Article

Twistors in TGD

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Abstract

In twistor Grassmannian approach to $\mathcal{N}=4$ SYM, twistors are replaced with supertwistors and the extreme elegance of the description of various helicity states using twistor space wave functions and M^8-H duality suggest that super-twistors are realized at the level of both M^8 and H. M^8 supertwistors are naturally realized at the level of momentum space.

1. Basic problem of twistor approach and mass as a relative notion in TGD framework

In TGD framework M^8-H duality allows to geometrize the notion of super-twistor in the sense that different components of super-field correspond to components of super-octonion each of which corresponds to a space-time surfaces satisfying minimal surface equations with string world sheets as singularities - this is geometric counterpart for masslessness.

In TGD particles are massless in 8-D sense and in general massive in 4-D sense but 4-D twistors are needed also now so that a modification of twistor approach is needed. The incidence relation for twistors suggests the replacement of the usual twistors with either non-commutative quantum twistors or with octo-twistors. Quantum twistors could be associated with the space-time level description of massive particles and octo-twistors with the description at imbedding space level. A possible alternative interpretation of quantum spinors is in terms of quantum measurement theory with finite measurement resolution in which precise eigenstates as measurement outcomes are replaced with universal probability distributions defined by quantum group. This has also application in TGD inspired theory of consciousness.

Twistor lift of TGD involves representation of space-time surfaces as 6-surfaces in twistor space of H having structure of S^2 bundle over space-time surface resulting in dimensional reduction. These 6-surfaces would be holomorphic and thus minimal surfaces represented in terms of polynomials having same degree as the corresponding M^8 octonionic polynomial by number theoretic universality.

2. Criticizing the notion of twistor space of M^4

I have assumed that what I call geometric twistor space of M^4 is simply $M^4\times S^2$. One can however consider standard twistor space CP_3 with metric signature (3,-3) as an alternative. This option reproduces the nice results of the earlier approach but the philosophy is different: there is no fundamental length scale but the hierarchy of causal diamonds (CDs) predicted by zero energy ontology (ZEO) gives rise to the breaking of the exact scaling invariance of M^8 picture. M^4 in H would not be be replaced with conformally compactified M^4 (M^4_{conf}) but conformally compactified cd (cd_{conf}) for which a natural identification is as $CP_{2,h}$ obtained from CP_2 by replacing second complex coordinate replaced with hypercomplex coordinate. The sizes of twistor spaces of cd_{conf} using CP_2 size as unit would reflect the hierarchy of size scales for CDs. The consideration on the twistor space of M^8 in similar picture leads to the identification of corresponding twistor space as HP_3 - quaternionic variant of CP_3: the counterpart of CD_8 would be HP_2.

The outcome of octo-twistor approach together with M^8-H duality leads to a nice picture view about twistorial description of massive states based on quaternionic generalization of twistor (super-)Grassmannian approach with twistor space identificed as $HP_{3,h}$, the quaternionic variant of $CP_{3,h}$. A radically new view is that descriptions in terms of massive and massless states are alternative options, and correspond to two different alternative twistorial descriptions and leads to the interpretation of p-adic thermodynamics as completely universal massivation mechanism having nothing to do with dynamics.

As a side product emerges a deeper understanding of ZEO based quantum measurement theory and consciousness theory relying on the universal roots of octonionic polynomials of M^8, which are not 4-D

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but analogs of 6-D branes. By M^8-H duality the finite sub-groups of $SU(2)$ of McKay correspondence appear quite concretely in the description of the measurement resolution of 8-momentum.

The parallel progress in the understanding SUSY in TGD framework in turn led to the identification of the super-counterparts of M^8, H and of twistor spaces modifying dramatically the physical interpretation of SUSY. Super-spinors in twistor space would provide the description of quantum states. Quaternionic super Grassmannians would be involved with M^8 description.

Keywords: Twister space, SUSY, mass, TGD framework.

1 Introduction

This article was inspired by a longer paper "*TGD view about McKay Correspondence, ADE Hierarchy, Inclusions of Hyperfinite Factors, and Twistors*". I found it convenient to isolate the part of paper related to twistors. In twistor Grassmannian approach to $\mathcal{N}=4$ SYM twistors are replaced with supertwistors and the extreme elegance of the description of various helicity states using twistor space wave functions suggests that super-twistors are realized at the level of M^8 geometry. These supertwistors are realized at the level of momentum space.

In TGD framework M^8-H duality allows to geometrize the notion of super-twistor in the sense that different components of super-field correspond to components of super-octonion each of which corresponds to a space-time surfaces satisfying minimal surface equations with string world sheets as singularities - this is geometric counterpart for masslessness.

1.1 Basic problem of twistor approach and mass as a relative notion in TGD framework

The basic problem of the ordinary twistor approach is that the states must be massless in 4-D sense. In TGD framework particles would be massless in 8-D sense. This leads to alternative descriptions depending on the choice of $M^{\subset}M^8$ and the 4-D mass of the particle depends on the choice of M^4. For M^4_L description $M^4_L \subset M^8$ is chosen so that states are massless in 4-D sense, and the description at momentum space level would be in terms of products of ordinary M^4 twistors and CP_2 twistors. For M^4_T description particles are massive in 4-D sense. How to generalize the twistor description to 8-D case?

The incidence relation for twistors suggests the replacement of the usual twistors with either non-commutative quantum twistors or with octo-twistors. Quantum twistors could be associated with the space-time level description of massive particles and octo-twistors with the description at imbedding space level. A possible alternative interpretation of quantum spinors is in terms of quantum measurement theory with finite measurement resolution in which precise eigenstates as measurement outcomes are replaced with universal probability distributions defined by quantum group. This has also application in TGD inspired theory of consciousness.

1.2 Criticizing the notion of twistor space of M^4

Twistor lift of TGD involves representation of space-time surfaces as 6-surfaces in twistor space of H having structure of S^2 bundle over space-time surface resulting in dimensional reduction. These 6-surfaces would be holomorphic and thus minimal surfaces represented in terms of polynomials having same degree as the corresponding M^8 octonionic polynomial by number theoretic universality.

