Article

Bianchi Type III String Cosmological Model in f(R,T) Modified Gravity Theory

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Abstract

We have analyzed Bianchi type III string cosmological model in $f(R,T)$ modified theory of gravity, where the gravitational Lagrangian is given by a arbitrary function of Ricci Scalar R and the trace of the tress-energy momentum tensor T, by consider $f(R,T) = R + 2f(T)$. To solve $f(R,T)$ modified field equations by using shear scalar (σ) proportional to expansion scalar (θ). We also discuss Hubble's parameter in terms of redshift, luminosity distance modulus and jerk parameter and observe that our solutions favor recent cosmological observations. The physical and geometrical properties of the model have been discussed.

Keywords: Bianchi Type III, string, cosmological model, modified gravity.

1 Introduction:

Recent observational and theoretical evidence prove that the universe is going through a phase of accelerated expansion [1-10]. It is generally supposed that this acceleration is due to dark matter and negative pressure, which is known as dark energy. The greatest problem of 21^{st} century authors are understanding the origin of dark energy and its nature. The theoretical models explain the nature of dark energy and the acceleration expansion i.e., cosmological constant (Padmanabhan [11]), quintessence (Farooq *et al.*[12]), phantom energy (Nojiri *et al.*[13]), k-essence (Chiba *et al.*[14], Pasqua *et al.*[15]), tachyon (Padmanabhan and Chaudhury [16]), Chapling gas (Bento *et al.*[17], Jamil [18]),and cosmological nuclear energy (Gupta and Pradhan [19]).

Kibble and Turok[20] have studied the importance of cosmic strings in the field of general relativity and their gravitational effect in 1982. Many aspects of the cosmic strings have been investigated by varies authors[21-25], when these strings coupled with perfect fluid and electromagnetic field in general relativity. As advancement of the studies a model was developed by Letelier[26] considering the uxsual general relativity in term of strings, it is the prime concern of many cosmological models as Bianchi type I and Kantowski-Sach. Krori *et al.*[27] studied the spatially homogeneous models of Bianchi type-II, VI_0, VII, and IX in presence of cosmic strings. Among the above models many have been generalized to null string and to perfect fluid strings under the context of source matter stress energy tensor for a perfect dust along with cosmic strength. The conservation law of dust and equation of motion of strings are able to be drawn in contemperary studies.

After the well testing of the Einstein general relativity theory of gravity, all observational local tests are passed by it up to the solar system scale. The Einstein gravity model of general theory of relativity becomes failure at large scales and needs modified theories of general relativity which have been developed and drawn much attention. On larger cosmological scales the modification in Einstein-Hilbert action

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may be an accurate description of a late time cosmic acceleration of the expanding universe. A natural combination of the early time inflation and late time acceleration is provided by $f(R)$ modified theory of gravity. To explain the accelerated expansion of the universe, other modified theories such as theory of scalar-Gauss-Bonnet gravity, so called $f(G)$(Nojiri *et al.*[28]) and theory of $f(T)$ gravity (Linder [29]), where T is the torsion have been proposed. Harko and Lobo [30] have proposed the maximal extension of the Hilbert-Einstein action by considering the gravitational Lagrangian as a arbitrary function of Ricci scalar R and of the matter Lagrangian L_m. A new generalized theory known as $f(R,T)$ gravity has been proposed by Harko et al. [31] in a recent paper. According to this theory, an arbitrary function of the scalar curvature R and the trace of the energy momentum tensor T is involved in gravitational Lagrangian. Recent studied P. K Sahoo *et al.*[39], Bianchi type string cosmological models in $f(R,T)$ gravity. Inspired by the above discussions, in this paper, We have analyzed Bianchi type III string cosmological model in $f(R,T)$ modified theory of gravity, where the gravitational Lagrangian is given by a arbitrary function of Ricci Scalar R and the trace of the tress-energy momentum tensor T, by consider $f(R,T) = R + 2f(T)$. To solve $f(R,T)$ modified field equations by using shear scalar (σ) proportional to expansion scalar (θ). We also discuss Hubble's parameter in terms of redshift, luminosity distance redshift, distance modulus redshift and jerk parameter and observe that our solutions favor recent cosmological observations. The physical and geometrical properties of the model have been discussed.

