Article

Theories of Quantum & Analog Gravity Represented in the Mandelbrot Set

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Abstract

The Mandelbrot Set displays features of the 5-d $\rightarrow$ 4-d transition in DGP gravity and in theories where a 5-d black hole gives rise to a 4-d white hole and spacetime bubble – which is our present-day cosmos – at (-0.75,0*i*). The ratio of radii here is 3:1 in imitation of Cartan's rolling ball analogy for Lie group G_2 symmetries. Holographic dualities are displayed across the boundary, linking the quantum mechanics in the precursor universe with the astrophysical realm in the current era, which we discuss. Then we examine the Misiurewicz point at about (-1.543689,0*i*) where is clearly depicted an analogy of BEC formation and Schwarzschild black hole event horizons, which forms an important part of the analog gravity program. The fact both of these dynamics are displayed in the same object indicates that the theories are not mutually-exclusive and could coexist in nature.

Keywords: AdS/CFT, 5D to 4D, Lie group, G2, Mandelbrot set, BEC analog gravity.

Introduction

While the Mandelbrot Set is known for its display of beautiful and intricate symmetries, it is asymmetrical on the whole. The intricacy of its patterning is due in part to the fact it contains both perfect symmetry in every flavor and global asymmetry, so that precisely symmetric forms must conform to asymmetrical boundaries. This interplay of local symmetry and global asymmetry in $\mathcal{M}$ makes it an ideal arena for the study of symmetry-breaking phenomena. The Mandelbrot Mapping Conjecture [1] (or MMC) states that we can map physical processes to locations in $\mathcal{M}$, where the cusp at (0.25,0*i*) represents the highest energy processes at the Planck scale and the extreme tip at (2,0*i*) depicts absolute zero temperature.

We also note that $\mathcal{M}$ in the complex numbers (2-d) is the shadow or projection of a higher-dimensional figure in the quaternions (4-d) and octonions (8-d). This makes it less surprising that $\mathcal{M}$ displays Cartan's rolling ball analogy for G_2 symmetries, where the ball's point of contact is at (-0.75,0*i*). This spot shows the fabric of spacetime folding back on itself during a 5-d $\rightarrow$ 4-d transition, as seen in DGP gravity [2] and explained in work by Pourhasan, Afshordi, and Mann [3], and Poplawski [4], where a black hole in the 5-d precursor became a white hole and expanding bubble, in 4-d spacetime – which is our cosmos. Black hole cosmologies were first

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proposed by Pathria [5] and Good [6] in 1972. My recent work [7] shows how $\mathcal{M}$ at (-0.75,0i) explains the AdS/CFT correspondence and 5-d → 4-d transition in details across the boundary between the cardioid and the circular region.

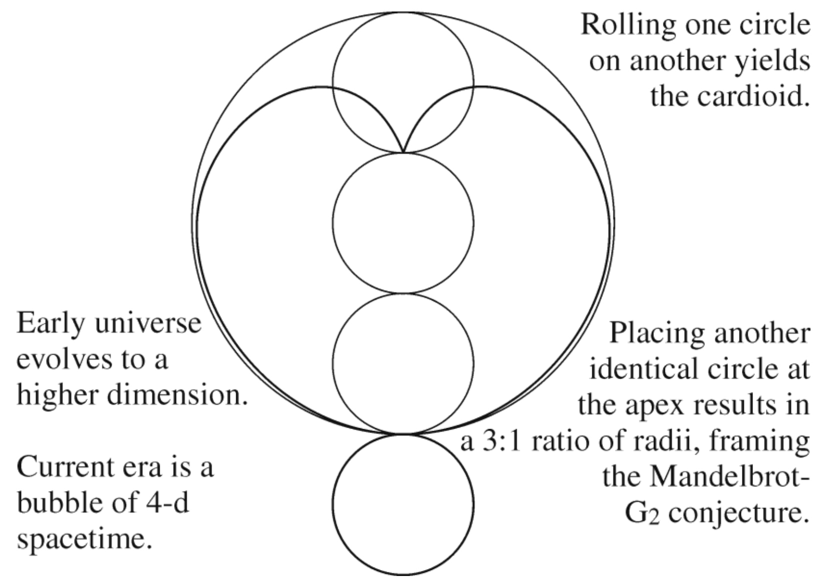

Fig. 1. The major geometry of the Mandelbrot Set can be constructed by rolling a circle on another of equal size, to form the cardioid, and then placing another circle of equal size at the extremum.

Under the above assumptions; our local universe is represented by the circular disc centered at (-1,0*i*), which in this model is a quaternionic bubble on an expanding sphere of the next higher dimension. And we can study the 5-d → 4-d transition, by looking at the pseudo-symmetric mirroring of forms across the boundary at (-0.75,0*i*). We see that a preserved symmetry on one side becomes a broken symmetry on the other, so weak interactions in the precursor universe are seen to couple or conformally map to electromagnetism in the current cosmological era. This analogy in $\mathcal{M}$ therefore suggests the 5-d → 4-d transition was involved in both matter-energy decoupling and electro-weak symmetry breaking.

This is understood by noting that the hypersurface of a 5-d sphere is 4-dimensional. We see that fermionic matter is constrained to move or flow in the periphery before the transition point, but in 4-d spacetime it can travel in the bulk. It also suggests nucleosynthesis and element building continued to evolve heavy and perhaps even super-heavy elements, before decoupling and the 5-d → 4-d dimensional shift, but that not all species which formed in the early universe were able to pass through the wormhole throat. The Mandelbrot Set clearly depicts how the fabric of spacetime turns inside-out during such a transition, but we can also show this using the octonions. If we designate the seven octonion imaginaries (i,j,k,I,J,K,L), we can assign I, J, K, and L to the

5-d parent or precursor, leaving the familiar *i*, *j*, and *k* for the three axes of rotation in our 4-d spacetime.