1. I have assumed that what I call geometric twistor space of M^4 is simply $M^4\times S^2$. It however turned out that one can consider standard twistor space CP_3 with metric signature (3,-3) as an alternative. This option reproduces the nice results of the earlier approach but the philosophy is different: there is no fundamental length scale but the hierarchy of causal diamonds (CDs) predicted by zero energy ontology (ZEO) gives rise to the breaking of the exact scaling invariance of M^8 picture. This forces

to modify M^8-H correspondence so that it involves map from M^4 to CP_3 followed by a projection to hyperbolic variant of CP_2.

M^4 in H would not be be replaced with conformally compactified M^4 (M^4_{conf}) but conformally compactified cd (cd_{conf}) for which a natural identification is as CP_2 with second complex coordinate replaced with hypercomplex coordinate. The sizes of twistor spaces of cd_{conf} using CP_2 size as unit would reflect the hierarchy of size scales for CDs. The consideration on the twistor space of M^8 in similar picture leads to the identification of corresponding twistor space as HP_3 - quaternionic variant of CP_3: the counterpart of CD_8 would be HP_2.

2. Octotwistors can be expressed as pairs of quaternionic twistors. Octotwistor approach suggests a generalization of twistor Grassmannian approach obtained by replacing the bi-spinors with complexified quaternions and complex Grassmannians with their quaternionic counterparts. Although TGD is not a quantum field theory, this proposal makes sense for cognitive representations identified as discrete sets of spacetime points with coordinates in the extension of rationals defining the adele [29] implying effective reduction of particles to point-like particles.

3. The outcome of octo-twistor approach together with M^8-H duality leads to a nice picture view about twistorial description of massive states based on quaternionic generalization of twistor (super-)Grassmannian approach with twistor space identificed as $HP_{3,h}$, the quaternionic variant of $CP_{3,h}$. A radically new view is that descriptions in terms of massive and massless states are alternative options, and correspond to two different alternative twistorial descriptions and leads to the interpretation of p-adic thermodynamics as completely universal massivation mechanism having nothing to do with dynamics.

   As a side product emerges a deeper understanding of ZEO based quantum measurement theory and consciousness theory relying on the universal roots of octonionic polynomials of M^8, which are not 4-D but analogs of 6-D branes. By M^8-H duality the finite sub-groups of $SU(2)$ of McKay correspondence appear quite concretely in the description of the measurement resolution of 8-momentum.

   The parallel progress in the understanding SUSY in TGD framework in turn led to the identification of the super-counterparts of M^8, H and of twistor spaces modifying dramatically the physical interpretation of SUSY. Super-spinors in twistor space would provide the description of quantum states. Quaternionic super Grassmannians would be involved with M^8 description.

1.3 What super-twistors are in TGD framework

What about super-twistors in TGD framework?

1. The parallel progress in the understanding SUSY in TGD framework [32] in turn led to the identification of the super-counterparts of M^8, H and of twistor spaces modifying dramatically the physical interpretation of SUSY. Super-spinors in twistor space would provide the description of quantum states. Super-Grassmannians would be involved with the construction of scattering amplitudes. Quaternionic super Grassmannians would be involved with M^8 description.

2. The great surprise from physics point of view is that in fermionic sector only quarks are allowed by $SO(1,7)$ triality and that anti-leptons are local 3-quark composites identifiable as spartners of quarks. Gauge bosons, Higgs and graviton would be also spartners and assignable to super-coordinates of imbedding space expressible as super-polynomials of quark oscillator operators. Super-symmetrization means also quantization of fermions allowing local many-quark states.

3. SUSY breaking would be caused by the same universal mechanism as ordinary massivation of massless states. The mass formulas would be supersymmetric but the choice of p-adic prime identifiable

as ramified prime of extension of rationals would depend on the state of super-multiplet. ZEO would make possible symmetry breaking without symmetry breaking as Wheeler might put it.

What about the interpretation of quantum twistors? They could make sense as 4-D space-time description analogous to description at space-time level. Now one can consider generalization of the twistor Grassmannian approach in terms of quantum Grassmannians.

2 Could standard view about twistors work at space-time level after all?

While asking what super-twistors in TGD might be, I became critical about the recent view concerning what I have called geometric twistor space of M^4 identified as $M^4 \times S^2$ rather than CP_3 with hyperbolic metric. The basic motivations for the identification come from M^8 picture in which there is number theoretical breaking of Poincare and Lorentz symmetries. Second motivation was that M^4_{conf} - the conformally compactified M^4 - identified as group $U(2)$ [1] (see `http://tinyurl.com/y35k5wwo`) assigned as base space to the standard twistor space CP_3 of M^4, and having metric signature (3,-3) is compact and is stated to have metric defined only modulo conformal equivalence class.

As found in the previous section, TGD strongly suggests that M^4 in $H = M^4 \times CP_2$ should be replaced with hyperbolic variant of CP_2, and it seems to me that these spaces are not identical. Amusingly, $U(2)$ and CP_2 are fiber and base in the representation of $SU(3)$ as fiber space so that the their identification does not seem plausible.

On can however ask whether the selection of a representative metric from the conformal equivalence class could be seen as breaking of the scaling invariance implied also by ZEO introducing the hierarchy of CDs in M^8. Could it be enough to have M^4 only at the level of M^8 and conformally compactified M^4 at the level of H? Should one have $H = cd_{conf} \times CP_2$? What cd_{conf} would be: is it hyperbolic variant of CP_2?

2.1 Getting critical

The only way to make progress is to become very critical now and then. These moments of almost despair usually give rise to a progress. At this time I got very critical about the TGD inspired identification of twistor spaces of M^4 and CP_2 and their properties.

2.1.1 Getting critical about geometric twistor space of M^4

Let us first discuss the recent picture and how to modify it so that it is consistent with the hierarchy of CDs. The key idea is that the twistor space and its base space represents CD so that one obtains scale hierarchy of twistor spaces as a realization of broken scale invariance giving rise to the p-adic length scale hierarchy.

