Exploration

TGD View on McKay Correspondence, ADE Hierarchy & Inclusions of Hyperfinite Factors

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Abstract

There are two mysterious looking correspondences involving ADE groups. McKay correspondence between McKay graphs characterizing tensor products for finite subgroups of $SU(2)$ and Dynkin diagrams of affine ADE groups is the first one. The correspondence between principal diagrams characterizing inclusions of hyper-finite factors of type II_1 (HFFs) with Dynkin diagrams for a subset of ADE groups and Dynkin diagrams for affine ADE groups is the second one. These correspondences are discussed from number theoretic point of view suggested by TGD and based on the interpretation of discrete subgroups of $SU(2)$ as subgroups of the covering group of quaternionic automorphisms $SO(3)$ (analog of Galois group) and generalization of these groups to semi-direct products $Gal(K) \triangleleft SU(2)_K$ of Galois group for extension K of rationals with the discrete subgroup $SU(2)_K$ of $SU(2)$ with representation matrix elements in K. The identification of the inclusion hierarchy of HFFs with the hierarchy of extensions of rationals and their Galois groups is proposed. A further mystery whether $Gal(K) \triangleleft SU(2)_K$ could give rise to quantum groups or affine algebras. In TGD framework the infinite-D group of isometries of "world of classical worlds" (WCW) is identified as an infinite-D symplectic group for which the discrete subgroups characterized by K have infinite-D representations so that hyper-finite factors are natural for their representations. Symplectic algebra SSA allows hierarchy of isomorphic sub-algebras SSA_n. The gauge conditions for SSA_n and $[SSA_n, SSA]$ would define measurement resolution giving rise to hierarchies of inclusions and ADE type Kac-Moody type algebras or quantum algebras representing symmetries modulo measurement resolution. A concrete realization of ADE type Kac-Moody algebras is proposed. It relies on the group algebra of $Gal(K) \triangleleft SU(2)_K$ and free field representation of ADE type Kac-Moody algebra identifying the free scalar fields in Kac-Moody Cartan algebra as group algebra elements defined by the traces of representation matrices (characters). A possible alternative interpretation of quantum spinors is in terms of quantum measurement theory with finite measurement resolution in which precise eigenstates as measurement outcomes are replaced with universal probability distributions defined by quantum group. This has also application in TGD inspired theory of consciousness.

Keywords: McKay Correspondence, ADE hierarchy, hyperfinite factors, TGD View.

1 Introduction

There are two mysterious looking correspondences involving ADE groups. McKay correspondence between McKay graphs characterizing tensor products for finite subgroups of $SU(2)$ and Dynkin diagrams of affine ADE groups is the first one. The correspondence between principal diagrams characterizing inclusions of hyper-finite factors of type II_1 (HFFs) with Dynkin diagrams for a subset of ADE groups and Dynkin diagrams for affine ADE groups is the second one.

I have considered the interpretation of McKay correspondence in TGD framework already earlier [4, 3] but the decision to look it again led to a discovery of a bundle of new ideas allowing to answer several key questions of TGD.

1. Asking questions about $M^8 - H$ duality at the level of 8-D momentum space [6] led to a realization that the notion of mass is relative as already the existence of alternative QFT descriptions in terms

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of massless and massive fields suggests (electric-magnetic duality). Depending on choice $M^4 \subset M^8$, one can describe particles as massless states in $M^4 \times CP_2$ picture (the choice is M_L^4 depending on state) and as massive states (the choice is fixed M_T^4) in M^8 picture. p-Adic thermal massivation of massless states in M_L^4 picture can be seen as a universal dynamics independent mechanism implied by ZEO. Also a revised view about zero energy ontology (ZEO) based quantum measurement theory as theory of consciousness suggests itself.

2. Hyperfinite factors of type II_1 (HFFs) [4, 3] and number theoretic discretization in terms of what I call cognitive representations [8] provide two alternative approaches to the notion of finite measurement resolution in TGD framework. One obtains rather concrete view about how these descriptions relate to each other at the level of 8-D space of light-like momenta. Also ADE hierarchy can be understood concretely.

