Delta h_1$ using $D(q_1)$. One could continue this process in the hope that the iteration converges. At each step one have group of equations $D_n q_n = 0$ for q_n and for h_{n+1}.

2. An objection is that the iteration could lead outside the extension of rationals if it involves infinite number of iterates. This could occur for space-time surface itself if the normal ordering terms affect the classical action and force to modify the preferred extremal and also cognitive representation at each step. Remaining inside the extension of rationals could also mean that the coefficients of the monomials at points of cognitive representation belong to the extension.

   It is not of course completely clear whether these equations make sense in the interior of CD or can be solved unlike the lowest equation. It however seems that for each independent monomial m_n the equation would be of form $D_0 m_n = ...$ so that other terms would define kind of sources term and the equation super-Dirac equation could be written as non-linear equation $D_0 q = -\Delta D(q)q$.

3. Each order of bosonic monomials would give its own group of equations making sense also for the cognitive representations and the same would be true for quark monomials and monomials of different orders would be coupled but different quark numbers in q (quarks and leptons) would decouple. These equations are analogous to those appearing in QFT in a gauge theory involving gauge fields and fermion fields.

For cognitive representations the situation is much simpler.

1. All that is needed is the transformation of the ordinary divergences of fermionic currents to a form in which derivative ∂_α is replaced with the linear action of super-gauge potential $A_{\alpha,s}$. Therefore there is no need to solve the non-linear modified Dirac equation in this case and it would become necessary only at the continuum limit. The full solution of non-linear super-Dirac equation would be necessary only in the continuum theory.

2. Could one think that q has vanishing derivatives at the points of cognitive representation: $\partial_\alpha q = 0$ implying $\Gamma^\alpha A_\alpha q = 0$ If the condition holds true then q would be effectively constant for cognitive representations and the situation would effectively reduce to that for option I. This condition is is diffeo-invariant but not gauge invariant. If the points of cognitive representation correspond to singularities of the space-time surface at which several roots of the octonionic polynomial co-incide, the tangent space at the level of M^8 parameterized by a point of CP_2 is not unique and the singular point is mapped to several points in H, and the conditions $\partial_\alpha q = 0$ would make sense at the level of M^8 at least.

3. If one assumes that the quarks correspond to singular points defined by intersections of roots also in the continuum case, one obtains discretization also in this case irrespective of whether one assumes $\partial_\alpha q = 0$ at singularities. Allowing analytic functions with rational Taylor coefficients one obtains also now roots which can be however transcendental and one can identify intersections of roots in the similar manner.

To sum up, there are many uncertainties involved but to my opinion the most satisfactory option is Option III. If one assumes that condition at continuum case, one would obtain also now the discretization.

2.4.4 What information is needed to solve the scattering amplitudes?

One can look the situation also from a more practical point of view. Are there any hopes of actually calculating something? Is it possible to have the information needed?

1. The condition that super-Dirac equation is satisfied would remove the need to have a lattice and cognitive representation would be enough. If the condition $\partial_\alpha q = 0$ holds true, the situation simplifies even more but this condition is not essential. The condition that the points of the cognitive representation assignable to quark oscillator operators correspond to singularities of space-time surface at which several space-time sheets intersect, would make the identification of these points of cognitive representation easier. Note that the notion of singular point makes sense also at the

continuum limit giving cognitive representation even in this case in terms of possibly transcendental roots of octonion analytic functions.

If the singular points correspond to solution to 4 polynomial conditions on octonionic polynomials besides the 4 conditions giving rise to the space-time surfaces. The intersections for two branches representing two roots of polynomial equation for space-time surface indeed involve 4 additional polynomial conditions so that the points would have coordinates in an extension of rationals, which is however larger than for the roots $t = r_n$. One could of course consider an additional condition requiring that the points belong to the extension defined by r_n but this seems un-necessary.

The octonionic coordinates used at M^8-side are unique apart from a translation of real coordinate and value of the radial light-like coordinate $t = r_n$ corresponds to a root of the polynomial defining the octonionic polynomial as its algebraic continuation. At this plane the space-time surfaces corresponding to polynomials defining external particles as space-time surfaces would intersect at partonic 2-surfaces containing the shared singular points defined as intersections.

