Essay

Comments on a Concept from General Relativity

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Abstract

It is not widely known that the mass-energy equivalence of special relativity and the energy-momentum equations that describe geodesics in General Relativity constrain the form of the time component of the metric tensor. It is possible to satisfy the constraint by considering gravitational fields to be real, independent entities that possess field energy densities and including these as source terms in the Einstein field equations. In this approach, the spacetime metric becomes a function of the gravitational potential rather than having to describe both geometry and gravity. This prevents the formation of event horizons and can eliminate the need for a cosmological constant (aka "dark energy"). The gravitational potentials and field energies offer a path toward alleviating the incompatibility of quantum mechanics and General Relativity.

Keywords: General relativity, concept, spacetime, mass-energy, dark energy, gravity.

1 Introduction

Einstein's theory of special relativity changed the way we regard space and time. In his theory of General Relativity he further showed that space and time must also be modified by the presence of gravitating objects. He recognized the need for a metric tensor description of gravity and eventually reached the conclusion that gravity could be entirely encapsulated in the geometry of spacetime. In his General Relativity theory, he considered all forms of mass-energy to be sources of gravitational fields excepting the energy of the gravitational field itself. Among the interesting consequences of this choice are the prediction of the black hole event horizon and the need for "dark energy" to drive the acceleration of the expansion of the cosmos. Although these have become part of the current mystique of relativistic physics, enthusiastically accepted by many physicists, they are not necessary features of a successful gravity theory. Both can be eliminated by considering gravity to be a real field in its own right rather than just a manifestation of spacetime geometry.

We can begin to explore this by embracing the other accepted features of General Relativity while restricting our attention to static fields and geodesic motions of test particles in matter free space. The geodesics will be of the form:

$$ds^2 = g_{00}c^2dt^2 + g_{11}(dx^1)^2 + g_{22}(dx^2)^2 + g_{33}(dx^3)^2 = c^2d\tau^2 \tag{1.1}$$

where the metric coefficients g_{ii} are functions of space coordinates (x^1, x^2, x^3) and no longer always have the values (1,-1,-1,-1) of special relativity. x^0 is the time, ct and c, the invariant speed of light. τ is proper time kept by a clock moving with the test particle.

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Since the mass of a particle depends upon its position in a gravitational field, we define m_0 to be the mass of a particle at rest at a location where the gravitational potential would be zero. The energy-momentum equation consistent with Eq. 1.1 then becomes

$$\Sigma_{i=0}^{3}\ g_{ii}(p^i)^2 = m_0^2c^2 = \Sigma_{i=0}^{3}\ g^{ii}(p_i)^2 \tag{1.2}$$

where $p^i \equiv m_0(dx^i/d\tau)$ are energy-momentum components. The left member of Eq. 1.2 merely recapitulates Eq. 1.1, but the covariant components are given by $p_k = g_{kk}p^k$, for which $p_0 = g_{00}p^0 = E/c$, where $E = mc^2$ is the conserved energy of a particle of mass m in geodesic motion.

If a particle of mass m is displaced quasistatically by $d\vec{r}$ by a force $\vec{F}$, the work done is $\vec{F}\cdot d\vec{r} = dE = c^2dm$. If the force is due to a gravitational field for which the potential is U; and rendered dimensionless by division by c^2, such that $\phi = U/c^2$, then the gravitational force on m is given by $\vec{F} = -mc^2\vec{\nabla}\phi$. The change of potential associated with a quasistatic displacement of the particle is defined as the work done by an opposing external force, thus

$$c^2dm = mc^2\vec{\nabla}\phi\cdot\vec{dr} = mc^2d\phi \tag{1.3}$$

which integrates to

$$m = m_0e^{\phi} \tag{1.4}$$

Thus the mass of a particle at rest will depend on its position within a gravitational field, but for accord with Eq. 1.1, the mass given by Eq. 1.4 needs to be multiplied by the Lorentz factor, $\gamma = 1/\sqrt{1 - v^2/c^2}$, where $v = \sqrt{\Sigma_{i=1}^{3}\ (-g_{ii}/g_{00})(dx^i/dt)^2}$ is the proper speed of a moving particle.