3. The description of 8-D twistors at momentum space-level is also a challenge of TGD. 8-D twistorializations in terms of octo-twistors (M_T^4 description) and $M^4 \times CP_2$ twistors (M_L^4 description) emerge at imbedding space level. Quantum twistors could serve as a twistor description at the level of space-time surfaces.

1.1 McKay correspondence in TGD framework

Consider first McKay correspondence in more detail.

1. McKay correspondence states that the McKay graphs characterizing the tensor product decomposition rules for representations of discrete and finite sub-groups of $SU(2)$ are Dynkin diagrams for the affine ADE groups obtained by adding one node to the Dynkin diagram of ADE group. Could this correspondence make sense for any finite group G rather than only discrete subgroups of $SU(2)$? In TGD Galois group of extensions K of rationals can be any finite group G. Could Galois group take the role of G?

2. Why the subgroups of $SU(2)$ should be in so special role? In TGD framework quaternions and octonions play a fundamental role at M^8 side of $M^8 - H$ duality [6]. Complexified M^8 represents complexified octonions and space-time surfaces X^4 have quaternionic tangent or normal spaces. $SO(3)$ is the automorphism group of quaternions and for number theoretical discretizations induced by extension K of rationals it reduces to its discrete subgroup $SO(3)_K$ having $SU(2)_K$ as a covering. In certain special cases corresponding to McKay correspondence this group is finite discrete group acting as symmetries of Platonic solids. Could this make the Platonic groups so special? Could the semi-direct products $Gal(K) \triangleleft SU(2)_K$ take the role of discrete subgroups of $SU(2)$?

1.2 HFFs and TGD

The notion of measurement resolution is definable in terms of inclusions of HFFs and using number theoretic discretization of X^4. These definitions should be closely related.

1. The inclusions $\mathcal{N} \subset \mathcal{M}$ of HFFs with index $\mathcal{M} : \mathcal{N} < 4$ are characterized by Dynkin diagrams for a subset of ADE groups. The TGD inspired conjecture is that the inclusion hierarchies of extensions of rationals and of corresponding Galois groups could correspond to the hierarchies for the inclusions of HFFs. The natural realization would be in terms of HFFs with coefficient field of Hilbert space in extension K of rationals involved.

   Could the physical triviality of the action of unitary operators $\mathcal{N}$ define measurement resolution? If so, quantum groups assignable to the inclusion would act in quantum spaces associated with the coset spaces $\mathcal{M}/\mathcal{N}$ of operators with quantum dimension $d = \mathcal{M} : \mathcal{N}$. The degrees of freedom below measurement resolution would correspond to gauge symmetries assignable to $\mathcal{N}$.

2. Adelic approach [7] provides an alternative approach to the notion of finite measurement resolution. The cognitive representation identified as a discretization of X^4 defined by the set of points with points having H (or at least M^8 coordinates) in K would be common to all number fields (reals and extensions of various p-adic number fields induced by K). This approach should be equivalent with that based on inclusions. Therefore the Galois groups of extensions should play a key role in the understanding of the inclusions.

How HFFs could emerge from TGD?

1. The huge symmetries of "world of classical words" (WCW) could explain why the ADE diagrams appearing as McKay graphs and principal diagrams of inclusions correspond to affine ADE algebras or quantum groups. WCW consists of space-time surfaces X^4, which are preferred extremals of the action principle of the theory defining classical TGD connecting the 3-surfaces at the opposite light-like boundaries of causal diamond $CD = cd \times CP_2$, where cd is the intersection of future and past directed light-cones of M^4 and contain part of $\delta M^4_\pm \times CP_2$. The symplectic transformations of $\delta M^4_+ \times CP_2$ are assumed to act as isometries of WCW. A natural guess is that physical states correspond to the representations of the super-symplectic algebra SSA.

2. The sub-algebras SSA_n of SSA isomorphic to SSA form a fractal hierarchy with conformal weights in sub-algebra being n-multiples of those in SSA. SSA_n and the commutator $[SSA_n, SSA]$ would act as gauge transformations. Therefore the classical Noether charges for these sub-algebras would vanish. Also the action of these two sub-algebras would annihilate the quantum states. Could the inclusion hierarchies labelled by integers $.. < n_1 < n_2 < n_3....$ with n_{i+1} divisible by n_i would correspond hierarchies of HFFs and to the hierarchies of extensions of rationals and corresponding Galois groups? Could n correspond to the dimension of Galois group of K.