2. The identification of cognitive representations goes beyond the recent knowhow in algebraic geometry. I have considered this problem in [35] in light of some recent number theoretic ideas. If the preferred extremals are images of octonionic polynomial surfaces and $M^8 - H$ duality the situation improves, and one might hope of having explicit representation of the images surfaces in H-side as minimal surfaces defined by polynomials.

2.5 About super-Taylor expansion of super-Kähler and super-Dirac actions

The study of the details of of the general vision reveals several new rather elegant features and clarifies the connections with QFT picture.

2.5.1 About the structure of bosonic and fermionic monomials

The super part of the imbedding space coordinates is H-vector and this allows to pose strong conditions on the form of the monomials.

1. One can construct the simplest monomials as bilinears of quarks and anti-quarks. Since oscillator operators are analogs of quark fields, one can construct analogs of left- and right-handed electroweak currents $\overline{q}(1 \pm \gamma_5)\gamma^k Qq$ involving charge matrix Q naturally assignable to electroweak interactions. The charge matrices Q should reflect the structure of CP_2 spinor connection so that analogs of electroweak currents would be in question. One can multiply the objects Hamiltonians HA_A of the isometries and even symplectic transformations at the boundary of CD.

2. One can obtain higher monomials of q and $\overline{q}$ by multiplying these vectorial currents by bi-linears, which are scalars and pseudo-scalars obtained by contracting some symmetry related vector field j_A^k of H with gamma matrices of H to give $\overline{q}(1 \pm \gamma_5)j_A^k Q\gamma_k q$ giving rise to analogs of scalar and pseudoscalar Higgs. The Killing vector fields of isometries of H and symplectic vector fields assignable to the Hamiltonians of $\delta CD \times CP_2$ are a natural choice for j_A^k.

   One can construct also scalar currents for which gamma matrices contract with gradient of Hamiltonian to give $\overline{q}(1 \pm \gamma_5)\gamma^k \partial_k H_A Q\gamma_k q$ as kind of duals of symplectic currents. These do not define symplectic transformations.

   These vector fields make sense at the boundaries of CD and this is enough (they could make sense also at shifted boundaries) since the field equations would allow to express monomials as linear superpositions of the monomials at boundary of CD. Oscillator would always be assigned with the boundaries of CD.

3. If the spin of graviton is assigned with spinor indices, the vector nature of the monomials excludes the analog of graviton. One can however consider also the possibility that the second spin index of graviton like state corresponds to the Hamilton of a symplectic isometry of S^2: for small enough size scales of CD this angular momentum would look like spin. In CP_2 degrees this would give rise to an analog of gluon. Also gluon with spin zero would be obtained.

   An alternative option is to assume that graviton corresponds to a non-local state with vectorial excitations at opposite throats of wormhole contact or at different wormhole contacts of closed flux tube. All these states are in principle possible and the question is which of them correspond to ordinary gravitons.

The super counterpart of Dirac spinor consists of odd monomials of quark spinor. Well-defined fermion number allows only monomials with quark number 1 and with definite H-chirality. One can have lepton like states as bilinears of local 3-quark composites appearing in the super-Kähler action determining the scattering amplitudes since super-Dirac action vanishes at mass shell.

1. In the bosonic case one has vectorial entities and now it is natural to require that one has an object transforming like spinor of H. This poses strong conditions on the monomials since one should have spin 1/2-isospin 1/2 representation.

2. The lowest monomial correponds to quark-antiquark current. What about leptonic analog. The number of oscillator operators at given point is 4+4=8. Leptonic part of super-Kähler action must have 3+3 indices. Therefore also leptonic bilinear seems to be possible and pairs of quarks and lepton like states are possible.

   Intuitively it is clear that leptonic term exists and corresponds to an entity completely antisymmetric in spin-isospin index pairs (s_3, i_3) of quark spinors. The construction of baryons without color symmetry indeed gives proton and neutron. In order to obtain Δ resonance from u and d quarks, one must have color degrees of freedom and perform anti-symmetrization in these.

   The general condition is that the tensor product of 3 8-D spin representation of $SO(1,7)$ contains 8-D representation in its decomposition. The existence of lepton representation is clear from the fact that the completely antisymmetric representation formed from 4 quarks is $SO(1,7)$ singlet and is product of lepton representation with 3 fold tensor product which must therefore contain spin-isospin 4-plet . The coupling to Kähler gauge potential would correspond to leptonic coupling, which is 3 times the quark coupling.