1. I have identified the twistor space of M^4 simply as $T(M^4) = M^4 \times S^2$. The interpretation would be at the level of octonions as a product of M^4 and choices of M^2 as preferred complex sub-space of octonions with S^2 parameterizing the directions of spin quantization axes. Real octonion axis would correspond to time coordinate. One could talk about the space of of light-like directions. Light-like vector indeed defines M^2. This view could be defended by the breaking of both translation and Lorentz invariance in the octonionic approach due to the choice of M^2 and by the fact that it seems to work.

   Remark: $M^8 = M^4 \times E^4$ is complexified to M^8_c by adding a commuting imaginary unit i appearing in the extensions of rationals and ordinary M^8 represents its particular sub-space. Also in twistor approach one uses often complexified M^4.

2. The objection is that it is ordinary twistor space identifiable as CP_3 with (3,-3) signature of metric is what works in the construction of twistorial amplitudes. CP_3 has metric as compact space and coset space. Could this choice of twistor space make sense after all as geometric twistor space?

   Here one must pause and recall that the original key idea was that Poincare invariance is symmetry of TGD for $X^4 \subset M^4 \times CP_2$. Now Poincare symmetry has been transformed to a symmetry acting at the level of M^8 in the moduli space of octonion structures defined by the choice of the direction of octonionic real axis reducing Poincare group to $T \times SO(3)$ consisting of time translations and rotations. Fixing of M^2 reducs the group to $T \times SO(2)$ and twistor space can be seen as the space for selections of quantization axis of energy and spin.

3. But what about the space H? The first guess is $H = M^4_{conf} \times CP_2$. According to [1] one has $M^4_{conf} = U(2)$ such that $U(1)$ factor is time- like and $SU(2)$ factor is space-like. One could understand $M^4_{conf} = U(2)$ as resulting by addition and identification of metrically 2-D light-cone boundaries at $t = \pm\infty$. This is topologically like compactifying E^3 to S^3 and gluing the ends of cylinder $S^3 \times D^1$ together to the $S^3 \times S^1$.

   The conformally compactified Minkowski space M^4_{conf} should be analogous to base space of CP_3 regarded as bundle with fiber S^2. The problem is that one cannot imagine an analog of fiber bundle structure in CP_3 having $U(2)$ as base. The identification $H = M^4_{conf} \times CP_2$ does not make sense.

4. In ZEO based breaking of scaling symmetry it is CD that should be mapped to the analog of M^4_{conf} - call it cd_{conf}. The only candidate is $cd_{conf} = CP_2$ with one hypercomplex coordinate. To understand why one can start from the following picture. The light-like boundaries of CD are metrically equivalent to spheres. The light-like boundaries at $t = \pm\infty$ are identified as in the case of M^4_{conf}. In the case of CP_2 one has 3 homologically trivial spheres defining coordinate patches. This suggests that cd_{conf} is simply CP_2 with second complex coordinate made hypercomplex. M^4 and E^4 differ only by the signature and so would do cd_{conf} and CP_2.

   The twistor spheres of CP_3 associated with points of M^4 intersect at point if the points differ by light-like vector so that one has and singular bundle structure. This structure should have analog for the compactification of CD. CP_3 has also bundle structure $CP_3 \to CP_2$. The S^2 fibers and base are homologically non-trivial and complex analogs of mutually orthogonal line and plane and intersect at single point. This defines the desired singular bundle structure via the assignment of S^2 to each point of CP_2.

   The M^4 points must belong to the interior of cd and this poses constraints on the distance of M^4 points from the tips of cd. One expects similar hierarchy of cds at the level of momentum space.

5. In this picture $M^4_{conf} = U(2)$ could be interpreted as a base space for the space of CDs with fixed direction of time axis identified as direction of octonionic real axis associated with various points of M^4 and therefore of M^4_{conf}. For Euclidian signature one would have base and fiber of the automorphism sub-group $SU(3)$ regarded as $U(2)$ bundle over CP_2: now one would have CP_2 bundle over $U(2)$. This is perhaps not an accident, and one can ask whether these spaces could be interpreted as representing local trivialization of $SU(3)$ as $U(2) \times CP_2$. This would give to metric cross terms between $U(2)$ and CP_2.

6. The proposed identification can be tested by looking whether it generalizes. What the twistor space for entire M^8 would be? $cd = CD_4$ is replaced with CD_8 and the discussion of the preceding chapter demonstrated that the only possible identification of the twistor space is now is as the 12-D hyperbolic variant of HP_3 whereas $CD_{8,conf}$ would correspond to 8-D hyperbolic variant of HP_2 analogous to hyperbolic variant of CP_2.

The outcome of these considerations is surprising.

1. One would have $T(H) = CP_3 \times F$ and $H = CP_{2,H} \times CP_{2,E}$ where first $CP_{2,H}$ has hyperbolic metric with metric signature $(1, -3)$ having M^4 as tangent space so that the earlier picture can be understood as an approximation. This would reduce the construction of preferred extremals of 6-D Kähler action in $T(H)$ to a construction of polynomial holomorphic surfaces and also the minimal surfaces with singularities at string world sheets should result as bundle projection. Since $M^8 - H$ duality must respect algebraic dynamics the maximal degree of the polynomials involved must be same as the degree of the octonionic polynomial in M^8.

2. The hyperbolic variant Kähler form and also spinor connection of hyperbolic CP_2 brings in new physics beyond standard model. This Kähler form would serve as the analog of Kähler form assigned to M^4 earlier, and suggested to explain the observed CP breaking effects and matter antimatter asymmetry for which there are two explanations [32].

Some comments about the Minkowskian signature of the hyperbolic counterparts of CP_3 and CP_2 are in order.

1. Why the metric of CP_3 could not be Euclidian just as the metric of F? The basic objection is that propagation of fields is not possible in Euclidian signature and one completely loses the earlier picture provided by $M^4 \times CP_2$. The algebraic dynamics in M^8 picture can hardly replace it.

2. The map assigning to the point M^4 a point of CP_3 involves Minkowskian sigma matrices but it seems that the Minkowskian metric of CP_3 is not explicitly involved in the construction of scattering amplitudes. Note however that the antisymmetric bi-spinor metric for the spin 1/2 representation of Lorentz group and its conjugate bring in the signature. $U(2, 2)$ as representation of conformal symmetries suggests $(2, 2)$ signature for 8-D complex twistor space with 2+2 complex coordinates representing twistors.