2 The basic equations:

The theory of suggests a modified gravity action given by

$$S = \frac{1}{16\pi}\int f(R,T)\sqrt{-g}d^4x + \int L_m\sqrt{-g}d^4x, \tag{2.1}$$

where ($f(R,T)$ is an arbitrary function of the Ricci scalar (R), and the trace (T) of the stress- energy tensor of the matter $T_{\mu\nu}$. L_m is the matter Lagrangian density. $T_{\mu\nu}$ is defined as

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta\sqrt{-g}L_m}{\delta g^{\mu\nu}}, \tag{2.2}$$

and its trace by $T = g^{\mu\nu}T^{\mu\nu}$. Assuming that the Lagrangian density L_m of matter depends only on the metric tensor components $g_{\mu\nu}$ and not on its derivatives leads to

$$T_{\mu\nu} = g_{\mu\nu}L_m - 2\frac{\partial L_m}{\partial g^{\mu\nu}}. \tag{2.3}$$

By varying the action S with respect to the metric tensor components $g^{\mu\nu}$, the gravitational field equations of $f(R,T)$ gravity is obtained (Harko et al.[31])as

$$f_R(R,T)R_{\mu\nu} - \frac{1}{2}f(R,T)g_{\mu\nu} + (g_{\mu\nu}\nu\Box - \nabla_\mu\nabla_\nu)f_R(R,T)$$
$$= 8\pi T_{\mu\nu} - f_T(R,T)T_{\mu\nu} - f_T(R,T)\Theta_{\mu\nu}, \tag{2.4}$$

where $\Box = \nabla^i\nabla_i, f_R(R,T) = \frac{\partial f(R,T)}{\partial R}, f_T = \frac{\partial f(R,T)}{\partial T}$ and ∇_i denotes the coveriant derivative. $\Theta_{\mu\nu}$ is given by

$$\Theta_{\mu\nu} = -2T_{\mu\nu} + g_{\mu\nu}L_m - 2g^{\alpha\beta}\frac{\partial^2 L_m}{\partial g^{\mu\nu}\partial g^{\alpha\beta}}. \tag{2.5}$$

Using matter Lagrangian L_m, the stress-energy tensor of the matter Lagrangian is given by

$$T_{ij} = (p+\rho)u_\mu u_\nu + pg_{\mu\nu} - \lambda\chi_\mu\chi_\nu, \tag{2.6}$$

where $u^\mu = (0,0,0,1)$ is the four velocity in the moving coordinates which satisfies the conditions $u^\mu u_\mu = -1$ and $u^\mu \nabla_\mu \nabla_\nu = 0$. Here ρ is the energy density, density for cloud of strings with particles to them, p denotes the pressure of the fluid and λ is the string tension density. The particle density ρ_p is defined as

$$\rho = \rho_p + \lambda, \tag{2.7}$$

λ may be positive or negative.
Moreover, the matter Lagrangian is not uniquely considered. So, the source term is described as a function of Lagrangian matter through different choices of it. Here we choose a perfect fluid matter as $L_m = -p$, which yields

$$\Theta_{\mu\nu} = -2T_{\mu\nu} - pg_{\mu\nu}. \tag{2.8}$$

Since the field equations in $f(R,T)$ gravity also depend on the physical nature of the matter field (through the tensor $\Theta_{\mu\nu}$), for each choice of f we obtain several theoretical models. Three explicit specification of the functional from of f has been considered

$$(i) f(R,T) = R + 2f(T), (ii) f(R,T) = f_1(R) + f_2(T),$$

$$(iii) f(R,T) = f_1(R) + f_2(R)f_3(T). \tag{2.9}$$

Let us assume $f(R,T) = (R) + 2f(T)$, the gravitational field equation (4) becomes

$$R_\mu \nu - \frac{1}{2}Rg_{\mu\nu} = 8\pi T_{\mu\nu} + 2f^{'}(T)T_{\mu\nu}[f(T) + 2pf^{'}(T)]g_{\mu\nu}. \tag{2.10}$$

Harko et al. have constructed some FRW cosmological models by using different forms of the $f(R,T)$ gravity.