The importance to understanding gravity is made clear by going further out on the real axis, and studying the Misiurewicz point $M_{3,1}$ at about (-1.543689,0*i*) where we see depicted both the quantum critical point of BEC formation and Schwarzschild black hole event horizons. A Schwarzschild black hole is an almost purely gravitational object, because it has no charge or spin, but it does radiate like a black body of a specific temperature. Beyond its mass and temperature, it also occupies space or has a characteristic size. This is seen to be the smallest space a given mass can occupy. For objects of aggregate molecular matter, the gravitational radius r_G defines only a tiny volume at their center, but a Schwarzschild object's gravitational radius r_S is the event horizon at its surface instead.

This phenomenology is realized by seeing these objects as a Bose-Einstein condensate of gravitons, in recent research by Dvali and Gomez [8]. The idea of a gravity-BEC connection goes back to Sakharov in 1967 [9], with many more explorations since then [10], however. This analogy is clearly represented in the Mandelbrot Set at $M_{3,1}$, where both the Schwarzschild event horizon and BEC condensation are depicted. Their common occurrence in $\mathcal{M}$ shows that a dimensional reduction from higher-dimensional spaces ending in a 5-d $\rightarrow$ 4-d transition can bring about processes which mimic the quantum critical point of BEC formation. This demonstrates that these two different approaches to unifying gravity can work in tandem or are compatible. We will explore the reasons why and the implications thereof.

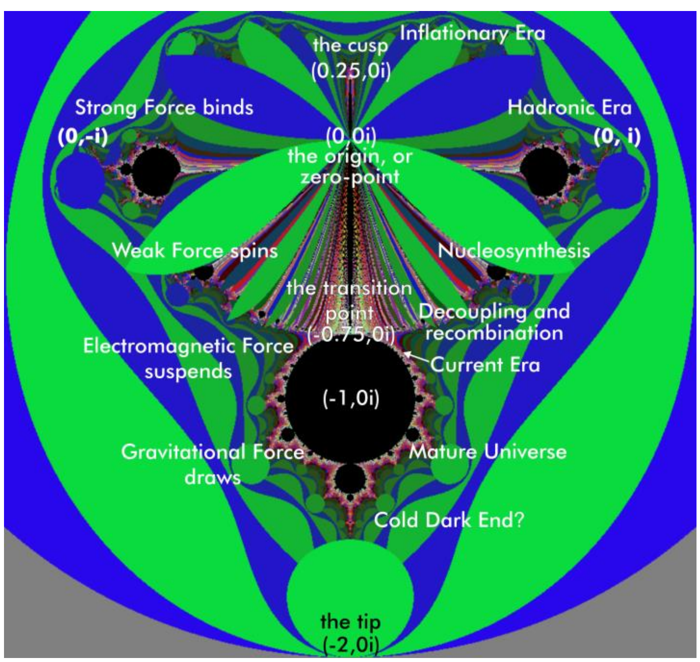

Fig. 2. The Mandelbrot Butterfly that appears when one colors in points where the iterand magnitude diminishes monotonically for 3 calculations is annotated to show relationships to the natural forces and cosmological eras, under the Mandelbrot Mapping Conjecture.

The Inherently Quantum Nature of $\mathcal{M}$ Explains why Gravity is Quantum Mechanical

How spacetime arises is often modeled using the tools of continuous spaces, though we imagine the origin of spacetime and gravity is quantum mechanical. The Mandelbrot Set is inherently and manifestly quantum mechanical, where every structure associated with it has a discrete period because it resolves after a definite (integer) number of iterations. A lot of what we associate with $\mathcal{M}$ is not a part of the set itself, but instead is located in color bands of the repeller sets surrounding it. The same pattern holds true with features of the Butterfly as they resolve only after a specific number of calculations or steps.

Similarly, points within the body of the set settle into patterns that enter into cycles, converging to a point, to repeat the same value over and over. Since the familiar 2-d representation of $\mathcal{M}$ in the complex numbers is a shadow or projection of a higher-dimensional figure; this pattern or sequence gives us insights into the progression of forms defining the dimensionality of the universe. Emergent evolution is seen to happen automatically in the context of octonionic embedding, but this is presumed to be part of a progression of defining events where the creation of spacetime gave it upper and lower limits for D in the seminal early universe, then converged on a specific dimensionality during or by the current cosmological epoch.

Of special interest in this regard are the Misiurewicz points, in that they exhibit a unique behavior, where they will endlessly repeat – after a set interval called a pre-period. For this reason; Misiurewicz points are called pre-periodic. But they are singular repelling points, so even a small deviation from the exact value will cause the calculations to assume a divergent trajectory instead, somewhat unpredictably. Ergo it is helpful that certain points of interest to gravity researchers are tractable analytically, allowing us to calculate algebraically exact and numerically precise values for the location of these points.

If we trace a path from the cusp along either edge of the cardioid; there is a progression of openings, circular bays or discs which have a characteristic period of repetition that remarkably is echoed by the number of branches in the Misiurewicz point that extends from it [11]. This starts at a high number and decrements by one each time, until just past $(0,\pm i)$ with a bay or disc where the period is 3, and then increases thereafter until reaching $(-0.75,0i)$, where it opens up into a circular bay or disc of period 2 centered at (-1,0*i*) that represents our 4-d bubble of spacetime. Going further along the real axis into the tail section or main antenna; we come to the primary Misiurewicz point for this bay $M_{3,1}$, at ~(-1.543689,0*i*), where we find represented in $\mathcal{M}$ and in its associated figures a number of analogies relevant to gravity. The author has for at least 20 years seen it as representing a Schwarzschild event horizon. But more recently it became apparent that it also represents the quantum critical point for BEC formation, the location where complexity is maximal, and more.

