Article

Modified Takabayasi String with Bulk Viscous Fluid & Variable Λ in Kantowski-Sachs Universe

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Abstract

The present study deals with Kantowski-Sachs cosmological model with modified Takabayasi string of the form $\rho = (1+\omega)\lambda + \Lambda(t)$ (where $\Lambda(t)$ depends on the Hubble parameter (H)) in the presence of bulk viscous fluid. To obtain the realistic model, we assume the condition that the shear scalar is proportional to expansion scalar, i.e., $A_1 = A_2^{\mu}$. Some physical and geometrical aspects of the model are discussed. The presence of viscous term prevents the universe to be empty. Also we obtained the expression for proper distance, luminosity distance, angular diameter distance and look back time.

Keywords: Kantowski-Sachs, cosmological model, modified Takabayasi string, variable cosmological term, bulk viscous fluid.

1. Introduction

The discovery of an accelerated expansion of the new theoretical models in the cosmology has significantly changed our view of the fate of the universe. Recent observations suggest that the universe is dominated by dark energy which can be characterized by an equation of state parameter $\omega(t)$ which is the ratio of the pressure to the density $\omega = \frac{p}{\rho}$. In the study of celebrated dark energy problem (Copeland et al. 2006), the ideal fluid with specific EOS remains to be the simplest possibility for the description of the current cosmic acceleration. Various examples of ideal fluid EOS may be considered for this purpose: constant EOS with negative pressure, imperfect EOS (Cardone et al. 2006), general EOS (Nojiri and Odintsov 2004; Stefancic 2005), inhomogeneous EOS (Nojiri and Odintsov 2005, 2006 and so on). Moreover, general relativity with ideal fluid of any type may be written in the equivalent form (Capozziello et al. 2006) as modified gravity (Nojiri and Odintsov 2003). Brevik et al. (2007) studied a model where there is an ideal fluid with an inhomogeneous equation of state of the form

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$p = (1+\omega)\rho + \Lambda(t)$ in which the parameters $\omega(t)$ and $\Lambda(t)$ are dependent linearly on the time t. In some works, the EOS is assumed to be periodic in time such that it could explain the so-called coincidence problem (Nojiri and Odintsov 2006; Saez-Gomez 2009; Hua-Hui et al. 2008).

In recent years cosmic strings have been studied to describe the early evolution of the universe. These cosmic strings have stress energy and couple to the gravitational field. The gravitational effects of string in general relativity have been studied by Letelier (1979) and Stachel (1980). Letelier (1983) studied relativistic cosmological solutions of cloud formed by massive strings in Bianchi type-I and Kantowski-Sachs space-times. Letelier (1983) studied a model of a cloud formed by massive strings instead of geometrical strings. Each massive string is formed by a geometrical strings with particles attached along its extension. Hence, the strings that form the cloud are the generalization of Takabayasi's realistic model of strings (p-strings). Therefore, p-string is the direct consequence of Takabayasi's string model. Letelier (1983) mentioned that the examples of equation of state for strings as follow:

a) Geometric string is $\rho = \lambda(\rho_p = 0)$
b) Takabayasi string is $\rho = (1+\omega)\lambda$
c) Barotropic equation $\rho = \rho(\lambda)$
d) The modified EOS for cloud of string $\rho = (1+\omega)\lambda + \Lambda(t)$

Beside the Bianchi type metrics, the Kantowski-Sachs models are also describing spatially homogeneous universes. These metrics represent homogeneous but anisotropically expanding (or contacting) cosmologies and provide models where the effects of anisotropic can be estimated and compared with all well-known Friedmann-Robertson-Walker class of cosmologies. Wang (2005) has obtained Kantowski-Sachs string cosmological model with bulk viscosity in general relativity. Kandalkar et al. (2009) have discussed Kantowski-Sachs viscous fluid cosmological model with a varying Λ. Rao et al.(2011) have studied various Bianchi type string cosmological models in the presence of bulk viscosity. Venkateswarlu et al. (2012) obtained Kantowski-Sachs String Cosmological Models in Sen-Dunn Theory of Gravitation. Samdurkar and Sen (2012) investigated the effect of bulk viscosity in on Bianchi Type V cosmological models with varying Λ general relativity. Samdurkar and Bawnerkar (2018) studied Effect of variable deceleration parameter and polytropic equation of state in Kantowski-Sachs universe.

