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Spatially Homogeneous Bianchi Type I Mesonic Models in Two-Fluid Cosmology

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Abstract

In this paper, we present exact solutions of Einstein's field equations in two-fluid cosmology in the presence of a zero-rest-mass scalar field within the framework of a spatially homogeneous Bianchi type-I space-time. In the two-fluid cosmology, one fluid represents the matter content of the universe and the other one to the model of cosmic microwave background radiation. Exact solution of field equations are obtained by assuming that the anisotropy in model is inversely proportional to m^{th} power of the average scale factor. Two classes of anisotropic cosmological models are obtained, one with the power-law expansion ($m \neq 3$) and the other one with exponential expansion ($m = 2$). The physical and kinematical behaviors of the models are also discussed in detail.

Keywords: Bianchi I universe, mesonic field, two-fluid cosmology.

1 Introduction

The simplest model of the observed universe is well represented by spatially homogeneous and isotropic Friedmann-Robertson-Walker (FRW) models, which are in some sense good global approximation of the present-day universe. But on smaller scales, the universe is neither homogeneous and isotropic nor do we expect that the universe in its early stages to have these features. At very early times in the evolution of the universe most of the matter and radiations currently observed are believed to have been created during the inflation. In fact, there are theoretical arguments from the recent experimental data which support the existences of an anisotropic phase approaching to isotropic phase leading to consider models of the universe with anisotropic background. The anomalies found in the cosmic microwave background (CMB) and large scale structure observations stimulated a growing interest in anisotropic cosmological models of the universe.

Spatially homogeneous and anisotropic cosmological models play significant roles in the description of large-scale behaviors of the universe. Bianchi spaces I-IX play important roles in constructing models of spatially homogeneous and anisotropic cosmologies [1]. Here we confine ourselves to Bianchi type-I space-time which is the simplest generalization of zero-curvature FRW model. In a Bianchi type-I model the spatial sections are flat but the expansion or contraction rates are direction dependent. Bianchi type-I model is extensively studied so for by several authors in different physical contexts.

Several cosmologists studied the two-fluid cosmological models in general relativity. Coley and Dunn [2] investigated Bianchi type VI_0 model in two-fluid cosmology. Pant and Oli [3] obtained two-fluid cosmological model for a Bianchi type-II space-time. Further, Oli [4, 5] presented Bianchi type-I space-times with and without cosmological and gravitational constants. Adhav et al.[6, 7] presented anisotropic two-fluid cosmological models of Bianchi type-III and V in two-fluid cosmology. Further, Adhav et al. [8]

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investigated a power-law scaling Kaluza-Klein cosmological model dominated by two interacting perfect fluids. Singh et al. [9] obtained a two-fluid cosmological model of Bianchi type-V with negative constant deceleration parameter. Samanta [10] investigated an anisotropic Bianchi type-III cosmological model with variable gravitational and cosmological constants. Samanta and Debata [11] presented a class of two-fluid models of the universe in a five-dimensional spherically symmetric space-time. Venkateswarlu and Sreenivas [12] studied anisotropic Kasner type two-fluids cosmological mode in the presence of zero-rest mass scalar field.

Motivated by above works, we have investigated a two-fluid cosmological models of Bianch type-I in the presence of zero-rest-mass scalar field. The paper is organized as follows: In Sect. 2, the metric and field equations are discussed. In Sect. 3, the exact solutions of the field equations are obtained by using the assumption that the anisotropy in the model is inversely proportional to a power function of average scale factor of the model which correspond to two classes of the universe, one with power-law expansion and the other one with exponential expansion. The physical and kinematical features of the models are discussed separately. Some concluding remarks are outlined in Sect.4.

2 The Metric and Field Equations

We consider the spatially homogeneous Bianchi type-I space-time of the form

$$ds^2 = dt^2 - A^2dx^2 - B^2dy^2 - C^2dz^2 \tag{2.1}$$

where A, B and C are functions of cosmic time t.

The Einstein's field equations in two-fluid cosmology in the presence of a zero-rest-mass scalar field in proper units ($8\pi G = c = 1$) are

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \left(\phi_{,\mu}\phi_{,\nu} - \frac{1}{2}g_{\mu\nu}\phi_{,\alpha}\phi^{,\alpha}\right) = -T_{\mu\nu} \tag{2.2}$$

where the energy-momentum tensor $T_{\mu\nu}$ for a two-fluid source is given by

$$T_{\mu\nu} = T^{(m)}_{\mu\nu} + T^{(r)}_{\mu\nu}. \tag{2.3}$$

Here $T^{(m)}_{\mu\nu}$ is the energy-momentum tensor for matter field and $T^{(r)}_{\mu\nu}$ is the energy-momentum tensor given by

$$T^{m}_{\mu\nu} = (\rho_m + p_m)u^{(m)}_{\mu}u^{(m)}_{\nu} - p_m g_{\mu\nu} \tag{2.4}$$

and

$$T^{r}_{\mu\nu} = \frac{4}{3}\rho_r u^{(r)}_{\mu}u^{r}_{\nu} - p_r g_{\mu\nu} \tag{2.5}$$

where ρ_m, p_m and ρ_r are matter energy density, matter pressure and radiation density respectively. In comoving coordinate system, the 4-velocity vectors can be taken as

$$u^{(m)}_{\mu} = (0, 0, 0, 1), \qquad u^{(r)}_{\mu} = (0, 0, 0, 1). \tag{2.6}$$

With the help of (2.3), (2.4), (2.5) and (2.6), the field equation (2.2) for the metric (2.1) lead to following system of equations:

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} + \frac{1}{2}\dot{\phi}^2 = -p_m - \frac{1}{3}\rho_r, \tag{2.7}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A}}{A}\frac{\dot{C}}{C}+\frac{1}{2}\dot{\phi}^2=-p_m-\frac{1}{3}\rho_r, \tag{2.8}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}}{A}\frac{\dot{B}}{B}+\frac{1}{2}\dot{\phi}^2=-p_m-\frac{1}{3}\rho_r, \tag{2.9}$$

$$\frac{\dot{A}}{A}\frac{\dot{B}}{B}+\frac{\dot{A}}{A}\frac{\dot{C}}{C}+\frac{\dot{B}}{B}\frac{\dot{C}}{C}+\frac{1}{2}\dot{\phi}^2=\rho_m+\rho_r. \tag{2.10}$$

The scalar function ϕ satisfies the wave equation $\phi^{;\mu}_{;\nu}=0$ which leads to

$$\ddot{\phi}+\dot{\phi}\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{C}{C}\right)=0. \tag{2.11}$$

An overdot denotes derivative with respect to t.

The average scale factor a and the spatial volume V of the metric (2.1) are defined by

$$V=a^3=ABC. \tag{2.12}$$

The physical parameters of dynamical interest in cosmology are the expansion scalar (θ), shear scalar (σ), Hubble parameter (H) and anisotropic parameter (A_m) given by

$$\theta=3\frac{\dot{a}}{a}=\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right), \tag{2.13}$$

$$\sigma^2=\frac{1}{2}\left[\left(\frac{\dot{A}}{A}\right)^2+\left(\frac{\dot{B}}{B}\right)^2+\left(\frac{\dot{C}}{C}\right)^2\right]-\frac{1}{6}\theta^2, \tag{2.14}$$

$$H=\frac{1}{3}(H_1+H_2+H_3), \tag{2.15}$$

$$A_m=\frac{1}{3}\sum_{\mu=1}^{3}\left(\frac{H_\mu-H}{H}\right)^2 \tag{2.16}$$

where H_1, H_2 and H_3 are directional Hubble parameters in the direction of x, y and z axes respectively.

An important observational quantity in cosmology is the deceleration parameter q defined by

$$q=-\frac{a\ddot{a}}{\dot{a}^2}. \tag{2.17}$$

The sign of q indicates whether the model inflates or not. The positive of q corresponds to standard decelerating models, whereas the negative sign indicates inflation.

3 Solution to the Field Equation

We have five equation (2.7)-(2.11) in seven unknown parameter A, B, C, ρ_m, p_m, ρ_r and ϕ. Thus to get a deterministic solution we require two additional conditions involving field variables and physical variables.

Subtracting (2.7) from (2.8), (2.8) from (2.9) and (2.9) from (2.7)and integrating the results, we obtain

$$\frac{\dot{A}}{A}-\frac{\dot{B}}{B}=\frac{k_1}{a^3}, \tag{3.1}$$

$$\frac{\dot{B}}{B} - \frac{\dot{C}}{C} = \frac{k_2}{a^3}, \tag{3.2}$$

$$\frac{\dot{A}}{A} - \frac{\dot{C}}{C} = \frac{k_3}{a^3} \tag{3.3}$$

where k_1, k_2 and k_3 are integration constants. Substitution of (3.1), (3.2) and (3.3) in (2.14) lead to

$$\sigma = \frac{k}{a^3} \tag{3.4}$$

where k is an arbitrary constant expressible in terms of k_1, k_2 and k_3.

Equations (2.7)-(2.10) suggest that

$$A^n = BC \tag{3.5}$$

where n is a positive constant. We set

$$B = A^{n/2} D, \qquad C = A^{n/2} D^{-1} \tag{3.6}$$

where D is a function of time t. From (3.2) and (3.6), we obtain

$$\frac{\dot{D}}{D} = \frac{K}{a^3}, \tag{3.7}$$

K being an arbitrary constant. The function D can be determined if the average scale factor a is explicitly known as a function of time. To determine a, we assume that the anisotropy $(\frac{\sigma}{\theta})$ is inversely proportional to a^m i.e.

$$\frac{\sigma}{\theta} = \frac{b}{a^m} \tag{3.8}$$

where $m(> 0)$ and b are constants [13]. Substituting (2.13) and (3.4) in (3.8) and solving the resulting equation, we can obtain

$$a = (c_1 t + c_2)^{1/(3-m)}, \qquad m \neq 3 \tag{3.9}$$

and

$$a = l \exp\left(\frac{kt}{3b}\right), \qquad m = 3 \tag{3.10}$$

where $c_1 = \frac{k(3-m)}{3b}$, c_2 and l are constants. Without loss of any generality, we take l unity.