   The signature of CP_3 metric is not explicitly visible in the construction of twistor amplitudes but analytic continuations are carried out routinely. One has also complexified M^4 and M^8 and one could argue that the problems disappear. In the geometric situation the signatures of the subspaces differ dramatically. As already found, analytic continuation could allow to define the variants of twistor spaces elegantly by replacing a complex coordinate with a hyperbolic one.

   Remark: For E^4 CP_3 is Euclidian and if one has $E^4_{conf} = U(2)$, one could think of replacing the Cartesian product of twistor spaces with $SU(3)$ group having $M^4_{conf} = U(2)$ as fiber and CP_2 as base. The metric of $SU(3)$ appearing as subgroup of quaternionic automorphisms leaving $M^4 \subset M^8$ invariant would decompose to a sum of M^4_{conf} metric and CP_2 metric plus cross terms representing correlations between the metrics of M^4_{conf} and CP_2. This is probably mere accident.

2.1.2 $M^8 - H$ duality and twistor space counterparts of space-time surfaces

It seems that by identifying $CP_{3,h}$ as the twistor space of M^4, one could develop $M^8 - H$ duality to a surprisingly detailed level from the conditions that the dimensional reduction guaranteed by the identification of the twistor spheres takes place and the extensions of rationals associated with the polynomials defining the space-time surfaces at M^8- and twistor space sides are the same. The reason is that minimal surface conditions reduce to holomorphy meaning algebraic conditions involving first partial derivatives in analogy with algebraic conditions at M^8 side but involving no derivatives.

1. The simplest identification of twistor spheres is by $z_1 = z_2$ for the complex coordinates of the spheres. One can consider replacing z_i by its Möbius transform but by a coordinate change the condition reduces to $z_1 = z_2$.

2. At M^8 side one has either $RE(P) = 0$ or $IM(P) = 0$ for octonionic polynomial obtained as continuation of a real polynomial P with rational coefficients giving 4 conditions (RE/IM denotes

real/imaginary part in quaternionic sense). The condition guarantees that tangent/normal space is associative.

Since quaternion can be decomposed to a sum of two complex numbers: $q = z_1 + Jz_2$ $RE(P) = 0$ correspond to the conditions $Re(RE(P)) = 0$ and $Im(RE(P)) = 0$. $IM(P) = 0$ in turn reduces to the conditions $Re(IM(P)) = 0$ and $Im(IM(P)) = 0$.

3. The extensions of rationals defined by these polynomial conditions must be the same as at the octonionic side. Also algebraic points must be mapped to algebraic points so that cognitive representations are mapped to cognitive representations. The counterparts of both $RE(P) = 0$ and $IM(P) = 0$ should be satisfied for the polynomials at twistor side defining the same extension of rationals.

4. $M^8 - H$ duality must map the complex coordinates $z_{11} = Re(RE)$ and $z_{12} = Im(RE)$ ($z_{21} = Re(IM)$ and $z_{22} = Im(IM)$) at M^8 side to complex coordinates u_{i1} and u_{i2} with $u_{i1}(0) = 0$ and $u_{i2}(0) = 0$ for $i = 1$ or $i = 2$, at twistor side.

   Roots must be mapped to roots in the same extension of rationals, and no new roots are allowed at the twistor side. Hence the map must be linear: $u_{i1} = a_iz_{i1} + b_iz_{i2}$ and $u_{i2} = c_iz_{i1} + d_iz_{i2}$ so that the map for given value of i is characterized by SL(2,Q) matrix $(a_i, b_i; c_i, d_i)$.

5. These conditions do not yet specify the choices of the coordinates (u_{i1}, u_{i2}) at twistor side. At CP_2 side the complex coordinates would naturally correspond to Eguchi-Hanson complex coordinates (w_1, w_2) determined apart from color $SU(3)$ rotation as a counterpart of $SU(3)$ as sub-group of automorphisms of octonions.

   If the base space of the twistor space $CP_{3,h}$ of M^4 is identified as $CP_{2,h}$, the hyper-complex counterpart of CP_2, the analogs of complex coordinates would be (w_3, w_4) with w_3 hypercomplex and w_4 complex. A priori one could select the pair (u_{i1}, u_{i2}) as any pair $(w_{k(i)}, w_{l(i)})$, $k(i) \neq l(i)$. These choices should give different kinds of extremals: such as CP_2 type extremals, string like objects, massless extremals, and their deformations.

String world sheet singularitees and world-line singularities as their light-like boundaries at the light-like orbits of partonic 2-surfaces are conjectured to characterize preferred extremals as surfaces of H at which there is a transfer of canonical momentum currents between Kähler and volume degrees of freedom so that the extremal is not simultaneously an extremal of both Kähler action and volume term as elsewhere. What could be the counteparts of these surfaces in M^8?

1. The interpretation of the pre-images of these singularities in M^8 should be number theoretic and related to the identification of quaternionic imaginary units. One must specify two non-parallel octonionic imaginary units e^1 and e^2 to determine the third one as their cross product $e^3 = e^1 \times e^2$. If e^1 and e^2 are parallel at a point of octonionic surface, the cross product vanishes and the dimension of the quaternionic tangent/normal space reduces from $D = 4$ to $D = 2$.

2. Could string world sheets/partonic 2-surfaces be images of 2-D surfaces in M^8 at which this takes place? The parallelity of the tangent/normal vectors defining imaginary units e_i, $i = 1, 2$ states that the component of e_2 orthogonal to e_1 vanishes. This indeed gives 2 conditions in the space of quaternionic units. Effectively the 4-D space-time surface would degenerate into 2-D at string world sheets and partonic 2-surfacesa as their duals. Note that this condition makes sense in both Euclidian and Minkowskian regions.

3. Partonic orbits in turn would correspond surfaces at which the dimension reduces to D=3 by light-likeness - this condition involves signature in an essential manner - and string world sheets would have 1-D boundaries at partonic orbits.