Motivated by the above investigations, we study the bulk viscous Kantowski-Sachs string cosmological model in the presence of time dependent cosmological term of the form $\Lambda = \beta H^2$ in general theory of relativity. The paper is organized as follows: In section 2, Metric and Field equations are mentioned. In section 3, we derive solution in the presence of bulk viscosity and time varying cosmological term by imposing the condition that the shear scalar is proportional to expansion scalar. Some physical and geometrical features are observed in section 4 and conclusion is given in the last section.

2. The metric & field equations

We consider metric in the form

$$ds^2 = -dt^2 + A_1^2 dr^2 + A_2^2 (d\theta^2 + \sin^2\theta d\psi^2) \tag{1}$$

where A_1 and A_2 are the functions of time t only.

The energy momentum tensor for a cloud of string along the x-direction in the presence of bulk viscous fluid is given by

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j - \xi\theta(u_i u_j + g_{ij}) \tag{2}$$

Here ρ is the energy density for a cloud string with particles attached to them , λ is the string tension density, H is the Hubble parameter, ξ is the coefficient of bulk viscosity, u^i the four-velocity of the particles and x^i is a unit space-like vector representing the direction of string. In a co-moving coordinate system, we have

$$u_i u^i = -x_i x^i = -1, u^i x_i = 0 \tag{3}$$

The Einstein field equations are

$$R_{ij} - \frac{1}{2} R g_{ij} = -T_{ij} \tag{4}$$

Here R_{ij} is the Ricci tensor, R is the Ricci scalar curvature and T_{ij} is the energy – momentum tensor.

For the metric (1) and energy momentum tensor (2) in co-moving system of co-ordinates, the above field equation yields

$$\frac{2\ddot{A}_2}{A_2} + \frac{\dot{A}_2^2}{A_2^2} + \frac{1}{A_2^2} = \lambda + \xi\theta \tag{5}$$

$$\frac{\ddot{A}_1}{A_1} + \frac{\ddot{A}_2}{A_2} + \frac{\dot{A}_1 \dot{A}_2}{A_1 A_2} = \xi\theta \tag{6}$$

$$\frac{2\dot{A}_1 \dot{A}_2}{A_1 A_2} + \frac{\dot{A}_2^2}{A_2^2} + \frac{1}{A_2^2} = \rho \tag{7}$$

an over dot indicates a derivative with respect to time *t*.

We define average scale factor *a(t)* and generalized Hubble parameter *H* for Kantowski-Sachs universe as

$$V = a^3 \tag{8}$$

$$H = \frac{\dot{a}}{a} \tag{9}$$

also expansion factor and shear scalar are

$$\theta = \frac{\dot{A}_1}{A_1} + \frac{2\dot{A}_2}{A_2} \tag{10}$$

$$2\sigma^2 = \sum_{i=1}^{3} H_i^2 - \frac{\theta^2}{3} \tag{11}$$

The mean anisotropic parameter and deceleration parameter are given by

$$\Delta_h = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 \tag{12}$$

$$q = \frac{-\ddot{a}a}{\dot{a}^2} \tag{13}$$

In order to get a deterministic solution we take the following plausible physical condition, the shear scalar σ is proportional to scalar expansion θ. This condition leads to

$$A_1 = A_2^{\mu} \quad ,\mu > 1 \tag{14}$$

Using (14), we can obtain the expressions for string and density as

$$\lambda = (1-\mu)\frac{\ddot{A}_2}{A_2} + (1-\mu^2)\frac{\dot{A}_2^2}{A_2^2} + \frac{1}{A_2^2} \tag{15}$$

and

$$\rho = (2\mu+1)\frac{\dot{A}_2{}^2}{A_2{}^2} + \frac{1}{A_2{}^2} \tag{16}$$

Here we consider the following EOS for a cloud of string

$$\rho = (1+\gamma)\lambda + \Lambda(t) \tag{17}$$

where the cosmological term is in the form

$$\Lambda = \beta H^2 \tag{18}$$

3. Solution of the field equations

By using (15), (16) and (18) in (17), we have

$$\frac{\ddot{A}_2}{A_2} + \left(\frac{\chi_0}{(\mu-1)(\gamma+1)}\right)\frac{\dot{A}_2{}^2}{A_2{}^2} = \frac{\gamma}{(\mu-1)(\gamma+1)}\frac{1}{A_2{}^2} \tag{19}$$

where

$$\chi_0 = \frac{1}{9}\{9[(\mu^2-1)(\gamma+1)+(2\mu+1)] - \beta(\mu+2)^2\}$$

which on integration gives

$$A_2 = \alpha_1 t + \alpha_2$$

Hence

$$A_1 = (\alpha_1 t + \alpha_2)^{\mu}$$

Therefore the metric (1) becomes

$$ds^2 = -dt^2 + (\alpha_1 t + \alpha_2)^{2\mu} dr^2 + (\alpha_1 t + \alpha_2)^2 (d\theta^2 + \sin^2\theta d\psi^2) \tag{20}$$

