Article

Simple Harmonic Oscillator in k-MOND

V. Thakare, M. R. Pund* & S. B. Rokde

Priyadarshini College of Engineering, Nagpur, India

Abstract

Pankovic and Kapor in 2010 proved a possibility that Milgrom's modified Newtonian dynamics, MOND can be interpreted as a theory with the modified kinetic terms of the usual classical Newtonian dynamics, simply called k-MOND. In this work we used this possibility and tried to find the Newtonian and k-MOND analogue of behaviour of Simple harmonic oscillator.

Keywords: Euler-Lagrange equation, kinetic energy, Lagrangian, MOND, potential energy, simple harmonic oscillator.

1. Introduction

A new Lagrangian functional of the simple harmonic oscillator has been proposed. The derived equation of motion is *almost* same as that of the conventional Lagrangian functional. The equation of motion is derived from Euler-Lagrange equation by performing partial derivatives on the Lagrangian functional of the second variation of the calculus of variations.

The simple harmonic oscillator model is very important in physics (Classical and Quantum). Harmonic oscillators occur widely in nature and are exploited in many manmade devices, such as clocks and radio circuits. They are the source of virtually all sinusoidal vibrations and waves. Highlighting the behavior of simple harmonic oscillator in view of the modified kinetic energy, due to Pankovic and Kapor in MOND theory by Milgrom, its Newtonian analogue is reclaimed.

When the uniform velocity of rotation of cluster of galaxies was first observed (Oort 1932 and Zwicky 1933) it was not in tune with Newtonian theory of gravity. The galaxies or stars sufficiently away from the centre of the cluster move with constant velocities more than predicted by the theory. The most widely accepted approach to explain this problem postulates the existence of the dark matter. However, even after seven decades, there is no convincing evidence of the dark matter. In an attempt to explain the observed uniform velocities of galaxies without dark matter hypothesis, Professor Milgrom in 1983 propounded an equation of motion which resulted into a theory, known as MOND (*Modified Newtonian Dynamics*), which is a phenomenological scheme whose basic premise is that the visible matter distribution in a galaxy or cluster of galaxies alone determines its dynamics. In MOND the Newton's second law of motion is generalised as

*Correspondence: Manisha R. Pund, J. D. Patil Sangludkar Mahavidyalaya, Daryapur, India. Email: pundmanisha@gmail.com

$$\mathbf{F} = m\mu(a/a_0)\mathbf{a} \ , \tag{1.1}$$

where μ is an interpolation function defined by

$$\mu(a/a_0) = \begin{cases} a/a_0 & a << a_0 \\ 1 & a >> a_0 \end{cases}.$$

For $a >> a_0$ above equation reduces to Newtonian one $\mathbf{F} = m\mathbf{a}$. The quantity a_0 is constant having the dimensions of an acceleration $\mathbf{a}$ and is evaluated as $a_0 \approx 2\times10^{-8}cm/s^2$ (Milgrom 1983, Bekenstein and Milgrom 1984). Even if MOND simply and elegantly describes important astronomical observational data, physical interpretation of the MOND is not simple at all. It is interesting that many of the researchers have directed their attention to probe into the various aspects of the MOND, from mathematical and physical point of views. Very recently, Pankovic and Kapor (2010) , have concluded that MOND can be interpreted as a theory with the modified kinetic term of the conventional Newtonian dynamics. In this try i go ahead with the implication of kinetic term with reference to the motion of a the simple harmonic oscillator.

2. Lagrangian With k-MOND

Consider a **Simple Harmonic Oscillator of** mass m attached to spring of constant k and displaced to a position x from equilibrium position.
Then kinetic energy in the Newtonian dynamics of simple harmonic oscillator of mass m in one dimension is written as.

$$T = \frac{1}{2}mv^2$$

And the Lagrangian of simple harmonic oscillator in one dimension is
written as

$$L = \frac{1}{2}mv^2 - \frac{1}{2}kx^2$$

The first term is kinetic energy and the second term is the potential energy of the simple harmonic oscillator.