3. Finite measurement resolution defined in terms of cognitive representations suggests a reduction of the symplectic group SG to a discrete subgroup SG_K, whose linear action is characterized by matrix elements in the extension K of rationals defining the extension. The representations of discrete subgroup are infinite-D and the infinite value of the trace of unit operator is problematic concerning the definition of characters in terms of traces. One can however replace normal trace with quantum trace equal to one for unit operator. This implies HFFs and the hierarchies of inclusions of HFFs [4, 3]. Could inclusion hierarchies for extensions of rationals correspond to inclusion hierarchies of HFFs and of isomorphic sub-algebras of SSA?

Quantum spinors are central for HFFs.

1. A possible interpretation of quantum spinors is in terms of quantum measurement theory with finite measurement resolution in which precise eigenstates as measurement outcomes are replaced with universal probability distributions defined by quantum group.

2. Quantum spinors have also a possible application in TGD inspired theory of consciousness [3]: the idea is that the truth value of Boolean statement is fuzzy. At the level of quantum measurement theory this would mean that the outcome of quantum measurement is not anymore precise eigenstate but that one obtains only probabilities for the appearance of different eigenstates. One might say that probability of eigenstates becomes a fundamental observable and measures the strength of belief.

3. In TGD particles are massless in 8-D sense and in general massive in 4-D sense but 4-D twistors are needed also now so that a modification of twistor approach is needed. The incidence relation for twistors suggests the replacement of the usual twistors with either non-commutative quantum twistors or with octo-twistors [11]. Quantum twistors could be associated with the space-time level description of massive particles.

2 McKay correspondence

Consider first McKay correspondence from TGD point of view.

2.1 McKay graphs

McKay graps are defined in the following manner. Consider group G which is discrete but not necessarily finite. If the group is finite there is a finite number of irreducible representations χ_I. Select preferred representation V - usually V is taken to be the fundamental representation of G and form tensor products $\chi_I \otimes V$. Suppose irrep χ_J appears n_{ij} times in the tensor product $\chi_I \otimes \chi_0$. Assign to each representation χ_I dot and connect the dots of χ_I and χ_J by n_{ij} arrows. This gives rise to MacKay graph.

Consider now the situation for finite-D groups of $SU(2)$. 2-D $SU(2)$ spinor representation as a fundamental representation is essential for obtaining the identification of McKay graphs as Dynkin diagrams of simply laced affine algebras having only single line connecting the roots (the angle between positive roots is 120 degrees) (see `http://tinyurl.com/z48d92t`).

1. For $SU(2)$ representations one has the basic rule $j_1 - 1/2 \leq j \leq j_1 + 1/2$ for the tensor product $j_1 \otimes 1/2$. This rule must be broken for finite subgroups of $SU(2)$ since the number of representations if finite so that branching point appears in McKay graph. In branching point the decomposition of $j_1 \otimes 1/2$ decomposes to 3 lower-dimensional representations of the finite subgroup takes place.

2. Simply lacedness means that given representation appears only once in $chi_I \otimes V$, when V is 2-D representation as it can be for a subgroup of $SU(2)$. Additional exceptional properties is the absence of loops ($n_{ii} = 0$) and connectedness of McKay graph.

3. One can consider binary icosahedral group (double covering of icosahedral group A_5 with order 60) as an example (for the McKay graph see `http://tinyurl.com/y2h55jwp`). The representations of A_5 are $1_A, 3_A, 3'_B, 4_A, 5_A$, where integer tells the dimension. Note that $SO(3)$ does not allow 4-D representation. For binary icosahedral group one obtains also the representations $2_A, 2'_B, 4_B, 6_A$. The McKay graph of E_8 contains one branching point in which one has the tensor product of 6-D and 2-D representations 6_A and 2_A giving rise to $5_A \oplus 3_C \oplus 4_B$.