3 The metric and field equations:

We consider the Bianchi type-III metric in the form

$$ds^2 = -dt^2 + A^2dx^2 + B^2e^{2x}dy^2 + C^2dz^2, \tag{3.1}$$

where A, B and C are functions of cosmic time t. This ensure that the model is totally anisotropic and spatially homogeneous. We define average scale factor (a) for Bianchi type-III space time as

$$a = (ABC)^{\frac{1}{3}}. \tag{3.2}$$

We define Hubble's parameter(H), expansion scalar(θ), shear scalar (σ), Anisotropy parameter$(\overline{A})$, and deceleration parameter(q) as

$$H = \frac{\dot{a}}{a}; \tag{3.3}$$

$$\theta = 3H; \tag{3.4}$$

$$\sigma^2 = \frac{1}{2}(\Sigma_{i=1}^3 H_i^2 - \frac{1}{3}\theta^2); \tag{3.5}$$

$$\overline{A} = \frac{1}{3}\Sigma_{i=1}^3(\frac{\Delta H_i}{H})^2 = \frac{2\sigma^2}{3H^2}; \tag{3.6}$$

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = \frac{d}{dt}(\frac{1}{H}) - 1; \tag{3.7}$$

where $\Delta H_i = H_i - H$ (i=x,y,z) represents the directional Hubble's parameters.
The equation(10) with (6) for line element (11) give rise to

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = -\left(\frac{16\pi + 3\mu}{2\mu}\right) p - \left(\frac{16\pi + 3\mu}{2\mu}\right) \lambda + \frac{\rho}{2}; \tag{3.8}$$

$$\frac{\ddot{C}}{C} + \frac{\ddot{A}}{A} + \frac{\dot{C}\dot{A}}{CA} = -\left(\frac{16\pi + 3\mu}{2\mu}\right) p - \frac{\lambda}{2} + \frac{\rho}{2}; \tag{3.9}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} = -\left(\frac{16\pi + 3\mu}{2\mu}\right) p - \frac{\lambda}{2} + \frac{\rho}{2}; \tag{3.10}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} - \frac{1}{A^2} = \left(\frac{16\pi + 3\mu}{2\mu}\right) \rho + \frac{\lambda}{2} + \frac{\rho}{2}; \tag{3.11}$$

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = 0. \tag{3.12}$$

4 Solutions of the field equations:

Integrating equation(22), we have

$$C = mB; \tag{4.1}$$

where m is constant. Without loss of generality, we assume $m = 1$. Hence equation (23) gives

$$A = B; \tag{4.2}$$

Equations(18)-(21) are reduced to

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = -\left(\frac{16\pi + 3\mu}{2\mu}\right) p - \left(\frac{16\pi + 3\mu}{2\mu}\right) \lambda + \frac{\rho}{2}; \tag{4.3}$$

$$\frac{\ddot{C}}{C} + \frac{\ddot{B}}{B} + \frac{\dot{C}\dot{B}}{CB} = -\left(\frac{16\pi + 3\mu}{2\mu}\right) p - \frac{\lambda}{2} + \frac{\rho}{2}; \tag{4.4}$$

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} - \frac{1}{B^2} = -\left(\frac{16\pi + 3\mu}{2\mu}\right) p - \frac{\lambda}{2} + \frac{\rho}{2}; \tag{4.5}$$

$$\frac{\dot{B}^2}{B^2} + 2\frac{\dot{B}\dot{C}}{BC} - \frac{1}{B^2} = \left(\frac{16\pi + 3\mu}{2\mu}\right) \rho + \frac{\lambda}{2} + \frac{\rho}{2}. \tag{4.6}$$

From equations (25) and (26), we get

$$\lambda = 0. \tag{4.7}$$

Again equations(25)-(28) are reduced to

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = -kp + \frac{\rho}{2}; \tag{4.8}$$