Because of the strong association we can make between $M_{3,1}$ and condensation; and since features of the lowest order Misiurewicz points in $\mathcal{M}$ extend to all of the higher-order points; we speculate that the disc-like forms populating the periphery of the Mandelbrot Butterfly represent regions in parameter space where material forms tend to congeal or condense with a specific pre-period. With this assumption; the MMC follows automatically, because the progression from highest to lowest temperature is seen to be a result of the sequence of periodic behaviors spelled out above. Instead of an object; we can see $\mathcal{M}$ as a map of vibratory phenomena. In my theory; features that resolve only after a large number of iterations correspond to a higher energy or temperature. So points near the cusp, requiring a very high iteration count to resolve, naturally represent high temperatures, while points in the neighborhood of (-2,0*i*) are rendered with very few calculations, in the single digits, and are seen as very cold. Furthermore; while having a similar progression, features of the Butterfly resolve sooner (after fewer calculations) than the regions of $\mathcal{M}$ they inhabit.

Since the discs surrounding the Butterfly abut a Misiurewicz point and contain a mini-$\mathcal{M}$; they encode a pre-periodic and periodic progression for congealing or condensing forms. What makes this matter more interesting is that $\mathcal{M}$ presents dimensional reduction and condensation as part of a larger pattern, once we see its connection to and representation of higher-dimensional reality. In the work of Kricker and Joshi [12], a higher-d generalization of $\mathcal{M}$ is used to map non-associative regions of the octonionic quadratic functions. If we see $\mathcal{M}$ in the complex numbers as a shadow or projection of a figure in the quaternions and octonions; it is easier to understand how it contains a model for Lie group G_2 and to see why it encodes a pattern that is encompassing in Math. We note first a hierarchy in the normed division algebras or number types:

$$\mathbb{O} \supset \mathbb{H} \supset \mathbb{C} \supset \mathbb{R}$$

The octonions contain the quaternions, which can be obtained by fixing 4 of the octonions' 7 axes of rotation or imaginaries. The quaternions in turn contain the complex numbers, which we get by fixing 2 of the quaternions' 3 axes of rotation, and they contain the real numbers, which are pure scalars with no freedom to vary. We note a similar hierarchy in the categories of forms and spaces:

$$Smooth \supset Top \supset Meas$$

Here we see that the smooth forms and spaces contain the topological ones, and these contain the forms that are measurable. And we see a partial resemblance to this same pattern in the phases of matter:

$$Gas \supseteq Liquid \supseteq Solid$$

Cosmological and Geometric Transitions

Geometrical and cosmological transitions play a large part in theories of gravity and cosmology based on $\mathscr{M}$. The relative strength of the fundamental forces is seen to be set by symmetry-breaking processes involving geometric transitions. This involves fixing the gauge of the forces through cosmological transitions such as the 5-d → 4-d dimensional shift in DGP gravity and related theories, which we see represented at (-0.75,0*i*) in $\mathscr{M}$. Gauge fixing for the electroweak transition relates to the size of the wormhole throat, if we assume a black hole → white hole phenomenology, because there is a dynamic balance between element building and the stricture of the opening for the bridge between the prior cosmos and our own. The reason is that the creation of heavy and super-heavy elements goes hand in hand with the trapping of spin in the spacetime fabric, which creates topological torsion.

According to R.M. Kiehn [13]; torsion is created whenever spin is trapped by structure, and here we are assuming that spin is trapped in the fabric of spacetime itself when particles or nuclei are created, which changes the shape or morphology of the background space over time through continuous topological evolution. So if more heavy and super-heavy elements are spawned during nucleosynthesis; this will tighten the wormhole throat at the transition so that fewer of the heaviest particles and nuclei will pass through.

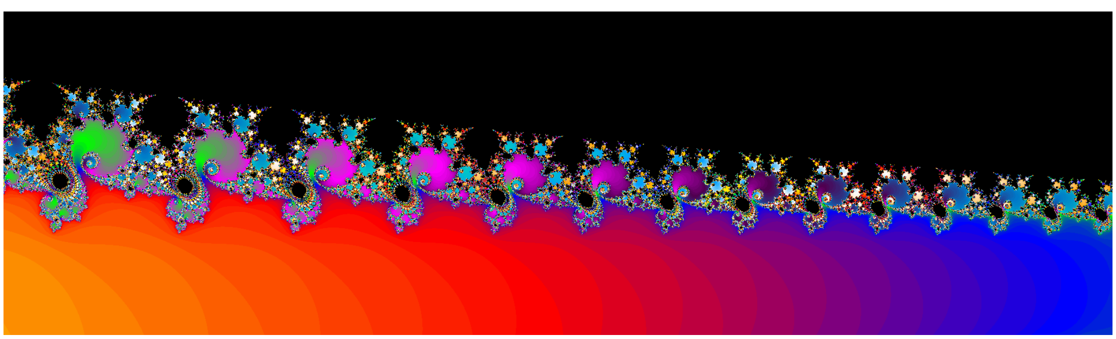

Fig. 3 A section of the Mandelbrot Set approaching (-0.75,0*i*), with spiral eyes left uncolored, shows tidal deformation due to increasing Weyl tensor values near the end of the precursor universe. This also shows how pockets of spin trapped in the fabric create topological torsion.

It is suggested in a recent paper by Christian and Diether [14] that the existence of matter in the present-day cosmos is proof of gravitational torsion, echoing earlier work of Kiehn [15]. That torsion can create particles as defects in the fabric formed by trapped spin follows naturally from the above assumptions with the addition that nucleosynthesis or element building is also wound up with torsion. This is illustrated in the forms along the periphery of $\mathscr{M}$ and the Mandelbrot Butterfly figure, as seen in Fig. 4 below. And it reflects the phenomenology mentioned earlier, where fermionic mass is pushed to the 'skin' or hypersurface of a higher-dimensional sphere, such that particles and nuclei are constrained to move along or across the brane face that contains the early universe.