McKay graphs can be defined for any finite group and they are not even unions of simply laced diagrams unless one has subgroups of $SU(2)$. Still one can wonder whether McKay correspondence generalizes from subgroups of $SU(2)$ to all finite groups. At first glance this does not seem possible but there might be some clever manner to bring in all finite groups.

The proposal has been that this McKay correspondence has a deeper meaning. Could simply laced affine ADE algebras, ADE type quantum algebras, and/or corresponding finite groups act as symmetry algebras in TGD framework?

2.2 Number theoretic view about McKay correspondence

Could the physical picture provided by TGD help to answer the above posed questions?

1. Could one identify discrete subgroups of $SU(2)$ with those of the covering group $SU(2)$ of $SO(3)$ of quaternionic automorphisms defining the continuous analog of Galois group and reducing to a discrete subgroup for a finite resolution characterized by extension K of rationals. The tensor products of 2-D spinor representation of these discrete subgroups $SU(2)_K$ would give rise to irreps appearing in the McKay graph.

2. In adelic physics [7] extensions K of rationals define an evolutionary hierarchy with effective Planck constant $h_{eff}/h_0 = n$ identified as the dimension of K. The action of discrete and finite subgroups of various symmetry groups can be represented as Galois group action making sense at the level of

X^4 [6] for what I have called cognitive representations. By $M^8 - H$ duality also the Galois group of quaternions and its discrete subgroups appear naturally.

This suggests a possible generalization of McKay correspondence so that it would apply to all finite groups G. Any finite group G can appear as Galois group. The Galois group $Gal(K)$ characterizing the extension of rationals induces in turn extensions of p-adic number fields appearing in the adele. Could the representation of G as Galois group of extension of rationals allow to generalize McKay correspondence?

Could the following argument inspired by these observations make sense?

1. $SU(2)$ is identified as spin covering of the quaternionic automorphism group. One can define the subgroups in matrix representation of $SU(2)$ based on complex numbers. One can replace complex numbers with the extension of rationals and speak of group $SU(2)_K$ identified as a discrete subgroup of $SU(2)$ having in general infinite order.

   The discrete finite subgroups $G \subset SU(2)$ appearing in the standard McKay correspondence correspond to extensions K of rationals for which one has $G = SU(2)_K$. These special extensions can be identified by studying the matrix elements of the representation of G and include the discrete groups Z_n acting as rotation symmetries of the Platonic solids. For instance, for icosahedral group Z_2,Z_3 and Z_5 are involved and correspond to roots of unity.

2. The semi-direct product $Gal \triangleleft SU(2)_K$ with group action

   $$(gal_1, g_1)(gal_2, g_2) = (gal_1 \circ gal_2, g_1(gal_1 g_2))$$

   makes sense. The action of $Gal \triangleleft SU(2)_K$ in the representation is therefore well-defined. Since all finite groups G can appear as Galois groups, it seems that one obtains extension of the McKay correspondence to semi-direct products involving all finite groups G representable as Galois groups.

3. A good guess is that the number of representations of $SU(2)_K$ involved is infinite if $SU(2)_K$ has infinite order. For $\tilde{A}_n$ and $\tilde{D}_n$ the branching occurs only at the ends of the McKay graph. For E_k, $k = 6, 7, 8$ the branching occurs in middle of the graph (see `http://tinyurl.com/y2h55jwp`). What happens for arbitrary G. Does the branching occur at all? One could argue that if the discrete subgroup has infinite order, the representation can be completed to a representation of $SU(2)$ in terms of real numbers so that the McKay graphs must be identical.

4. A concrete realization of ADE type Kac-Moody algebras is proposed. It relies on the group algebra of $Gal(K) \triangleleft SU(2)_K$ and free field representation of ADE type Kac-Moody algebra identifying the free scalar fields in Kac-Moody Cartan algebra as group algebra elements defined by the traces of representation matrices (characters).

5. A possible interpretation of quantum spinors is in terms of quantum measurement theory with finite measurement resolution in which precise eigenstates as measurement outcomes are replaced with universal probability distributions defined by quantum group [3]. TGD inspired theory of consciousness is a possible application.