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} - \frac{1}{B^2} = -kp + \frac{\rho}{2}; \tag{4.9}$$

$$\frac{\dot{B}^2}{B^2}+2\frac{\dot{B}\dot{C}}{BC}-\frac{1}{B^2}=k\rho+\frac{\rho}{2}; \tag{4.10}$$

where $k=\left(\frac{16\pi+3\mu}{2\mu}\right)$.
Equations (30)-(32) are three equations with four unknown namely, B,C, p, ρ. Now to get the determinate solutions, we require one more condition.
We assume that the expansion scalar (θ) is proportional to shear scalar (σ), and using (22), we get

$$\frac{1}{\sqrt{3}}\left(\frac{\dot{B}}{B}-\frac{\dot{C}}{C}\right)=\alpha_0\left(2\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right); \tag{4.11}$$

where α_0 is constant,which yields to

$$\frac{\dot{B}}{B}=n\frac{\dot{C}}{C}; \tag{4.12}$$

where $n=\frac{2\alpha_0\sqrt{3}+1}{1-\alpha_0\sqrt{3}}$,from equation (34), we have

$$B=C^n. \tag{4.13}$$

This condition is explained to Thorne[32], the observations of the velocity red-shift relation for extra-galactic source says that Hubble expansion of the universe is isotropic at that time within$\simeq$ 30 present Kristian and Sachs [33], Kantowski and Sachs[34]. To put more precisely, red-shift studies the limit

$$\frac{\sigma}{H}\leq 0.3;$$

on the ratio of the shear tensor (σ) and Hubble parameter (H) in the nearest of our galaxy today.
From equations(30) and (31), we have

$$\frac{\ddot{B}}{B}+\frac{\dot{B}^2}{B^2}-\frac{\ddot{C}}{C}-\frac{\dot{B}\dot{C}}{BC}-\frac{1}{B^2}=0. \tag{4.14}$$

Using equations(35) in equation (36), we obtain

$$\frac{\ddot{C}}{C}+2n\left(\frac{\dot{C}}{C}\right)^2=\frac{1}{(n-1)C^{2n}}. \tag{4.15}$$

Solving equation (37), we get

$$C=\left[\frac{nt}{\sqrt{n^2-1}}+c\right]^{\frac{1}{n}}. \tag{4.16}$$

From equations(24),(35) and (38), we have

$$B=\left[\frac{nt}{\sqrt{n^2-1}}+c\right]; \tag{4.17}$$

$$A=\left[\frac{nt}{\sqrt{n^2-1}}+c\right]. \tag{4.18}$$

The scale factor a is defined as

$$a=\left[\frac{nt}{\sqrt{n^2-1}}+c\right]^{\frac{n+2}{3n}}. \tag{4.19}$$

Hence the metric (11) take the from

$$ds^2=-dt^2+\left[\frac{nt}{\sqrt{n^2-1}}+c\right]^2dx^2+\left[\frac{nt}{\sqrt{n^2-1}}+c\right]e^{2x}dy^2 + \left[\frac{nt}{\sqrt{n^2-1}}+c\right]^{\frac{2}{n}}dz^2. \tag{4.20}$$

5 The physical and geometrical properties of the model:

The spatial volume (V), Hubble's parameter (H), expansion scalar (θ), shear scalar (σ), anisotropy parameter $\overline{A}$, and deceleration parameter (q) are given by

$$V = \left[\frac{nt}{\sqrt{n^2-1}} + c\right]^{\frac{2n+1}{n}}; \tag{5.1}$$

$$H = \frac{(2n+1)}{3\sqrt{n^2-1}\left[\frac{nt}{\sqrt{n^2-1}} + c\right]}; \tag{5.2}$$

$$\theta = \frac{(2n+1)}{\sqrt{n^2-1}\left[\frac{nt}{\sqrt{n^2-1}} + c\right]}; \tag{5.3}$$

$$\sigma^2 = \frac{(n-1)}{3(n+1)\left[\frac{nt}{\sqrt{n^2-1}} + c\right]^2}; \tag{5.4}$$

$$\overline{A} = 2\left(\frac{n-1}{2n+1}\right)^2; \tag{5.5}$$

$$q = \frac{n-1}{2n+1}. \tag{5.6}$$

The energy density and pressure are given by

$$\rho = \frac{(2kn + k + \frac{1}{2})}{(k^2 + \frac{1}{4})(n^2-1)} \frac{1}{\left[\frac{nt}{\sqrt{n^2-1}} + c\right]^2}; \tag{5.7}$$

$$p = \frac{(n - k + \frac{1}{2})}{(k^2 + \frac{1}{4})(n^2-1)} \frac{1}{\left[\frac{nt}{\sqrt{n^2-1}} + c\right]^2}. \tag{5.8}$$