This creates a condition where fermionic mass pushes the boundaries of spacetime outward, and gravity becomes repulsive, in the time leading up to the 5d $\rightarrow$ 4-d transition. That is; fermionic mass and gravity push or expand during the 5-d precursor phase, but they pull or contract massive bodies in the current era. This is largely because the fabric of space turns inside-out during the 5d black hole $\rightarrow$ 4-d white hole transition, just as we see in $\mathscr{M}$ at (-0.75,0*i*). If we think about this in terms of the octonions; 4 axes of rotation labeled (I,J,K,L) end up in the parent universe, and the remaining 3 become the familiar (i,j,k) axes of our 4-d spacetime, or the quaternion imaginaries. Thus there are no common basis elements between the bulk of the parent universe and the empty space of the 3-d vacuum in ours. This explains why some particle species native to higher dimensions, such as sterile neutrinos [16], are not seen in our universe though they apparently influence particle interactions here [17].

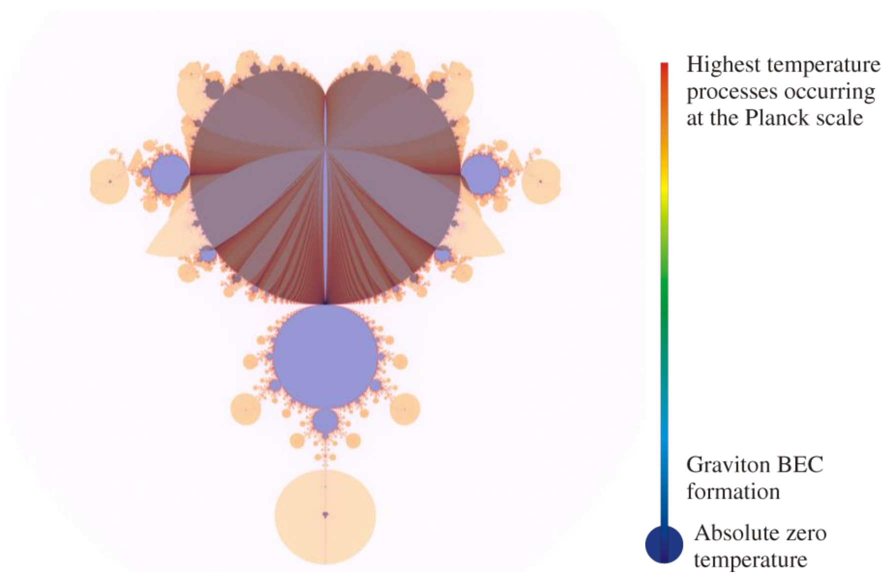

Fig. 4. The Mandelbrot Butterfly in this image by Paul Bourke shows the analogy of this figure with a cosmic thermometer which explains the action of gravity if we assume gravitons form a condensate.

Gravity Analogs in $\mathcal{M}$

As a catalog for symmetry-breaking phenomena; the Mandelbrot Set adds a lot to the toolkit for analog gravity researchers. On the whole; it is asymmetrical, but it displays a remarkable interplay between local symmetry and global asymmetry at the branching Misiurewicz points which occur at all of the bulbs or bays surrounding the cardioid, and around the entire periphery of $\mathcal{M}$. These points appear perfectly symmetrical and exactly resemble their Julia Set counterparts at their centers, but conform to the asymmetrical background at the edges, as was first observed by Tan Lei [18].

The Misiurewicz point $M_{3,1}$ is of special interest to gravity researchers, because it is the 0^{th} order or ground state point representing archetypal features that extend to all of the other branching or inflecting Misiurewicz points in $\mathcal{M}$. Misiurewicz points in $\mathcal{M}$ are seen to come in three flavors, branching, inflecting, and terminal points. Misiurewicz points occur on all the dendritic branches that extend from the discs or bays along the periphery of $\mathcal{M}$ as described above, delineating a sharp change or transition from one region to the next. The scale factor goes to zero at all Misiurewicz points, and it increases thereafter if it is not a terminal point. We see a progression of self-similar forms that repeat in smaller and smaller copies when approaching any Misiurewicz point, where they all converge and to which they are all reduced.

At the point designated $M_{3,1}$; we observe a phase inversion for the self-similar forms entering and leaving it. This resembles rows of telephone poles on both sides of a long road that meet at infinity, and converge to a point, but then reappear and grow in reverse phase on the other side. Since $M_{3,1}$ has only one avenue of entrance and exit; this is equivalent to what is seen at the event horizon of a Schwarzschild black hole, but it is also the same as the quantum critical point of BEC formation.

This gives us unique insights into the dynamism of analog gravity by allowing us to peer behind the event horizon boundary algorithmically, and to see the process of condensation dynamically. Furthermore; we can use the Mandelbrot-Julia correspondence to study the progression through the quantum critical point in Butterfly Julia animations that show the way forms merge and coalesce. Since we can examine this process in an archetypical setting in $\mathcal{M}$; we have the ability to proceed past any impasse impeding progress. And with the understanding that $\mathcal{M}$ in the complex numbers is the shadow or projection of a higher-dimensional figure in the quaternions and octonions; we can see and show how quantum and analog gravity can coexist, or are compatible.