3. Quarks and lepton monomials have also satellites obtained by adding scalars and pseudo-scalars constructible as quark-anti-quark bi-linears in the manner already discussed. The interpretation as analogs of Higgs fields might make sense.

2.5.2 Normal ordering terms from contractions of oscillator operators

Normal ordering terms from contractions of oscillator operators is a potential problem. In the discretization based on cognitive representations this problem disappears.

1. Contraction terms could induce discrete coupling constant evolution by renormalizing the local monomials. Infinite number these terms would spoil number theoretical vision since a sum over infinite number of terms in general leads outside the extension of rationals involved. If the number of contractions is finite, there are no problems. This is the case in the number theoretical vision since contraction involves always a pair of points. If the rule for construction of S-matrix holds true these points are at opposite boundaries of CD. In the general case they can be at the same boundary. The number of contracted points cannot be larger than the number of points in cognitive representation, which is finite in the generic situation.

This would give discrete coupling constant evolution as function of extension of rationals since the contractions renormalize the coefficients of the 4+4 terms in the local composites of oscillator operators. The original proposal that additional symmetries are needed to obtain discrete coupling constant evolution is not needed.

2. One could argue that algebraic numbers as a limit for extension is enough to get the continuum limit since the points of cognitive representation would be dense subset of 3-surface. For continuum theory 3-D delta functions would replace Kronecker deltas in anti-commutators implying in ordinary QFT divergences coming as powers of 3-D delta function at zero.

   In the proposed vision one can allow contractions even in the continuum case. The monomials in the interior are linear multilocal composites of those at either boundary of CD involving 3-D integration over boundary points. Contractions associated with two monomials in the interior means an appearance of delta function cancelling the second integration so that there is no divergence.

2.5.3 About the super-Taylor expansions of spinor connection and -curvature

There are also questions related to the details of the expansion of super Kähler function in powers of monomials of quark oscillator operators.

1. The rule is that one develops Kähler function as Taylor series with argument shifted by super-part of the super-coordinate. This involves expansion in powers of coordinate gradients and also the expansion of Kähler gauge potential. In the case of modified Dirac action one must expand also the spinor connection of CP_2.

   A potential problem is that the Taylor expansions of Kähler gauge potential and spinor connection have infinite number of terms. Since the monomials in the interior can be expressed linearly in terms of those at boundary of CD by classical field equations, number theoretic discretization based on cognitive representation implies that only a finite number of terms are obtained by using normal ordering and the fact that the number of oscillator operators at same point is 4+4=8. Normal ordering terms would represent radiative corrections giving rise to renormalization depending on the extension of rationals.

2. Is this enough or should one modify the Taylor expansion of Kähler gauge potential A? The idea that $A_k dh^k$ is the basic entity suggests that one must form super Taylor series for both A_k and dh^k. This would give $A_k dh^k \rightarrow A_k \partial_k \delta h^k + A_l \partial_(\delta h^l))dh^k$. By performing an infinitesimal super gauge transformation $A_l \rightarrow A_l + \partial_l(A_l \delta h^k)$ one obtains $A_k \rightarrow A_k + J_{kl}\Delta h^k_s$, where Δh^k_s denotes super part of super-coordinate. The next term would vanish by covariant constancy of J_{kl}.

   The same trick could be applied to spinor connection and since also spinor curvature is covariantly constant, one would obtain only 2 terms in the expansion also in the continuum case. This provides an additional reason for why S $(= CP_2)$ must be constant curvature space.

   This applies also to M^4: in fact, twistor approach strongly suggests that also M^4 has the analog of covariantly constant Kähler form. This conforms with the breakdown of Poincare symmetry at M^8 level forced by the selection of the octonion structure. Poincare invariance is gained by integrating over the moduli space of octonion structures in the construction of scattering amplitudes. What is remarkable that one could use the irreps of Lorentz group at boundaries of CD, which for obvious reasons are much more natural than than those of Poincare group.

3. In the case of imbedding metric the same trick would give only the c-number term and only the gradients of imbedding space coordinates would contribute to the super counterpart of the induced metric. In this case general gauge super-coordinate transformation would allow to treat the components of metric as constants.