In the case of a particle momentarily at rest in a gravitational field, $p_0 = E/c = mc$ and all other components of momentum are zero. This is a special state of motion that is accessible to a particle in geodesic motion under the influence of gravity; for example, at the apex of flight of a particle that travels in a direction opposite to that of the gravitational force. At this position Eq. 1.2 yields

$$g^{00}E^2/c^2 = g^{00}m^2c^2 = m_0^2c^2 \tag{1.5}$$

Substituting for m from Eq. 1.4 leads immediately to $g^{00}e^{2\phi} = 1$. Noting that $g_{00} = 1/g^{00}$ for a diagonal metric we obtain the interesting and insufficiently appreciated result that special relativity and the definition of a potential require

$$g_{00} = e^{2\phi} \tag{1.6}$$

One interesting consequence of Eq. 1.6 is that the distantly observed redshift, z, of radiation originating from the surface of a gravitating mass, M, of radius, R, is an exact exponential function given by $z = e^{-\phi} - 1$, where $\phi = -GM/(c^2R)$. Einstein noted that the expression for the redshift must "in all strictness" be an exponential function of the potential (Einstein 1907, as translated by Schwartz 1977).

2 Spatial Dependence of the Exponential Metric:

To complete the metric, we adopt an isotropic form in which all spatial components are equally affected by the presence of gravitating matter.

$$ds^2 = e^{2\phi}c^2dt^2 - e^{\lambda}(dx^2 + dy^2 + dz^2) \tag{2.1}$$

This form would seem to be generally necessary for consistency with the Hughes-Drever experiments (Hughes, Robinson & Beltran-Lopez 1960, Drever 1961) that revealed the isotropy of inertia. The function $\lambda(x, y, z)$ can be constrained by requiring that photons and other waves be permitted to pass through matter free space without gravity causing dispersion. The requirement that wave speeds in matter free space be independent of wavelength or direction of propagation relative to the direction of the gravitational field can be enforced by requiring a "harmonic coordinate condition", (Weinberg 1972, p.

163) $\partial_k(\sqrt{-g}\ g^{kj}) = 0$, where $\sqrt{-g}$ is the determinant of the metric tensor. This condition ensures that the phase speed of light will be the same in every direction[2]. With g_{00} given by Eq. 1.6, this requires $\partial_i(e^{\phi+\lambda/2}) = 0$. With the boundary condition that $g_{ii} = 1$ where $\phi = 0$, we require $\lambda = -2\phi$ and $g_{ii} = -e^{-2\phi}$ for $i = 1, 2, 3$ and

$$ds^2 = e^{2\phi}c^2dt^2 - e^{-2\phi}(dx^2 + dy^2 + dz^2) \tag{2.2}$$

This metric was first proposed by Hüseyin Yilmaz (Yilmaz 1958, 1971) (excepting that Yilmaz used a potential which is the negative of the ϕ used here) A very interesting feature of this metric is that the geodesics of particles traveling in this matter-free space reduce to Newtonian equations of motion. Using one of the standard forms of the geodesic equations with $u^i = dx^i/ds$ and $g_{ii} = e^{-2\phi}$ for $i = 1, 2, 3$,

$$\frac{du_k}{ds} = (1/2)(\partial_k g_{ij})u^iu^j = (1/2)(-2\partial_k\phi)\delta_{ij}g_{ij}u^iu^j = -\partial_k\phi \tag{2.3}$$

It is also of considerable interest, that we have not restricted ϕ to be the potential of a single source. It might well be a superposition of the potentials of multiple sources. This would provide interactive many body solutions of the equations of motion within the context of a relativistic metric gravity theory. Such solutions have not previously been found for General Relativity.

3 Astrophysical Tests

Changing to spherical coordinates produces a metric form that is extremely useful for astrophysical applications[3] .

$$ds^2 = e^{2\phi}c^2dt^2 - e^{-2\phi}[(dr)^2 + (rd\theta^2)^2 + (r\ sin\theta d\Phi)^2] \tag{3.1}$$

The discussion to this point has not included any restriction on the form of ϕ, however, considering the success of Newtonian gravity theory within our solar system, it should be obvious that the relevant potential for the solar system would be $\phi = -GM/(c^2r)$, where G is the Newtonian gravitational constant and M, the mass of the sun. With this potential, the geodesics of Eq. 3.1 pass all of the early solar system tests of relativistic gravity theory that were offered as confirmation for General Relativity. In fact, sufficient accuracy is achieved in describing the perihelion advance of Mercury, the bending of starlight that grazes the sun and the Shapiro time delay of microwaves reflected from Venus if the exponential functions are expanded to only second order in ϕ.