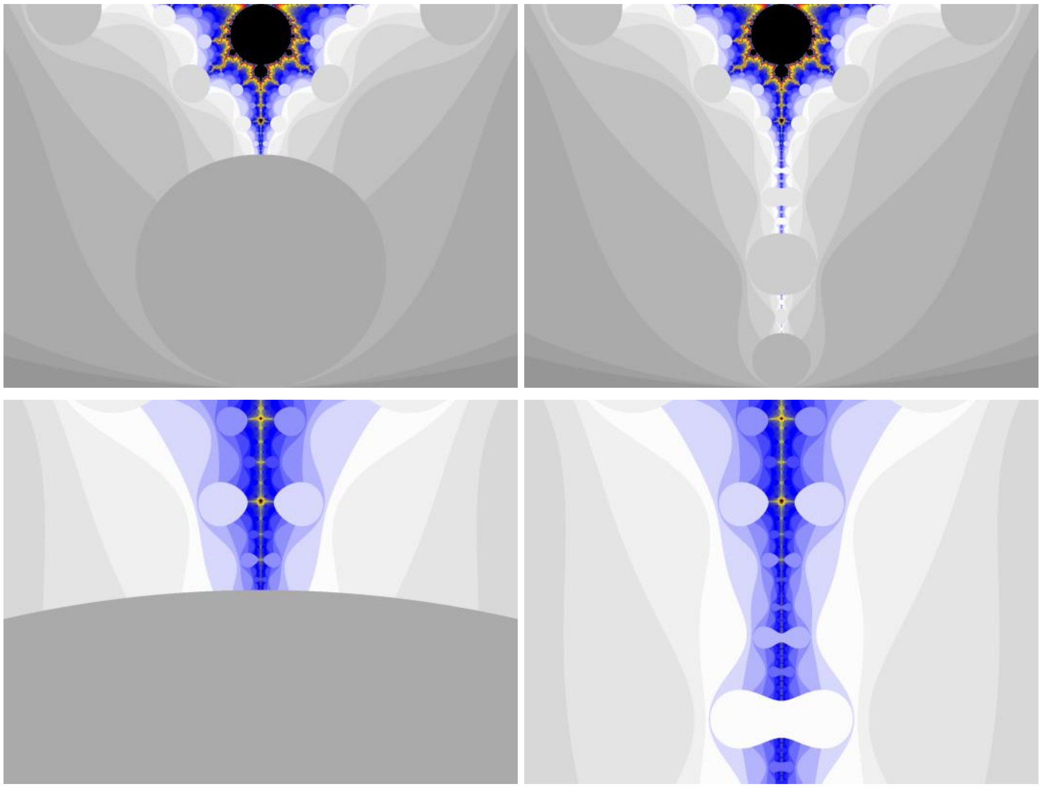

Fig. 5. The largest disc at the base of the Butterfly is on left and magnified below at $M_{3,1.}$ One layer is removed by suppressing the lowest order solution on right, to show coalescence or condensation.

Additionally; the Mandelbrot Set at $M_{3,1}$ presents us with analogs for even more ways to model the behavior or phenomenology of Schwarzschild event horizons, each one giving us new insights into the nature of gravity. As explained above; a Schwarzschild black hole is a good way to model pure gravity, because it has no spin or charge, only having mass and a characteristic radius r_S, that sets its size or event horizon. However; it was found by Hawking to radiate like a black body at a temperature T that decreases as it gains mass. When Hawking and Bekenstein calculated the entropy they found that it varies proportionally to the horizon area times Planck's constant, signaling the area is quantized. Specifically:

$$S_{BH} = \frac{kA}{4L_P^2} = \frac{Akc^3}{4G\hbar}$$

This set the stage for various kinds of quantum gravity theory that treat BH horizons as a quantum-mechanical rather than classical boundary surface, and it also opens the door to a thermodynamic or entropic treatment of gravity, along lines developed by Jacobson [19], Verlinde [20], and Padmanabhan [21]. In my view; it is the natural endpoint of all such theories that gravity's action is like a process of condensation [22], which links the above back to the model by Dvali and Gomez [23] that sees black holes as a graviton condensate.

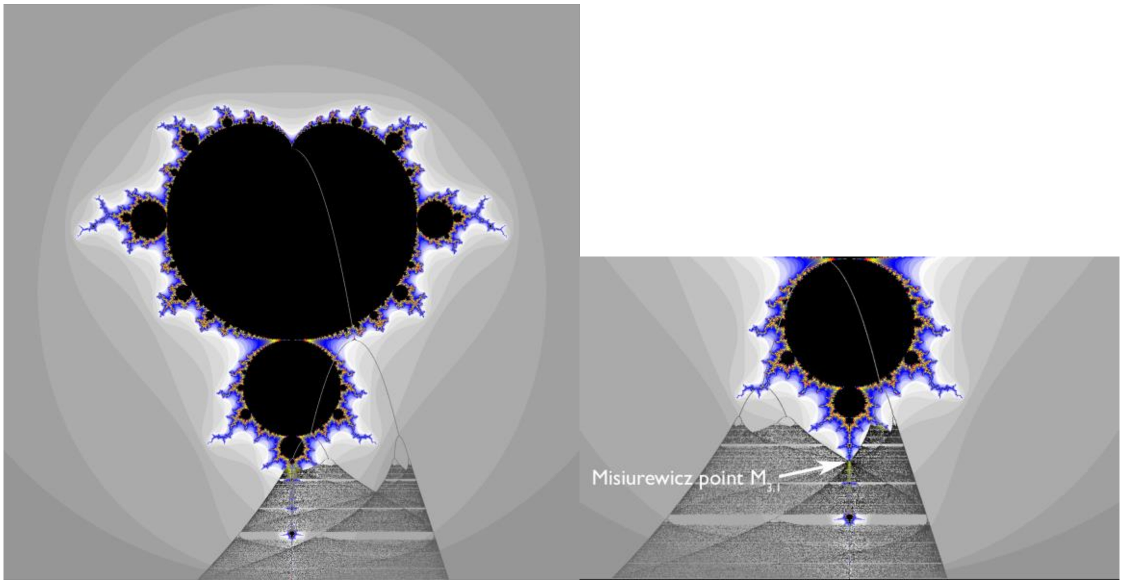

Fig. 6. The Mandelbrot Set with the bifurcation diagram for $c = c^2 + c$ superimposed, then shifted, magnified, and annotated to show how the band merging point coincides with $M_{3,1}$, demonstrates how $\mathcal{M}$ recreates the logistic map and the progression to chaos.