   Also the notion of quantum twistor [11] can be considered. In TGD particles are massless in 8-D sense and in general massive in 4-D sense but 4-D twistors are needed also now so that a modification of twistor approach is needed. The incidence relation for twistors suggests the replacement of the usual twistors with non-commutative quantum twistors.

3 ADE diagrams and principal graphs of inclusions of hyperfinite factors of type II$_1$

Dynkin diagrams for ADE groups and corresponding affine groups characterize also the inclusions of hyperfinite factors of type II$_1$ (HFFs) [3].

3.1 Principal graphs and Dynkin diagrams for ADE groups

1. If the index $\beta = \mathcal{M} : \mathcal{N}$ of the Jones inclusion satisfies $\beta < 4$, the affine Dynkin diagrams of $SU(n)$ (discrete symmetry groups of n-polygons) and E_7 (symmetry group of octahedron and cube) and $D(2n+1)$ (symmetries of double 2n+1-polygons) are not allowed.

2. Vaughan Jones [2] (see `http://tinyurl.com/y8jzvogn`) has speculated that these finite subgroups could correspond to quantum groups as kind of degenerations of Kac-Moody groups. Modulo arithmetics defined by the integer n defining the quantum phase suggests itself strongly. For $\beta = 4$ one can construct inclusions characterized by extended Dynkin diagram and any finite sub-group of $SU(2)$. In this case affine ADE hierarchy appear as principal graphs characterizing the inclusions. For $\beta < 4$ the finite measurement resolution could reduce affine algebra to quantum algebra.

3. The rule is that for odd values of n defining the quantum phase the Dynkin diagram does not appear. If Dynkin diagrams correspond to quantum groups, one can ask whether they allow only quantum group representations with quantum phase $q = exp(i\pi/n)$ equal to even root of unity.

3.2 Number theoretic view about inclusions of HFFs and preferred role of $SU(2)$

Consider next the TGD inspired interpretation.

1. TGD suggests the interpretation in terms of representations of $Gal(K(G)) \triangleleft G$ for finite subgroups G of $SU(2)$, where $K(G)$ would be an extension associated with G. This would generalize to subgroups of $SU(2)$ with infinite order in the case of arbitrary Galois group. Quantum groups have finite number of representations in 1-1-correspondence with terms of finite-D representations of G. Could the representations of $Gal(K(G)) \triangleleft G$ correspond to the representations of quantum group defined by G?

   This would conform with the vision that there are two manners to realize finite measurement resolution. The first one would be in terms of inclusions of hyper-finite factors. Second would be in terms cognitive representations defining a number theoretic discretization of X^4 with imbedding space coordinates in the extension of rationals in which Galois group acts.

   In fact, also the discrete subgroup of infinite-D group of symplectic transformations of $\Delta M^4_+ \times CP_2$ would act in the cognitive representations and this suggests a far reaching implications concerning the understanding of the cognitive representations, which pose a formidable looking challenge of finding the set of points of X^4 in given extension of rationals [10].

2. This brings in mind also the model for bio-harmony in which genetic code is defined in terms of Hamiltonian cycles associated with icosahedral and tetrahedral geometries [5, 9]. One can wonder why the Hamiltonian cycles for cubic/octahedral geometry do not appear. On the other hand, according to Vaughan the Dynkin diagram for E_7 is missing from the hierarchy of so principal graphs characterizing the inclusions of HFFs for $\beta < 4$ (a fact that I failed to understand). Could the genetic code directly reflect the properties of the inclusion hierarchy?

How would the hierarchies of inclusions of HFFs and extensions of rationals relate to each other?

1. I have proposed that the inclusion hierarchies of extensions K of rationals accompanied by similar hierarchies of Galois groups $Gal(K)$ could correspond to a fractal hierarchy of sub-algebras of hyperfinite factor of type II_1. Quantum group representations in ADE hierarchy would somehow correspond to these inclusions. The analogs of coset spaces for two algebras in the hierarchy define would quantum group representations with quantum dimension characterizing the inclusion.