To higher orders, this metric with a Newtonian potential correctly describes even the relativistic geodesic orbits of particles in astrophysical accretion disks. The differences of results obtained from this metric and the Schwarzschild metric are too small to be resolved by the presently available observational data. For example, there is an innermost marginally stable circular orbit for particles in an accretion disk. The geodesics of Eq. 3.1 show this orbit radius to be $5.24GM/c^2$, compared to a radius of $6GM/c^2$ for the Schwarzschild metric. The energy that can be dissipated via viscous heating and radiation as a particle spirals in to this radius is 5.5% of the rest mass energy of the particle according to the geodesics of Eq. 3.1. The energy dissipation in a Schwarzschild metric is 5.7% of the rest mass.

There is a photon orbit for the metric of Eq. 3.1 with a radius of $2GM/c^2$, compared to $3GM/c^2$ for the Schwarzschild metric. The shadow of a dark object small enough to live within its photon orbit for

[2] A plane wave of the form $\psi = e^{i(\omega t - \mathbf{k}.\mathbf{r})}$)propagating in the space of Eq. 2.1 should satisfy a generalized d'Alembertian equation, $\Box^2\psi = (1/\sqrt{-g})\partial_i(\sqrt{-g}\ g^{ij}\partial_j\psi) = 0$. This will generate nonzero terms of the form $\psi k_j\partial_i(\sqrt{-g}\ g^{ij})$ that will make the phase speed of light depend on its direction of travel unless $\partial_i(\sqrt{(-g}\ g^{ij}) = 0$. This harmonic gauge condition is assumed to hold for the metric of Eq. 2.1 and must also hold if ψ were to represent a gravitational wave.

[3] Although Eqs. 2.1, 2.2 and 3.1 are static metrics, it is possible to permit time dependence while maintaining a harmonic coordinate requirement that gravity not produce dispersion of photons. Define functions ϕ_0 and ϕ_1 such that $4\phi_0 = 3\lambda/2 - \phi$ and $4\phi_1 = \lambda/2 + \phi$ and require that $\partial_0\phi_0 = 0$ and $\vec{\nabla}\phi_1 = 0$. Thus ϕ_0 depends only on (x, y, z) while ϕ_1 depends only upon time, t. The metric can then be written as $ds^2 = e^{(2\phi_0+6\phi_1)}c^2dt^2 - e^{(-2\phi_0+2\phi_1)}(dx^2 + dy^2 + dz^2)$. Time dependence such as this would be necessary for a cosmological metric that would evolve in time (Robertson 2015).

Eq 3.1 would have a diameter of $4e^1 GM/c^2 = 10.873GM/c^2$, compared to $6\sqrt{3}GM/c^2 = 10.392GM/c^2$ for a Schwarzschild black hole. The recently published image of the central object of M87 is a great achievement, but it is apparently not capable of establishing that the object is a black hole.

The metric Eq. 3.1 with the Newtonian potential appears to be applicable to masses of arbitrarily small radius. Radiation from extremely compact objects might be so gravitationally redshifted that their surface radiations would be undetectable, but lacking event horizons they would not be black holes (Robertson & Leiter 2003, 2006, Robertson 2016). But despite the apparent ability of the metric Eq. 3.1 to describe the solar system and relativistic astrophysics, it has been excluded by fiat and not allowed as a solution of the Einstein field equations.

4 Field Equations and Gravitational Field Energy

The Einstein field equations can be written as

$$G_i^j = -(8\pi G/c^4)T_i^j \tag{4.1}$$

Where G_i^j is the Einstein tensor, G the Newtonian gravitational force constant and T_i^j is a stress-energy tensor that represents the gravitational sources. Evaluating the G_0^0 component of the Einstein tensor for the isotropic metric of Eq. 2.1 yields exactly

$$G_0^0 = e^{-\lambda}(\lambda'' + 2\lambda'/r + \lambda'^2/4) = -(8\pi G/c^4)T_0^0 \tag{4.2}$$

Where primes represent derivatives with respect to r.

If the Newtonian potential, $\lambda/2 = -\phi = GM/(c^2 r)$ of matter-free space is substituted into Eq. 4.2 and the equations for other components of G_i^j, it produces
$T_0^0 = -e^{2\phi}(1/(8\pi G))(GM/r^2)^2 = T_2^2 = T_3^3 = -T_1^1$. This shows that there is a non-zero field stress-energy tensor in the matter-free space of a Newtonian gravitational field. If we define the field strength as the force per unit mass on a test particle, there is a g-field equal to $-GM/r^2$. The Newtonian stress-energy tensor components are thus proportional to $(1/2)\epsilon_g g^2$, where $\epsilon_g = 1/(4\pi G)$, and completely analogous to an electric field energy density $(1/2)\epsilon_0 E^2$. Considering this field energy density to be physically real and including it as a source in the right member of the Einstein field equations is all that is needed for Eq. 3.1 with a Newtonian potential to become an acceptable solution of the Einstein field equations.