But the same line of reasoning points to information-theoretic relations, since if the quantum properties of particles absorbed are subsumed in a black hole's bulk, and what is emitted comes out as purely thermal radiation – this breaks a fundamental postulate of quantum mechanics called unitarity. So a large number of theories have been developed that attempt to reconcile the thermal spectrum prediction of Hawking with the assumptions of QM. Some of this theoretical work has been fueled by the suggestion of 't Hooft [24] that we can view the event horizon as a 2-d surface where the laws of 3-d Physics are written, minus gravity, which is like a hologram. This idea was later generalized by Juan Maldacena [25] and others to extend the concept to a holographic mapping between 4-d and 5-d, as well as to suggest that we can see the laws of our Physics (again minus gravity) projected on a 2-d surface at infinity – which is the AdS/CFT correspondence.

The idea that the holographic correspondence is connected to the 5-d $\rightarrow$ 4-d transition is featured in papers by Pourhasan, Afshordi, and Mann about a 5-d black hole that empties into a white hole in our universe that creates our 4-d spacetime. And the author has written about how insights into the AdS/CFT correspondence can be gleaned by looking at $\mathcal{M}$ [26], on either side of (-0.75,0*i*) where the boundary of $\mathcal{M}$ folds back on itself, and which is the point of contact for Cartan's rolling ball. The bifurcation diagram for $c = c^2 + c$ splits here and at every location along the real axis where the boundary folds back, illustrating the progression to chaos, and highlighting the significance of Misiurewicz point $M_{3,1}$, where the bands merge.

This ties in nicely with some of the work cited above, including many different flavors of emergent or entropic gravity. The theory is in close agreement with the approach of Jacobson and that of Padmanabhan, which cast gravity in thermodynamic terms. This derives in part from representation of the spreading and sharing model for entropy [27] in $\mathcal{M}$, via the relation at the Misiurewicz points first spelled out by Tan Lei, where symmetry is nearly exact at the center of a branching feature and form diverges as we go further out. But the correspondence of $\mathcal{M}$ to its bifurcation diagram, in combination with the MMC, vividly depicts how entropy and thermodynamic cooling from the expanse of space are connected to the action of gravity – when we examine $\mathcal{M}$ and its family of forms around $M_{3,1}$.

The progression to chaos is not identical to the increase of entropy, even if we see $M_{3,1}$ as a Rindler horizon, the gravitational horizon r_G, or the Schwarzschild radius r_S of a black hole. It is a closer match to recent work by Susskind and colleagues [28] that suggests a black hole's event horizon is a place of maximal informational and computational complexity. Plainly; the place in $\mathcal{M}$ where computational difficulty is at a maximum is the cardioid cusp at (0.25,0*i*), representing the Planck scale, where calculating the features of $\mathcal{M}$ or the repeller sets in the periphery of its local neighborhood requires large numbers of iterations to resolve. But the Misiurewicz point $M_{3,1}$ is infinitely repeating when iterated, just like points in the body of $\mathcal{M}$, and it is known by its bifurcation diagram to encode maximal informational complexity, which makes it very special indeed.

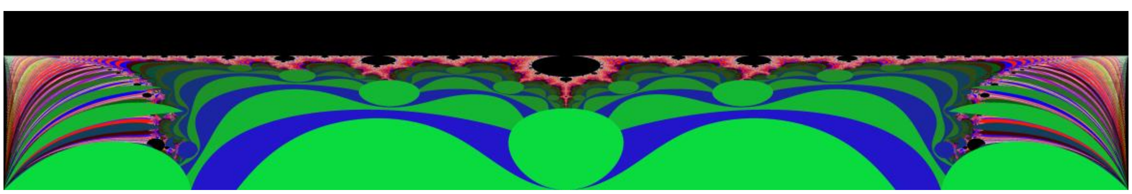

Fig. 7. Butterfly where concentric circles at (-1,0*i*) are mapped to rows of pixels shows asymptotic flatness of space on top edge, and green discs representing macroscopic masses in gravity wells below.

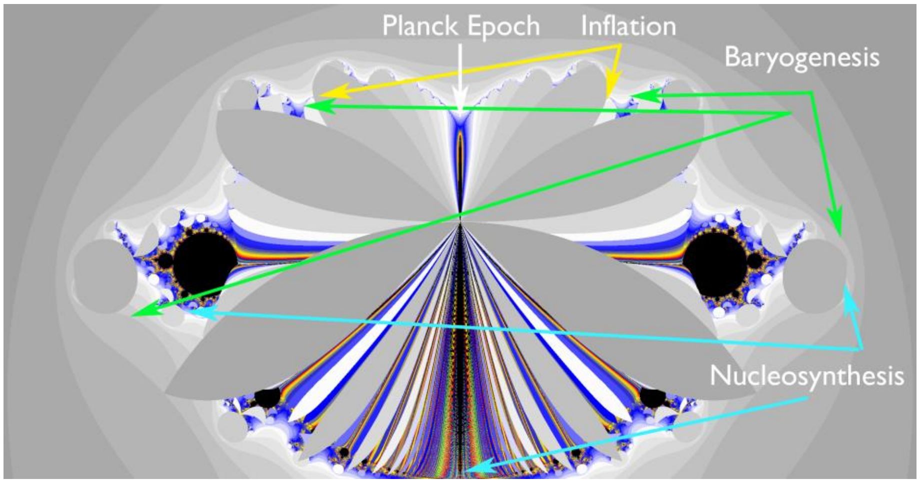

Fig. 8. The cardioid section of the Mandelbrot Butterfly is annotated to show the cosmological progression and the creation of form in the early universe. This illustrates how particles emerge in the periphery and are swept along the edge by the expanding fireball during nucleosynthesis.