2. The quantum group in question would correspond to a quantum analog of finite-D group of $SU(2)$ which would be in completely unique role mathematically and physically. The infinite-D group in question could be the symplectic group of $\delta M^4_+ \times CP_2$ assumed to act as isometries of WCW. In the hierarchy of Galois groups the quantum group of finite group $G \subset SU(2)$ would correspond to Galois group of an extension K.

3. The trace of unit matrix defining the character associated with unit element is infinite for these representations for factors of type I. Therefore it is natural to assume that hyper-finite factor of type II_1 for which the trace of unit matrix can be normalized to 1. Sub-factors would have trace of projector with trace smaller than 1.

4. Do the ADE diagrams for groups $Gal(K(G)) \triangleleft G$ indeed correspond to quantum groups and affine algebras? Why the finite groups should give rise to affine/Kac-Moody algebras? In number theoretic vision a possible answer would be that depending on the value of the index β of inclusion the symplectic algebra reduces in the number theoretic discretization by gauge conditions specifying the inclusion either to Kac-Moody group ($\beta = 4$) or to quantum group ($\beta < 4$).

What about subgroups of groups other than $SU(2)$? According to Vaughan there has been work about inclusion hierarchies of $SU(3)$ and other groups and it seems that the results generalize and finite subgroups of say $SU(3)$ appear. In this case the tensor products of finite sub-groups McKay graphs do not however correspond to the principal graphs for inclusions. Could the number theoretic vision come in rescue with the replacement of discrete subgroup with Galois group and the identification of $SU(2)$ as the covering for the Galois group of quaternions?

3.3 How could ADE type quantum groups and affine algebras be concretely realized?

The questions discussed are following. How to understand the correspondence between the McKay graph for finite group $G \subset SU(2)$ and ADE (affine) group Dynkin diagram for $\beta < 4$ ($\beta = 4$)? How the nodes of McKay grap representing the irreps of finite group can correspond to the positive roots of a Dynkin diagram, which are essentially vectors defined by eigenvalues of Cartan algebra generators for complexified Lie-algebra basis.

The first thing that comes in mind is the construction of representation of Kac-Moody algebra using scalar fields labelled by Cartan algebra generators (see `http://tinyurl.com/y9lkeelk`): these representations are discussed by Edward Frenkel [1]. The charged generators of Kac-Moody algebra in the complement of Cartan algebra are obtained by exponentiating the contractions of the vector formed by these scalar fields with roots to get $\alpha \cdot \Phi = \alpha_i \Phi^i$. The charged field is represented as a normal ordered product $: exp(i\alpha \cdot \Phi) :$. If one can assign to each irrep of G a scalar field in a natural manner one could achieve this.

Neglect first the presence of the group algebra of $Gal(K(G)) \triangleleft G$. The standard rule for the dimensions of the representations of finite groups reads as $\sum_i d_I^2 = n(G)$. For double covering of G one obtains this rule separately for integer spin representations and half-odd integers spin representations. An interesting possibility is that this could be interpreted in terms of supersymmetry at the level of group algebra in which representation of dimension d_I appears d_I times.

The group algebra of G and its covering provide a universal manner to realize these representations in terms of a basis for complex valued functions in group (for extensions of rationals also the values of the functions must belong to the extension).

1. Representation with dimension d_I occurs d_I times and corresponds to $d_I \times d_I$ representation matrices D^I_{mn} of representation χ_I, whose columns and rows provide representations for left- and right-sided action of G. The tensor product rules for the representations χ_I can be formulated as double tensor products. For basis states $|J,n\rangle$ in χ_I and $|J,n\rangle$ in χ_J one has

$$|I,m\rangle_{\otimes}|J,n\rangle = c^{K,p}_{I,m|J,n}|K,p\rangle \ \ ,$$

where $c^{K,p}_{J,n|J,n}$ are Glebch-Gordan coefficients.