2.1.3 Getting critical about implicit assumptions related to the twistor space of CP_2

One can also criticize the earlier picture about implicit assumptions related the twistor spaces of CP_2.

1. The possibly singular decomposition of F to a product of S^2 and CP_2 would has a description similar to that for CP_3. One could assign to each point of CP_2 base homologically non-trivial sphere intersecting it orthogonally.

2. I have assumed that the twistor space $T(CP_2) = F = SU(3)/U(1) \times U(1)$ allows Kaluza-Klein type metric meaning that the metric decomposes to a sum of the metrics assignable to the base CP_2 and fiber S^2 plus cross terms representing interaction between these degrees of freedom. It is easy to check that this assumption holds true for Hopf fibration $S^3 \to S^2$ having circle $U(1)$ as fiber (see `http://tinyurl.com/qbvktsx`). If Kaluza-Klein picture holds true, the metric of F would decompose to a sum of CP_2 metric and S^2 metric plus cross terms representing correlations between the metrics of CP_2 and S^2.

3. One should demonstrate that $F = SU(3)/U(1) \times U(1)$ has metric with the expected Kaluza-Klein property. One can represent $SU(3)$ matrices as products XYZ of 3 matrices. X represents a point of base space CP_2 as matrix, Y represents the point of the fiber $S^2 = U(2)/U(1) \times U(1)$ of F in similar manner as $U(2)$ matrix, and the Z represents $U(1) \times U(1)$ element as diagonal matrix [1](see `http://tinyurl.com/y6c3pp2g`).

   By dropping $U(1) \times U(1)$ matrix one obtains a coordinatization of F. To get the line element of F in these coordinates one could put the coordinate differentials of $U(1) \times U(1)$ to zero in an expression of $SU(3)$ line element. This should leave sum of the metrics of CP_2 and S^2 with constant scales plus cross terms. One might guess that the left- and righ-invariance of the $SU(3)$ metric under $SU(3)$ implies KK property.

Also CP_3 should have the KK structure if one wants to realize the breaking of scaling invariance as a selection of the scale of the conformally compactified M^4. In absence of KK structure the space-time surface would depend parametrically on the point of the twistor sphere S^2.

2.2 The nice results of the earlier approach to M^4 twistorialization

The basic nice results of the earlier picture should survive in the new picture.

1. Central for the entire approach is twistor lift of TGD replacing space-time surfaces with 6-D surfaces in 12-D $T(M^4) \times T(CP_2)$ having space-time surfaces as base and twistor sphere S^2 as fiber. Dimensional reduction identifying twistor spheres of $T(M^4)$ an $T(CP_2)$ and makes these degrees of freedom non-dynamical.

2. Dimensionally reduced action 6-D Kähler action is sum of 4-D Kähler action and a volume term coming from S^2 contribution to the induced Kähler form. On interpretation is as a generalization of Maxwell action for point like charge by making particle a 3-surface.

   The interpretation of volume term is in terms of cosmological constant. I have proposed that a hierarchy of length scale dependent cosmological constants emerges. The hierarchy of cosmological constants would define the running length scale in coupling constant evolution and would correspond to a hierarchy of preferred p-aic length scales with preferred p-adic primes identified as ramified primes of extension of rationals.

3. The twistor spheres associated $M^4 \times S^2$ and F were assumed to have same radii and most naturally same Euclidian signature: this looks very nice since there would be only single fundamental length equal to CP_2 radius determining the radius of its twistor sphere. The vision to be discussed would

be different. There would be no fundamental scale and length scales would emerge through the length scale hierarchy assignable to CDs in M^8 and mapped to length scales for twistor spaces.

The identification of twistor spheres with same radius would give only single value of cosmological constant and the problem of understanding the huge discrepancy between empirical value and its naive estimate would remain. I have argued that the Kähler forms and metrics of the two twistor spheres can be rotated with respect to each other so that the induced metric and Kähler form are rotated with respect to each other, and the magnetic energy density assignable to the sum of the induced Kähler forms is not maximal.

The definition of Kähler forms involving preferred coordinate frame would gives rise to symmetry breaking. The essential element is interference of real Kähler forms. If the signatures of twistor spheres were opposite, the Kähler forms differ by imaginary unit and the interference would not be possible.

Interference could give rise to a hierarchy of values of cosmological constant emerging as coefficient of the Kähler magnetic action assignable to $S^2(X^4)$ and predict length scale dependent value of cosmological constant and resolve the basic problem related to the extremely small value of cosmological constant.

4. One could criticize the allowance of relative rotation as adhoc: note that the resulting cosmological constant becomes a function depending on S^2 point. For instance, does the rotation really produce preferred extremals as minimal surfaces extremizing also Kähler action except at string world sheets? Each point of S^2 would correspond to space-time surface X^4 with different value of cosmological constant appearing as a parameter. Moreover, non-trivial relative rotation spoils the covariant constancy and $J^2(S^2) = -g(S^2)$ property for the S^2 part of Kähler form, and that this does not conform with the very idea of twistor space.

5. One nice implication would be that space-time surfaces would be minimal surfaces apart from 2-D string world sheet singularities at which there is a transfer of canonical momentum currents between Kähler and volume degrees of freedom. One can also consider the possibility that the minimal surfaces correspond to surfaces give as roots of 3 polynomials of hypercomplex coordinate of M^2 and remaining complex coordinates.

   Minimal surface property would be direct translation of masslessness and conform with the twistor view. Singular surfaces would represent analogs of Abelian currents. The universal dynamics for minimal surfaces would be a counterpart for the quantum criticality. At M^8 level the preferred complex plane M^2 of complexified octonions would represent the singular string world sheets and would be forced by number theory.

   Masslessness would be realized as generalized holomorphy (allowing hyper-complexity in M^2 plane) as proposed in the original twistor approach but replacing holomorphic fields in twistor space with 6-D twistor spaces realized as holomorphic 6-surfaces.

2.3 ZEO and twistorialization as manners to introduce scales in M^8 physics

M^8 physics as such has no scales. One motivation for ZEO is that it brings in the scales as sizes of causal diamonds (CDs).

2.3.1 ZEO generates scales in M^8 physics

Scales are certainly present in physics and must be present also in TGD Universe.