The most prominent features of the Mandelbrot Butterfly are its wings and the discs that populate the periphery of the figure. At the cusp; the wings are seen to push directly on the boundary or repeller sets, which are thought in the theory to represent the fabric of space or spacetime. The wings are later seen to represent creation/annihilation pathways for sub-atomic particles, while the discs along the periphery are the material forms that congeal out of the energy soup. As shown in Fig. 8 above; the ratcheting action of the wings during the inflationary phase gives rise to pockets of empty space that allow the formation of subatomic particles.

The largest pocket and disc, to the far left and right, represents the creation of anti-protons and protons, or hydrogen nuclei. In the cardioid section of the Butterfly we see the early universe evolving sub-atomic particles and then nuclei as it unfolds to a 5-d volume. Then in the circular region that is our 4-d bubble, aggregate objects give way to black holes. This is depicted in Figure 9 below. The theory predicts that rapidly spinning black holes should exist early in the post-decoupling cosmos and that slower spinning BHs will come to predominate over time. Accordingly; spiral galaxies that are tightly wound earlier on or in the current era should be seen to unwind like clock springs over cosmic time, eventually turning into gently inflected star streams instead of spirals. A similar result would be expected on the basis of the continuing expansion of space over time. But it is graphically depicted in $\mathcal{M}$, and though we can bracket our location using the MMC, this feature helps to more precisely determine the location of the current era in that figure using visible spiral galaxies as a guide.

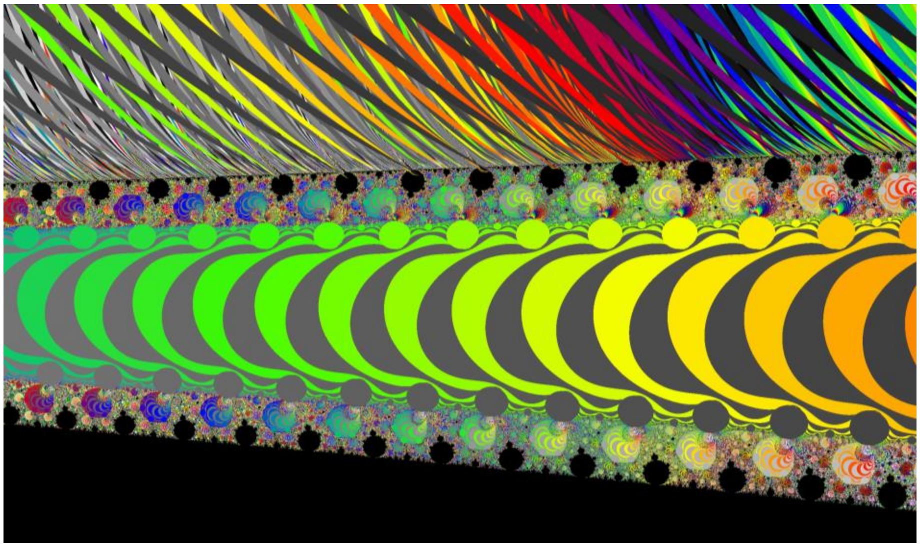

Fig. 9. Element building continues above the fold, in the final phase of the 5-d parent or precursor, with time moving right to left. Macroscopic masses are below, with time moving left to right, and rapidly spinning BHs in the earliest post-decoupling phase give way to slower spinning BHs over time.

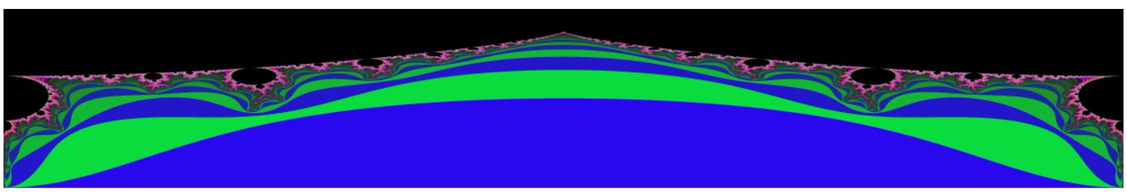

Fig. 10. The Mandelbrot Set where concentric circles about (0,0*i*) are mapped to rows of pixels shows how it fundamentally breaks symmetry, allowing material form to evolve in the open spaces.

One of the most interesting analogies to come out of this research is the idea that the event horizon of a black hole is like the amplitude null or virtual ground point in an inverting op-amp circuit. In such a circuit, a high gain amplifier is used in an inverting voltage divider configuration, to allow a specific gain and a higher bandwidth to be obtained easily. This results in a null point, or virtual ground, where the input resistor and the resistor carrying the inverted feedback signal meet – which is also the input line of the amplifier. Here the two signals (incoming and inverted phase) cancel out, because what is coming back is a backwards mirror of what is coming in. If one could probe along the length of the input resistor; one would find the amplitude of the signal progressively diminished, until arriving at zero, and a progressively increasing signal similarly probing the feedback resistor.

This feature is of special interest, and it is mimicked in $\mathcal{M}$ at $M_{3,1}$. Here too we see a kind of reverse mirror effect. I stated earlier that what is seen as we approach $M_{3,1}$ is relatively uninteresting – two rows of telephone poles diminish to a point, then appear in reverse phase on the other side – which is easily missed. But this spot holds clues to properties extending by self-similarity to all the branching or inflecting Misiurewicz points in the Mandelbrot Set. And this makes for a most interesting analogy for Schwarzschild event horizons, because we can state how and see why the wavefunction evolves as it does on the other side of the event horizon. The fact it is an inverse mirror makes the horizon appear perfectly black and suck things in. The incoming and returning signal combine to bring everything at the edge to zero.