It is actually accepted that a gravitational field energy must exist in the weak field Newtonian limit of General Relativity and the parts of the Einstein tensor that would describe it have even been identified (Weinberg 1972, Sec. 7-2, 7-6, Eq. 7.6.4), but it is somehow believed that the field equations would also be compatible with $\nabla^2\lambda = 0$ and with $G_i^j \equiv 0$ giving the solution for the metric. As shown by Eq. 4.2, this is clearly not possible due to the presence of the term $\lambda'^2/4$. Weinberg's Eq. 7.6.4 correctly identifies this term as a field energy term, but the Newtonian potential is not then a solution of the General Relativity field equations which require $G_i^j \equiv 0$ in matter-free space.

Since it is now obviously not necessary to exclude gravitational field energy density in a successful astrophysical gravity theory, one would expect to find some other strong reason for this exclusion that is still part of General Relativity. But maintaining the exclusion instead seems to require us to accept unphysical event horizons and a mysterious "dark energy" that dominates the cosmos.

5 The Event Horizon

Since Eq. 1.6 depends only on the definition of a potential, we can use it to find the potential applicable to mass M located at $r = 0$ for the isotropic metric of General Relativity. (Weinberg 1972, p. 181)

$$g_{00} = e^{2\phi_{GR}} = \frac{(1 - GM/2c^2 r)^2}{(1 + GM/2c^2 r)^2} \tag{5.1}$$

from which we see that the gravitational potential ϕ_{GR} for the isotropic metric of General Relativity must be

$$\begin{aligned} \phi_{GR} &= ln((1 - GM/2c^2r)/(1 + GM/2c^2r)) \\ &\approx -GM/c^2r - (GM/c^2r)^3/12 - (GM/c^2r)^5/80 \ \ldots \end{aligned} \tag{5.2}$$

This is a Newtonian potential at lowest order, but it diverges as an event horizon is approached at $r = GM/2c^2$. A particle in geodesic free fall in this potential would have constant energy, E, while subject to a gravitational force given by $\vec{F} = -E\vec{\nabla}\phi_{GR}$. This force becomes infinite as the event horizon is approached. This seems very unphysical since it would occur at a location for which neither gravitating mass nor extreme curvature of spacetime exist. The requirement that T_i^j exclude the energy density of the gravitational field, leading to a requirement that $G_i^j \equiv 0$, is what produced ϕ_{GR} rather than the Newtonian potential.

6 Discussion

Yilmaz has proposed that the field energy tensor of the gravitational field should be included as part of T_i^j. If this is added explicitly as a source term in the right member of the field equations for matter-free space, then the Newtonian potential provides an exact solution of the Einstein field equations for the metric of Eq. 3.1. Once this is accepted for weak field conditions it is difficult to see why we should ever expect the field energy to become negligible for the stronger fields of progressively more compact masses. Thus there would appear to be no reason to continue to use metrics with event horizons. As discussed above, Eq. 3.1 with a Newtonian potential is capable of encompassing most of relativistic astrophysics.

Yilmaz provided some general expressions (Yilmaz 1992) for use with metrics with time dependence or potentials more complicated than that of Newtonian gravity. These have been used (Robertson 2015) as source terms for the field equations for a Yilmaz cosmological metric that correctly describes the acceleration of the expansion of the cosmos without need of a cosmological constant (aka "dark energy").

In the field equations for a matter continuum, it is not always necessary to add separate field stress-energy source terms to the right member of the field equations if it is recognized that field energy contributes to mass-energy density or fluid pressure terms in the right members of the field equations. In fact, it is often difficult to disentangle the various source term contributions.

Rather than regarding gravitation to be a mere manifestation of spacetime geometry, perhaps we should consider that gravity might be a real quantum field with its quantum effects manifest in the gravitational potentials. As described here, this leaves most of the mathematical structure of General Relativity intact, but it removes spacetime geometry from its central role in gravity theory. The fabric of spacetime is certainly affected by gravity via the gravitational potential, but the underlying cause of a gravitational potential is a matter that can be considered separately from spacetime geometry. There is no need for gravity theory to be concerned with event horizons, "firewalls" or other singular effects that arise from the exclusion of a field stress-energy tensor. While minor in its other mathematical effects, this might still seem to many physicists to be a major conceptual change. Much work remains for a younger generation of physicists to extend and polish the Yilmaz theory for application to the interiors of mass distributions or to circumstances for which harmonic coordinate conditions might not apply.

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