4. Some physical & geometrical properties of the models

The expression for density is given by

$$\rho = \frac{(2\mu+1)\,\alpha_1^2+1}{(\alpha_1\,t+\alpha_2)^2} \tag{21}$$

Expression for string is given by

$$\lambda = \frac{(1-\mu^2)C_1^2+1}{(\alpha_1\,t+\alpha_2)^2} \tag{22}$$

Expression for the bulk coefficient is

$$\xi = \frac{\mu^2\,C_1}{(\mu+2)(\alpha_1 t+\alpha_2)} \tag{23}$$

Expression for the cosmological term is

$$\Lambda = \frac{\beta(\mu+2)^2\,C_1^2}{9(\alpha_1 t+\alpha_2)^2} \tag{24}$$

Expression for Spatial volume is as follows

$$V = (\alpha_1 t+\alpha_2)^{\mu+2} \tag{25}$$

Expression for expansion factor can be found as

$$\theta = \frac{(\mu+2)C_1}{(\alpha_1 t+\alpha_2)} \tag{26}$$

Expression for shear scalar can be found as

$$\sigma^2 = \frac{(\mu-1)^2\,C_1^2}{3(\alpha_1 t+\alpha_2)^2} \tag{27}$$

Expression for mean anisotropic parameter is

$$\Delta_h = \frac{2(\mu-1)^2}{(\mu+2)^2} \tag{28}$$

The deceleration parameter is given by

$$q = -1+\frac{3}{\mu+2} \tag{29}$$

The sign of q indicates whether the model inflates or not. A positive sign of q corresponds to the standard decelerating model whereas the negative sign of q indicates inflation. Here it can be seen clearly from (29) that q lies between - 1 to 0, which represents universe is accelerating.

The Jerk parameter is given by

$$J = \frac{(\mu-1)(\mu-4)}{(\mu+2)^2} \tag{30}$$

Our Finding:

- The spatial volume V is finite at t=0, expands as t increases and becomes infinitely large at t tends to infinity.

- The average scale factor $a = (\alpha_1 t + \alpha_2)^{\frac{\mu+2}{3}}$, hence it has point type singularity at $t = -\frac{\alpha_2}{\alpha_1}$.

From (21), it is observed that the energy density is a function of time t and always decrease positively with the expansion. At the initial stage $t \to 0$, the universe has infinitely large i.e. $\rho \to \infty$.

We also measure the physical parameters of the model (20) such as proper distance, luminosity distance, angular diameter and Look back time.

Proper Distance

$$d(z) = \frac{(\mu+2)}{(1-\mu)} \frac{\left((1+z)^{\frac{1-\mu}{\mu+2}} - 1\right)}{H_0 (1+z)^{\frac{1-\mu}{\mu+2}}} \tag{31}$$

Luminosity Distance

$$d_L = \frac{(\mu+2)}{(1-\mu)} \frac{\left((1+z)^{\frac{1-\mu}{\mu+2}} - 1\right)}{H_0 (1+z)^{\frac{1-\mu}{\mu+2}-1}} \tag{32}$$

Angular Diameter Distance

$$d_A = H_0^{-1} \frac{(\mu+2)}{(1-\mu)} \frac{\left((1+z)^{\frac{1-\mu}{\mu+2}} - 1\right)}{(1+z)^{\frac{1-\mu}{\mu+2}+1}} \tag{33}$$

Look back Time

$$H_0(t_0 - t) = \frac{\mu+2}{3}\left(1 - (1+z)^{-\frac{3}{\mu+2}}\right) \tag{34}$$

5. Conclusion

In this paper we have investigated the modified EOS Takabayasi string of the form $\rho = (1+\gamma)\,\lambda + \Lambda(t)$ in the presence of bulk viscous fluid and obtained the solution of the equations with varying cosmological term $\Lambda = \beta H^2$ in the frame of Kantowski-Sachs universe. It is observed that scalar expansion (θ), Hubble parameter (H), shear scalar (σ), density (ρ), bulk viscosity coefficient (ξ) and cosmological term (Λ) decreases with increase of time. The spatial volume increases as time increases and becomes infinite as t tends to infinity. Also for the given universe, jerk comes out to be constant. We find $\frac{\sigma}{\theta}$ = constant, which shows that the model is anisotropy and becomes isotropic for $\mu = 1$. We have also obtained the expression for proper distance, luminosity distance, angular diameter distance and look back time.

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