2. For the tensor product of matrices D^I_{mn} and D^J_{mn} one must apply this rule to both indices. The orthogonality properties of Glebsch-Gordan coefficients guarantee that the tensor product contains only terms in which one has same representation at left- and right-hand side. The orthogonality rule is

$$\sum_{m,n} c^{K,p}_{I,m|J,n} c^{K,q}_{I,r|J,s} \propto \delta_{K,L} \ \ .$$

3. The number of states is $n(G)$ whereas the number $I(G)$ of irreps corresponds to the dimension of Cartan algebra of Kac-Moody algebra or of quantum group is smaller. One should be able to pick only one state from each representation D^I.

   The condition that the state X of group algebra is invariant under automorphism gXg^{-1} implies that the allowed states as function in group algebra are traces $Tr(D^I)(g)$ of the representation matrices. The traces of representation matrices indeed play fundamental role as automorphism invariants. This suggests that the scalar fields Φ_I in Kac-Moody algebra correspond to Hilbert space coefficients of $Tr(D^I)(g)$ as elements of group algebra labelled by the representation. The exponentiation of $\alpha \cdot \Phi$ would give the charged Kac-Moody algebra generators as free field representation.

4. For infinite sub-groups $G \subset SU(2)$ $d(G)$ is infinite. The traces are finite also in this case if the dimensions of the representations involved are finite. If one interprets the unit matrix as a function having value 1 in entire group $Tr(Id)$ diverges. Unit dimension for HFFs provide a more natural notion of dimension $d = n(G)$ of group algebra $n(G)$ as $d = n(G) = 1$. Therefore HFFs would emerge naturally.

It is easy to take into account $Gal(K(G))$. One can represent the elements of semi-direct product $Gal(K(G)) \triangleleft G$ as functions in $Gal(K(G)) \times G$ and the proposed construction brings in also the tensor products in the group algebra of $Gal(K(G))$. It is however essential that group algebra elements have values in K. This brings in tensor products of representations Gal and G and the number of representations is $n(Gal) \times n(G)$. The number of fields Φ_I as also the number of Cartan algebra generators of ADE Lie algebra increases from $I(G)$ to $I(Gal) \times I(G)$. The reduction of the extension of coefficient field for the Kac-Moody algebra from complex numbers to K splits the Hilbert space to sectors with smaller number of states.

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References

[1] Frenkel E. Free Field Realizations in Representation Theory and Conformal Field Theory. Available at: `http://www.mathunion.org/ICM/ICM1994.2/Main/icm1994.2.1256.1269.ocr.pdf`.

[2] Jones V. In and around the origin of quantum groups. Available at: `http://arxiv.org/abs/math/0309199`, 2003.

[3] Pitkänen M. Evolution of Ideas about Hyper-finite Factors in TGD. In *Hyper-finite Factors and Dark Matter Hierarchy*. Online book. Available at: http://www.tgdtheory.fi/tgdhtml.html/neuplanck1#vNeumannnew, 2006.

[4] Pitkänen M. Was von Neumann Right After All? In *Hyper-finite Factors and Dark Matter Hierarchy*. Online book. Available at: http://www.tgdtheory.fi/tgdhtml/neuplanck1.html#vNeumann, 2006.

[5] Pitkänen M. Geometric theory of harmony. Available at: http://tgdtheory.fi/public_html/articles/harmonytheory.pdf, 2014.

[6] Pitkänen M. Does M^8-H duality reduce classical TGD to octonionic algebraic geometry? Available at: http://tgdtheory.fi/public_html/articles/ratpoints.pdf, 2017.

[7] Pitkänen M. Philosophy of Adelic Physics. Available at: http://tgdtheory.fi/public_html/articles/adelephysics.pdf, 2017.

[8] Pitkänen M. TGD view about coupling constant evolution. Available at: http://tgdtheory.fi/public_html/articles/ccevolution.pdf, 2018.

[9] Pitkänen M. An overall view about models of genetic code and bio-harmony. Available at: http://tgdtheory.fi/public_html/articles/gcharm.pdf, 2019.

[10] Pitkänen M. Scattering amplitudes and orbits of cognitive representations under subgroup of symplectic group respecting the extension of rationals . Available at: http://tgdtheory.fi/public_html/articles/symplorbsm.pdf, 2019.

[11] Pitkänen M. Twistors in TGD. Available at: http://tgdtheory.fi/public_html/articles/twistorTGD.pdf, 2019.