1. In TGD Universe CP_2 scale plays the role of fundamental length scale, there is also the length scale defined by cosmological constant and the geometric mean of these two length scales defining a scale of order 10^{-4} meters emerging in the earlier picture and suggesting a biological interpretation.

The fact that conformal inversion $m^k \rightarrow R^2 m^k/a^2$, $a^2 = m^k m_k$ is a conformal transformation mapping hyperboloids with $a \geq R$ and $a \leq R$ to each other, suggests that one can relate CP_2 scale and cosmological scale defined by Λ by inversion so that cell length scale would define one possible radius of cd_{conf}.

2. In fact, if one has $R(cd_{conf}) = x \times R(CP_2)$ one obtains by repeated inversions a hierarchy $R(k) = x^k R$ and for $x = \sqrt{p}$ one obtains p-adic length scale hierarchy coming as powers of $\sqrt{p}$, which can be also negative. This suggests a connection with p-adic length scale hypothesis and connections between long length scale and short length scale physics: they could be related by inversion. One could perhaps see Universe as a kind of Leibnizian monadic system in which monads reflect each other with respect to hyperbolic surfaces $a = constant$. This would conform with the holography.

3. Without additional assumptions there is a complete scaling invariance at the level of M^8. The scales could come from the choice of 8-D causal diamond CD_8 as intersection of 8-D future and past directed light-cones inducing choice of cd in M^4. CD serves as a correlate for the perceptive field of a conscious entity in TGD inspired theory of consciousness and is crucial element of zero energy ontology (ZEO) allowing to solve the basic problem of quantum measurement theory.

2.3.2 Twistorial description of CDs

Could the map of the surfaces of 4-surfaces of M^8 to $cd_{conf} \times CP_2$ by a modification of $M^8 - H$ correspondence allow to describe these scales? If so, compactification via twistorialization and $M^8 - H$ correspondence would be the manner to describe these scales as something emergent rather than fundamental.

1. The simplest option is that the scale of cd_{conf} corresponds to that of CD_8 and CD_4. One should also understand what CP_2 scale corresponds. The simplest option is that CP_2 scale defines just length unit since it is difficult to imagine how this scale could appear at M^8 level. cd_{conf} scale squared would be multiple or CP_2 scale squared, say prime multiple of it, and assignable to ramified primes of extension of rationals. Inversions would produce further scales. Inversion would allow kind of hologram like representation of physics in long length scales in arbitrary short length scales and vice versa.

2. The compactness of cd_{conf} corresponds to periodic time assignable to over-critical cosmologies starting with big bang and ending with big crunch. Also CD brings in mind over-critical cosmology, and one can argue that the dynamics at the level of cd_{conf} reflects the dynamics of ZEO at the level of M^8.

2.3.3 Modification of H and $M^8 - H$ correspondence

It is often said that the metric of M^4_{conf} is defined only modulo conformal scaling factor. This would reflect projectivity. One can however endow projective space CP_3 with a metric with isometry group $SU(2,2)$ and the fixing of the metric is like gauge choice by choosing representative in the projective equivalence class. Thus CP_3 with signature (3,-3) might perhaps define geometric twistor space with base cd_{conf} rather than M^4_{conf} very much like the twistor space $T(CP_2) = F = SU(3)/U(1) \times U(1)$ at the level. Second projection would be to M^4 and map twistor sphere to a point of M^4. The latter bundle structure would be singular since for points of M^4 with light-like separation the twistor spheres have a common point: this is an essential feature in the construction of twistor amplitudes.

New picture requires a modification of the view about H and about $M^8 - H$ correspondence.

1. H would be replaced with $cd_{conf} \times CP_2$ and the corresponding twistor space with $CP_3 \times F$. $M^8 - H$ duality involves the decomposition $M^2 \subset M^4 \subset M^8 = M^4 \times CP_2$, where M^4 is quaternionic subspace containing preferred place M^2. The tangent or normal space of X^4 would be characterized by

a point of CP_2 and would be mapped to a point of CP_2 and the point of CP_2 - or rather point plus the space S^2 or light-like vectors characterizing the choices of M^2 - would mapped to the twistor sphere S^2 of CP_3 by the standard formulas.

$S^2(cd_{conf})$ would correspond to the choices of the direction of preferred octonionic imaginary unit fixing M^2 as quantization axis of spin and $S^2(CP_2)$ would correspond to the choice of isospin quantization axis: the quantization axis for color hyperspin would be fixed by the choice of quaternionic $M^4 \subset M^8$. Hence one would have a nice information theoretic interpretation.

2. The M^4 point mapped to twistor sphere $S^2(CP_3)$ would be projected to a point of cd_{conf} and define $M^8 - H$ correspondence at the level of M^4. This would define compactification and associate two scales with it. Only the ratio $R(cd_{conf})/R(CP_2)$ matters by the scaling invariance at M^8 level and one can just fixe the scale assignable to $T(CP_2)$ and call it CP_2 length scale.

One should have a concrete construction for the hyperbolic variants of CP_n.

1. One can represent Minkowski space and its variants with varying signatures as sub-spaces of complexified quaternions, and it would seem that the structure of sub-space must be lifted to the level of the twistor space. One could imagine variants of projective spaces CP_n, $n = 2, 3$ as and HP_n, $n = 2, 3$. They would be obtained by multiplying imaginary quaternionic unit I_k with the imaginary unit i commuting with quaternionic units. If the quaternions λ involved with the projectivization $(q_1, ..., q_n) \equiv \lambda(q_1, ..., q_n)$ are ordinary quaternions, the multiplication respects the signature of the subspace. By non-commutativity of quaternions one can talk about left- and right projective spaces.

2. One would have extremely close correspondence between M^4 and CP_2 degrees of freedom reflecting the $M^8 - H$ correspondence. The projection $CP_3 \to CP_2$ for E^4 would be replaced with the projection for the hyperbolic analogs of these spaces in the case of M^4. The twistor space of M^4 identified as hyperbolic variant of CP_3 would give hyperbolic variant of CP_2 as conformally compactified cd. The flag manifold $F = SU(3)/U(1) \times U(1)$ as twistor space of CP_2 would also give CP_2 as base space.