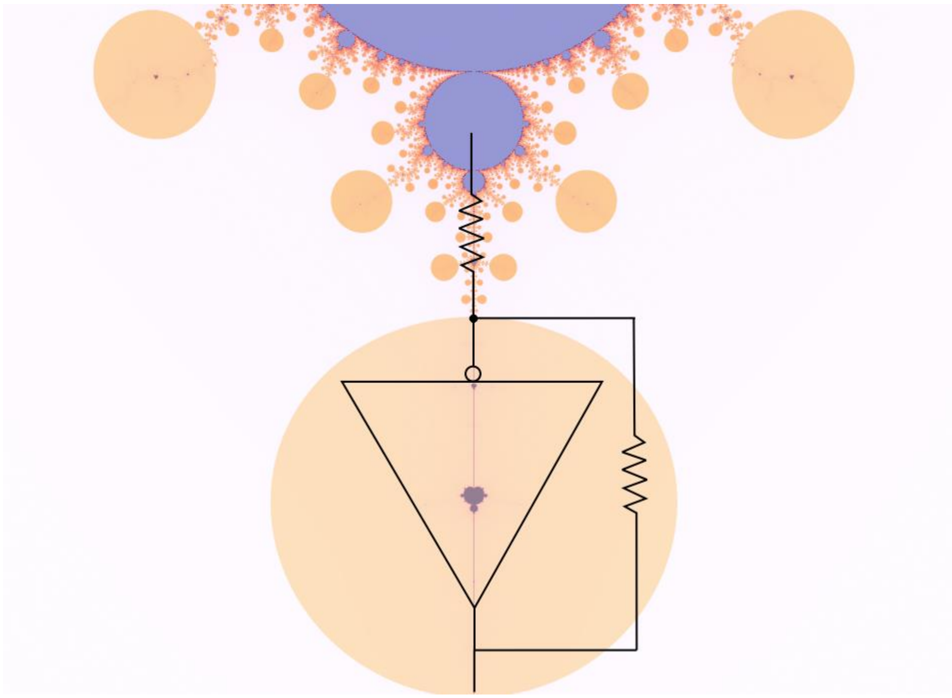

Fig. 11. Base of the Butterfly figure from Paul Bourke is overlaid with an inverting feedback op-amp circuit, to show the null point analogy. A Schwarzschild event horizon is modeled here as the point of extinction, where the input and feedback resistors meet.

Coexistence

The fact that various theories of Quantum, Entropic, and BEC Analog Gravity are simultaneously represented in the Mandelbrot Set shows that they can coexist or are compatible possibilities, and not mutually-exclusive options. This allows us to contemplate how the pieces fit together, by seeing visual representations of the relationships between processes in $\mathcal{M}$. Too often; researchers work to affirm or promote a specific theory, so once people have put in time working in String Theory, Loop Quantum Gravity, or Causal Sets, they are not likely to collaborate with researchers who have devoted their time to competing theories. Beverly Berger made comments in her plenary panel at GR21, which were echoed by Lee Smolin in the Quantum Gravity sessions, that there is a lot to be gained by people in each camp of researchers sharing their results and finding ways to use advances from other groups or approaches.

The Mandelbrot Set allows us to see several approaches at work at once, which signifies that competing approaches *can* work together in nature. We observe at (-0.75,0i) features of the 5-d $\rightarrow$ 4-d transition in DGP gravity (for Dvali, Gabadadze, and Porrati) and explained phenomenologically as a black hole in a prior universe feeding a white hole and bubble that becomes our cosmos, in the work of Pourhasan, Afshordi, and Mann – as well as Poplawski and a few others. Knowing the details of this transition can provide deep insights into the AdS/CFT correspondence among other things.

But at Misiurewicz point $M_{3,1}$; we find a remarkable confluence of representations because it depicts analogies to a Schwarzschild event horizon, the quantum critical point or critical surface for BEC formation, the place of maximal complexity, and an inverting amplifier's virtual ground. I made the connection with a Schwarzschild horizon around 20 years ago, and more recently became aware it points to a connection with BEC formation. This was a finding I was reluctant to share initially, but later I became aware the idea of linking black holes with BECs was older and richer than my conception of it. In addition to recent work by Dvali and Gomez, which was my doorway; there is a wealth of research linking gravity to BEC formation, going all the way back to a paper by Sakharov in '67, then to Sudarshan [29], and later to Chapline and Laughlin [30] among others.

By this point, Barceló, Liberati, and Visser cite hundreds of researchers working in this area [31]. And then there is the connection of this location in $\mathcal{M}$ with the band merging point in the bifurcation diagram, as first noted by Grossman and Thomae [32]. Misiurewicz point $M_{3,1}$ is relevant to the study of gravity because it shows how divergent trajectories converge at a gravitational horizon or quantum critical surface. It permits an exact solution because it satisfies the solvable equation $((c^2+c)^2+c)^2+c=(c^2+c)^2+c$, as shown in Peitgen and Richter [33]. But remarkably; this analogy connects $\mathcal{M}$ to a broad spectrum of theories involving emergent or entropic gravity, and especially to recent work by Susskind on complexity at an event horizon. We find all of these examples vividly represented in $\mathcal{M}$ and its associated figures. And recent work by Tiozzo [34] proving the monotonicity of entropy in the Mandelbrot Set shows that these results can be extended.

Declaration

This paper is a companion to the author's presentation for GR22 session D4 - Quantum fields in curved spacetime, semi-Classical gravity, Quantum gravity phenomenology, and theoretical aspects of analogue gravity.

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