We notice that spatial volume (V) is zero at $t = -\frac{c}{\frac{n}{\beta\sqrt{n^2-1}}} = t_1$, and expansion scalar ($\theta$) is infinite which indicates that the universe starts evolving with zero volume and infinite rate of expansion at time $t = t_1$. The scale factor is zero at $t = t_1$, and therefore the model exhibits point type singularity. The density (ρ) , pressure (p) are zero at $t = t_1$. As $t \to \infty$, the spatial volume (V) tends to infinite, Hubble's parameter (H) and expansion scalar (θ), shear scalar (σ) and density (ρ) , pressure (p) are tends to zero. The density (ρ) , pressure (p) both are decreasing functions of cosmic times. The anisotropy ($\overline{A}$) is constant, hence the model posses anisotropic behaviour in whole span of evolution of the universe. The deceleration parameter (q) is positive $n > 1$, which indicates the universe is decelerating phase of expansion and deceleration parameter (q) is negative for $n < 1$, which indicates the universe is accelerating phase of expansion. Recent observations of SNeIa and CBMR agreement accelerating models and present value q_0 of the decelerating parameter are $-1.27 < q < 2$. Redshift survey of galaxy furnish the nature of $q_0 \sim 0.3$, with an upper limit of q_0 strictly less than ~ 0.7. Current observations show that the deceleration parameter of the cosmology is in the range $-1 \leq q \leq 0$, i.e., $q_0 = -0.77$, we take $n = 0.2$ in equation (48), we obtain $q_0 = -0.92$, which lies the present range of q.

6 Expressions for some observable parameters:

(a) $H(z)$ and $\mu(z)$ parameters:
The Hubble parameter (H) provides an estimate of the age of an expanding universe. If also indicates the expanding rate of the universe. From equation (44), we have

$$H = \frac{(2n+1)}{3\sqrt{n^2-1}\left[\frac{nt}{\sqrt{n^2-1}}+c\right]}.$$

Hence

$$\frac{H}{H_0} = \frac{\left[\frac{nt_0}{\sqrt{n^2-1}}+c\right]}{\left[\frac{nt}{\sqrt{n^2-1}}+c\right]}; \tag{6.1}$$

where H_0 is the present value of Hubble's parameter. We have two important observable properties of the universe are related to the scale factor: the cosmological redshift and the Hubble law. One point of confusion is the belief that the cosmological redshifts are ordinary Doppler shifts resulting from motions of the galaxies through space. After all, cosmologists talk about *expansion velocities*, and the Hubble law relates just such a velocity to a distance. An analogy with Doppler shifts can be useful particularly for near by galaxies where the redshifts are small, but the cosmological redshift is really more akin to the gravitational redshift discussed in connection with black holes. Both cosmological redshifts arise directly from a metric. The cosmological redshift is produced by photons traversing expanding space. It does not occur because of reception of the light, as the case for the conventions Doppler shift, but as the light traverse through space. The formula

$$\frac{a_0}{a} = 1+z; \tag{6.2}$$

tells us only the radio of the scale factor today, when the light is received, compared to the scale factor at the time, the light was emitted. It tells us nothing about how the expansion (or contraction) proceeded with time.
From equations (52) and (41), we have

$$\frac{a_0}{a} = 1+z = \frac{\left[\frac{nt_0}{\sqrt{n-1}}+c\right]^{\frac{2n+1}{3n}}}{\left[\frac{nt}{\sqrt{n^2-1}}+c\right]^{\frac{2n+1}{3n}}}. \tag{6.3}$$

Equations (53) and (51), can be written in the form of Hubble parameter and redshift parameter.