The general solution of field equations at the level of $T(H)$ would correspond to holomorphy in general sense for the 6-surfaces defined by 3 vanishing conditions for holomorphic functions - 6 real conditions. Effectively this would mean the knowledge of the exact solutions of field equations also at the level of H: TGD would be an integrable theory. Scattering amplitudes would in turn constructible from these solutions using ordinary partial differential equations [32].

1. The first condition would identify the complex coordinates of $S^2(cd_{conf})$ and $S^2(CP_2)$: here one cannot exclude relative rotation represented as a holomorphic transformation but for $R(cd_{conf}) \gg R(CP_2)$ the effect of the rotation is small.

2. Besides this there would be vanishing conditions for 2 holomorphic polynomials. The coordinate pairs corresponding to $M^2 \subset M^4$ would correspond to hypercomplex behavior with hyper complex coordinate $u = \pm t - z$. t and z could be assigned with $U(1)$ fibers of Hopf fibrations $SU(2) \to S^2$.

3. The octonionic polynomial $P(o)$ of degree $n = h_{eff}/h_0$ with rational coefficients fixes the extension of rationals and since the algebraic extension should be same at both sides, the polynomials in twistor space should have same degree. This would give enormous boos concerning the understanding of the proposed cancellation of fermionic Wick contractions in SUSY scattering amplitudes forced by number theoretic vision [32].

2.3.4 Possible problems related to the signatures

The different signatures for the metrics of the twistor spheres of cd_{conf} and CP_2 can pose technical problems.

1. Twistor lift would replace X^4 with 6-D twistor space X^6 represented as a 6-surface in $T(M^4) \times T(CP_2)$. X^6 is defined by dimensional reduction in which the twistor spheres $S^2(cd_{conf})$ and $S^2(CP_2)$ are identified and define the twistor sphere $S^2(X^4)$ of X^6 serving as a fiber whereas space-time surface X^4 serves as a base. The simplest identification is as $(\theta,\phi)_{S^2(M^4)} = (\theta,\phi)_{S^2(CP_2)}$: the same can be done for the complex coordinates $z_{S^2(M^4_{conf})} = z_{S^2(CP_2))})$. An open question is whether a Möbius transformation could relate the complex coordinates. The metrics of the spheres are of opposite sign and differ only by the scaling factors $R^2(cd_{conf})$ and $R^2(CP_2)$.

2. For cd_{conf} option the signatures of the 2 twistor spheres would be opposite (time-like for cd_{conf}). For $R(cd_{conf})/R(CP_2) = 1$. $J^2 = -g$ is the only consistent option unless the signature of space is not totally positive or negative and implies that the Kähler forms of the two twistor spheres differ by i. The magnetic contribution from $S^2(X^4)$ would give rise to an infinite value of cosmological constant proportional to $1/\sqrt{g_2}$, which would diverge $R(cd_{conf})/R(CP_2) = 1$. There is however no need to assume this condition as in the original approach.

2.4 Hierarchy of length scale dependent cosmological constants in twistorial description

At the level of M^8 the hierarchy of CDs defines a hierarchy of length scales and must correspond to a hierarchy of length scale dependent cosmological constants. Even fundamental scales would emerge.

1. If one has $R(cd_{conf})/R(CP_2) >> 1$ as the idea about macroscopic cd_{conf} would suggest, the contribution of $S^2(cd_{conf})$ to the cosmological constant dominates and the relative rotation of metrics and Kähler form cannot affect the outcome considerably. Therefore different mechanism producing the hierarchy of cosmological constants is needed and the freedom to choose rather freely the ratio $R(cd_{conf})/R(CP_2)$ would provide the mechanism. What looked like a weakness would become a strength.

2. $S^2(cd_{conf}$ would have time-like metric and could have large scale. Is this really acceptable? Dimensional reduction essential for the twistor induction however makes $S^2(cd_{conf})$ non-dynamical so that time-likeness would not be visible even for large radii of $S^2(cd_{conf})$ expected if the size of cd_{conf} can be even macroscopic. The corresponding contribution to the action as cosmological constant has the sign of magnetic action and also Kähler magnetic energy is positive. If the scales are identical so that twistor spheres have same radius, the contributions to the induced metric cancel each other and the twistor space becomes metrically 4-D.

3. At the limit $R(cd_{conf}) \to RCP_2)$ cosmological constant coming from magnetic energy density diverges for $J^2 = -G$ option since it is proportional to $1/\sqrt{g_2}$. Hence the scaling factors must be different. The interpretation is that cosmological constant has arbitrarily large values near CP_2 length scale. Note however that time dependence is replaced with scale dependence and space-time sheets with different scales have only wormhole contacts.

It would seem that this approach could produce the nice results of the earlier approach. The view about how the hierarchy of cosmological constants emerges would change but the idea about reducing coupling constant evolution to that for cosmological constant would survive. The interpretation would be in terms of the breaking of scale invariance manifesting as the scales of CDs defining the scales for the twistor spaces involved. New insights about p-adic coupling constant evolution emerge and one finds a new "must" for ZEO. $H = M^4 \times CP_2$ picture would emerge as an approximation when cd_{conf} is replaced with its tangent space M^4. The consideration of the quaternionic generalization of twistor space suggests natural identification of the conformally compactified twistor space as being obtained from CP_2 by making second complex coordinate hyperbolic. This need not conform with the identification as $U(2)$.

3 How to generalize twistor Grassmannian approach in TGD framework?

One should be able to generalize twistor Grassmannian approach in TGD framework. The basic modification is replacement of 4-D light-like momenta with their 8-D counterparts. The octonionic interpretation encourages the idea that twistor approach could generalize to 8-D context. Higher-dimensional generalizations of twistors have been proposed but the basic problem is that the index raising and lifting operations for twistors do not generalize (see `http://tinyurl.com/y24lkwce`).