Following Pankovic and Kapor (2010) we adopt the form of kinetic energy as

$$T = \frac{1}{2}v^2\left(\frac{a}{a+a_0}\right) \tag{2.1}$$

Then the Lagrangian L of the system has taken the form,

$$L = T - V = \frac{1}{2}mv^2\left(\frac{a}{a+a_0}\right) - \frac{1}{2}kx^2$$

Since $L = L(x, v, a)$, it satisfies the generalized Euler-Lagrange equation

$$\frac{d}{dt}\left(\frac{\partial L}{\partial v}\right) - \frac{d^2}{dt^2}\left(\frac{\partial L}{\partial a}\right) = -\frac{\partial V}{\partial x} \tag{2.2}$$

After computation we get equation as follows,

$$\frac{d}{dt}\left(\frac{\partial T}{\partial v}\right) = \frac{m\,a^2}{(a+a_0)} + O_1 \tag{2.3}$$

and $$\frac{d^2}{dt^2}\left(\frac{\partial T}{\partial a}\right) = \frac{ma_0\,a^2}{(a+a_0)^2} + O_2 \tag{2.4}$$

where the respective first terms at the right sides of (2.3), (2.4) are leading terms, while O_1 and O_2 represent non-leading terms whose forms are

$$O_1 = \frac{ma_0 v\dot{a}}{(a+a_0)^2}$$

and $$O_2 = \left[\frac{ma_0 v\dot{a}}{(a+a_0)^2} - \frac{4ma_0 va\dot{a}}{(a+a_0)^3} - \frac{ma_0 v^2\dot{a}}{(a+a_0)^3} + \frac{3ma_0 v^2\dot{a}^2}{(a+a_0)^4}\right]$$

Neglecting these approximate values,the Euler-Lagranges equation (2.2) assumes the form

$$\frac{ma^2}{(a+a_0)} - \frac{ma^2 a_0}{(a+a_0)^2} = -\frac{\partial v}{\partial x}$$

$$\frac{ma^2}{(a+a_0)} - \frac{ma^2 a_0}{(a+a_0)^2} = -\frac{\partial v}{\partial x} \tag{2.5}$$

3. Lagrangian in two different regions

Considering $a >> a_0$ i.e $\frac{a_0}{a} = h,$ h $\rightarrow 0$ for Newtonian regime (2.5) turns out approximately to

$$ma = -\frac{\partial V}{\partial x} \tag{3.1}$$

In case when $a << a_0$ dynamical equation (2.5) becomes more and more different from usual Newtonian classical dynamical equation.

Suppose, however that a is initially very small, i.e much smaller than a_0. In this

case (2.1) approximates to

$$T = \frac{1}{2} m \frac{v^2 a}{a_0}.$$

And by using above Kinetic Energy, we obtain

$$\frac{d}{dt}\left(\frac{\partial T}{\partial v}\right) = \frac{d}{dt}\frac{2mva}{2a_0},$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial v}\right) = \frac{m}{a_0}\left[v\dot{a} + a^2\right]$$

In the same case $\frac{d^2}{dt^2}\left(\frac{\partial T}{\partial a}\right)$ can be neglected and then (2.2) becomes

$$\frac{ma^2}{a_0} = -\frac{\partial V}{\partial \theta}. \tag{3.2}$$

Obviously this expression corresponds to (1.1), i.e. to MOND dynamics for small acceleration.

For $a >> a_0$, (2.1) comes out to be

$$T = \frac{1}{2} m v^2.$$

Using above expression for Kinetic Energy, we deduce that

$$\frac{d}{dt}\left(\frac{\partial T}{\partial v}\right) = \frac{d}{dt} mv \ ,$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial v}\right) = ma$$

$$\text{and } \frac{d^2}{dt^2}\left(\frac{\partial T}{\partial a}\right) = 0.$$

In this case (2.2) assumes the form

$$ma = -\frac{\partial V}{\partial x} \tag{3.3}$$

Therefore Newtonian analogue of simple harmonic oscillator is $ma = -\frac{\partial V}{\partial x}$ and the k-MOND analogue of simple harmonic oscillator is $\frac{ma^2}{a_0} = -\frac{\partial V}{\partial x}$

4. Conclusion

We have demonstrated the k-MOND analogue and restored the Newtonian analogue for a simple harmonic oscillator.

Received June 1, 2019; Accepted June 26, 2019

References

1. Oort J, Bull. Astron. Inst. Neth. **6,** 249 1932
2. Zwicky F, Helv. Phys. Acta **6,** 110 1933
3. Milgrom M, Astrophysical Journal **270,** 365 1983
4. Milgrom M, Astrophysical Journal **270,** 371 1983
5. Milgrom M, Astrophysical Journal **270,** 384 1983
6. Bekenstein J D and Milgrom M, Astrophysical Journal **286, 7** 1984
7. Pankovic V and Kapor D V, arXiv: 1012.3533v1 [physics.gen-ph] 2010