$$H = H_0(1+z)^{\frac{3n}{2n+1}}. \tag{6.4}$$

We define distance modulus (μ) as

$$\mu(z) = 5logd_L + 25; \tag{6.5}$$

where d_L is Luminosity distance, defined by

$$d_L = r_1(1+z)a_0. \tag{6.6}$$

A photon emitted by a source with coordinate $r = r_1$ and $t = t_1$ and received at a time t by an observer located at $r = 0$, then find r_1 from the following relation:

$$r_1 = \int_t^{t_0}\frac{dt}{a} = \frac{3\sqrt{n^2-1}}{(n-1)}\left\{\left[\frac{nt_0}{\beta\sqrt{n^2-1}}+c\right]^{\frac{n-1}{3n}} - \left[\frac{nt}{\sqrt{n-1}}+c\right]^{\frac{n-1}{3n}}\right\}. \tag{6.7}$$

From equations (56) and (57), we can obtain the expression for distance modulus.
(b)The Jerk Parameter (j) **:**
The dimensionless jerk parameter (j) third derivative of the scale factor with respect to cosmic time t(Chiba and Nakamura[35], Blandford *et al.* [36], Visser [37], Sahni[38]) and provides a perfect diagnosis of how much a dark energy model is closed to ΛCDM dynamics. A deceleration to acceleration transition occurs for models with a positive value of j_0 and negative value of q_0. Flat ΛCDM models have a constant jerk $j = 1$. The jerk parameter (j) is defined as

$$j(t) = \frac{\dddot{a}}{aH^3} = \left(\frac{a^2H^2}{2H^2}\right)'', \tag{6.8}$$

over dot and primes denote derivatives with respect to cosmic time t and scale factor respectively.
The jerk parameter (j) appears in the fourth term of a Taylor expansion of the scale factor around a_0

$$\frac{a(t)}{a_0} = 1 + H_0(t-t_0) - \frac{1}{2}q_0H_0^2(t-t_0)^2 + \frac{1}{6}j_0H_0^3(t-t_0)^3 + O[(t-t_0)^4]. \tag{6.9}$$

Equation (58) can be written as

$$j(t) = q + 2q^2 - \frac{\dot{q}}{H}. \tag{6.10}$$

From equations (44),(48) and (60), we have

$$j(t) = \frac{(n-1)(4n-1)}{(2n+1)^2}. \tag{6.11}$$

For a flat ΛCDM model, jerk parameter (j) has the value $j = 1 \ \ at \ \ n = 0$.

7 Conclusion:

In this paper, We have analyzed Bianchi type III string cosmological model in $f(R,T)$ modified theory of gravity, where the gravitational Lagrangian is given by a arbitrary function of Ricci Scalar R and the trace of the tress-energy momentum tensor T, by consider $f(R,T) = R + 2f(T)$. To solve $f(R,T)$ modified field equations by using shear scalar (σ) proportional to expansion scalar (θ). We observe that spatial volume (V) is zero at $t = -\frac{\frac{c}{n}}{\beta\sqrt{n^2-1}} = t_1$, and expansion scalar (θ) is infinite which indicates that the universe starts evolving with zero volume and infinite rate of expansion at time $t = t_1$. The scale factor is zero at $t = t_1$, and therefore the model exhibits point type singularity. The density (ρ) , pressure (p) are zero at $t = t_1$. As $t \to \infty$, the spatial volume (V) tends to infinite, Hubble's parameter (H) and expansion scalar (θ), shear scalar (σ) and density (ρ) , pressure (p) are tends to zero. The density (ρ) , pressure (p) both are decreasing functions of cosmic times. The anisotropy $(\overline{A})$ is constant, hence the model posses anisotropic behaviour in whole span of evolution of the universe. The deceleration parameter (q) is positive $n > 1$, which indicates the universe is decelerating phase of expansion and deceleration parameter (q) is negative for $n < 1$, which indicates the universe is accelerating phase of expansion. Recent observations of SNeIa and CBMR agreement accelerating models and present value q_0 of the decelerating parameter are $-1.27 < q < 2$. Redshift survey of galaxy furnish the nature of $q_0 \sim 0.3$, with an upper limit of q_0 strictly less than ~ 0.7. Current observations show that the deceleration parameter of the cosmology is in the range $-1 \leq q \leq 0$, i.e., $q_0 = -0.77$, we take $n = 0.2$, we obtain $q_0 = -0.92$, which lies the present range of q.
We also discussed Hubble's parameter in terms of redshift, luminosity distance modulus and jerk parameter and observe that our solutions favor recent cosmological observations.

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