1. For octonionic twistors as pairs of quaternionic twistors index raising would not be lost working for M_T option and light-like M^8 momenta can be regarded sums of M^4_T and E^4 parts as also twistors. Quaternionic twistor components do not commute and this is essential for incidence relation requiring also the possibility to raise or lower the indices of twistors. Ordinary complex twistor Grassmannians would be replaced with their quaternionic countparts. The twistor space as a generalization of CP_3 would be 3-D quaternionic projective space $T(M^8) = HP_3$ with Minkowskian signature (6,6) of metric and having real dimension 12 as one might expect.

   Another option realizing non-commutativity could be based on the notion of quantum twistor to be also discussed.

2. Second approach would rely on the identification of $M^4 \times CP_2$ twistor space as a Cartesian product of twistor spaces of M^4 and CP_2. For this symmetries are not broken, $M^4_L \subset M^8$ depends on the state and is chosen so that the projection of M^8 momentum is light-like so that ordinary twistors and CP_2 twistors should be enough. $M^8 - H$ relates varying M^4_L based and M^4_T based descriptions.

3. The identification of the twistor space of M^4 as $T(M^4) = M^4 \times S^2$ can be motivated by octonionic considerations but might be criticized as non-standard one. The fact that quaternionic twistor space HP_3 looks natural for M^8 forces to ask whether $T(M^4) = CP_3$ endowed with metric having signature (3,3) could work in the case of M^4. In the sequel also a vision based on the identification $T(M^4) = CP_3$ endowed with metric having signature (3,3) will be discussed.

3.1 Twistor lift of TGD at classical level

In TGD framework twistor structure is generalized [26, 27, 24, 31]. The inspiration for TGD approach to twistorialization has come from the work of Nima Arkani-Hamed and colleagues [13, 7, 8, 10, 16, 14, 4]. The new element is the formulation of twistor lift also at the level of classical dynamics rather than for the scattering amplitudes only [26, 24, 27, 31].

1. The 4-D light-like momenta in ordinary twistor approach are replaced by 8-D light-like momenta so that massive particles in 4-D sense become possible. Twistor lift of TGD takes places also at the space-time level and is geometric counterpart for the Penrose's replacement of massless fields with twistors. Roughly, space-time surfaces are replaced with their 6-D twistor spaces represented as 6-surfaces. Space-time surfaces as preferred extremals are minimal surfaces with 2-D string world sheets as singularities. This is the geometric manner to express masslessness. X^4 is simultaneously also extremal of 4-D Kähler action outside singularities: this realizes preferred extremal property.

2. One can say that twistor structure of X^4 is induced from the twistor structure of $H = M^4 \times CP_2$, whose twistor space $T(H)$ is the Cartesian product of geometric twistor space $T(M^4) = M^4 \times CP_1$

and $T(CP_2) = SU(3)/U(1) \times U(1)$. The twistor space of M^4 assigned to momenta is usually taken as a variant of CP_3 with metric having Minkowski signature and both CP_1 fibrations appear in the more precise definition of $T(M^4)$. Double fibration [15] (see `http://tinyurl.com/yb4bt74l`) means that one has fibration from $M^4 \times CP_1$ - the trivial CP_1 bundle defining the geometric twistor space to the twistors space identified as complex projective space defining conformal compactification of M^4. Double fibration is essential in the twistorialization of TGD [25].

3. The basic objects in the twistor lift of classical TGD are 6-D surfaces in $T(H)$ having the structure of twistor space in the sense that they are CP_1 bundles having X^4 as base space. Dimensional reduction to CP_1 bundle effectively eliminates the dynamics in CP_1 degrees of freedom and its only remnant is the value of cosmological constant appearing as coefficient of volume term of the dimensionally reduced action containing also 4-D Kähler action. Cosmological term depends on p-adic length scales and has a discrete spectrum [31, 30].

CP_1 has also an interpretation as a projective space constructed from 2-D complex spinors. Could the replacement of these 2-spinors with their quantum counterparts defining in turn quantum CP_1 realize finite quantum measurement resolution in M^4 degrees of freedom? Projective invariance for the complex 2-spinors would mean that one indeed has effectively CP_1.

3.2 Octonionic twistors or quantum twistors as twistor description of massive particles

For M^4_T option the particles are massive and the one encounters the problem whether and how to generalize the ordinary twistor description.

3.3 Basic facts about twistors and bi-spinors

It is convenient to start by summarizing the basic facts about bi-spinors and their conjugates allowing to express massless momenta as $p^{aa'} = \lambda^a \tilde{\lambda}^{a'}$ with $\tilde{\lambda}$ defined as complex conjugate of λ and having opposite chirality (see `http://tinyurl.com/y6bnznyn`).

1. When λ is scaled by a complex number $\tilde{\lambda}$ suffers an opposite scaling. The bi-spinors allow the definition of various inner products

$$\begin{aligned} \langle \lambda, \mu \rangle &= \epsilon_{ab} \lambda^a \mu^b \ , \\ \left[\tilde{\lambda}, \tilde{\mu}\right] &= \epsilon_{a'b'} \tilde{\lambda}^{a'} \tilde{\mu}^{b'} \ , \\ p \cdot q &= \langle \lambda, \mu \rangle \left[\tilde{\lambda}, \tilde{\mu}\right] \ , \ (q_{aa'} = \mu_a \tilde{\mu}_{a'}) \ . \end{aligned} \tag{3.1}$$

2. Spinor indices are lowered and raised using antisymmetric tensors $\epsilon^{\alpha\beta}$ and $\epsilon_{\dot{\alpha}\dot{\beta}}$. If the particle has spin one can assign it a positive or negative helicity $h = \pm 1$. Positive helicity can be represented by introducing artitrary negative (positive) helicity bispinor μ_a ($\mu_{a'}$) not parallel to λ_a ($\mu_{a'}$) so that one can write for the polarization vector

$$\begin{aligned} \epsilon_{aa'} &= \frac{\mu_a \tilde{\lambda}_{a'}}{\langle \mu, \lambda \rangle} \ , \ \text{positive helicity} \ , \\ \epsilon_{aa'} &= \frac{\lambda_a \tilde{\mu}_{a'}}{\left[\tilde{\mu}, \tilde{\lambda}\right]} \ , \ \text{negative helicity} \ . \end{aligned} \tag{3.2}$$