Now, from (2.12) and (3.5), we get

$$A = a^{\frac{3}{(n+1)}}. \tag{3.11}$$

3.1 Model for $m \neq 3$

In this section we use (3.9) to derive a model of the universe. From (3.9) and (3.11), the scale factor A has the solution

$$A = (c_1 t + c_2)^{3/(n+1)(3-m)}. \tag{3.12}$$

Using (3.9) in (3.7) and integrating, we obtain

$$D = \exp\left[\frac{K(m-3)}{mc_1}(c_1 t + c_2)^{m/(m-3)}\right]. \tag{3.13}$$

Hence, the solution to the scale factors B and C are given by

$$B = (c_1 t + c_2)^{3n/2(n+1)(3-m)} \exp\left[\frac{K(m-3)}{mc_1}(c_1 t + c_2)^{m/(m-3)}\right], \tag{3.14}$$

$$C = (c_1 t + c_2)^{3n/2(n+1)(3-m)} \exp\left[-\frac{K(m-3)}{mc_1}(c_1 t + c_2)^{m/(m-3)}\right]. \tag{3.15}$$

For this model the directional Hubble's parameters and mean Hubble parameter have values given by

$$H_1 = \frac{c_1}{(n+1)(3-m)(c_1t+c_2)}, \tag{3.16}$$

$$H_2 = \frac{3nc_1}{2(n+1)(3-m)(c_1t+c_2)} + \frac{K}{(c_1t+c_2)^{\frac{3}{(3-m)}}}, \tag{3.17}$$

$$H_2 = \frac{3nc_1}{2(n+1)(3-m)(c_1t+c_2)} - \frac{K}{(c_1t+c_2)^{\frac{3}{(3-m)}}}, \tag{3.18}$$

$$H = \frac{c_1}{(3-m)(c_1t+c_2)}. \tag{3.19}$$

The expansion scalar shear scalar, anisotropy parameter and deceleration parameter have expressions as

$$\theta = \frac{3c_1}{(3-m)(c_1t+c_2)} \tag{3.20}$$

$$\sigma = \frac{k}{(c_1t+c_2)^{3/(3-m)}}, \tag{3.21}$$

$$A_m = \left(\frac{n-2}{n+1}\right)^2 + \frac{2K^2(m-3)^2}{3c_1^2(c_1t+c_2)^{2m/3-m}}, \tag{3.22}$$

$$q = 2-m. \tag{3.23}$$

We observe that $q=0$ when $m=2$; $q<0$ when $m>2$ and $q>0$ when $m<2$. Thus the model corresponding to an accelerating universe for $m>0$ and a decelerating universe for $m<2$.

The general solution of (2.11) is

$$\phi = \frac{\phi_0}{(c_1t+c_2)^{m/3-m}} \tag{3.24}$$

where ϕ_0 is an arbitrary constant.

We now assume the relation between pressure and energy density of the matter field through the "gamma-law" equation of state

$$p_m = (\gamma-1)\rho_m, \qquad 1 \leq r \leq 2 \tag{3.25}$$

Substituting the values of A, B and C in (2.7)-(2.10) and using (3.25), we find that

$$\rho_m = \frac{1}{(4-3\gamma)}\left[\frac{80n^2c_1^2+36nc_1^2-36n(n+1)(3-m)c_1^2}{4(n+1)^2(3-m)^2(c_1t+c_2)^2} + \frac{2K^2}{(c_1t+c_2)^2} + \frac{4\phi_0^2}{2(c_1t+c_2)^{6/3-m}}\right], \tag{3.26}$$

$$p_m = \frac{(\gamma-1)}{(4-3\gamma)}\left[\frac{80n^2c_1^2+36nc_1^2-36n(n+1)(3-m)c_1^2}{4(n+1)^2(3-m)^2(c_1t+c_2)^2} + \frac{2K^2}{(c_1t+c_2)^2} + \frac{4\phi_0^2}{2(c_1t+c_2)^{6/3-m}}\right], \tag{3.27}$$

$$\rho_r = \frac{3}{(3\gamma-4)}\left[\frac{18nc_1^2-9n^2c_1^2-12n(n+1)(3-m)c_1^2}{4(n+1)^2(3-m)^2(c_1t+c_2)^2} + \frac{\gamma(9n^2c_1^2+36nc_1^2)}{4(n+1)^2(3-m)^2(c_1t+c_2)^2} + \frac{(2-\gamma)K^2}{(c_1t+c_2)^2} + \frac{\gamma\phi_0^2}{2(c_1t+c_2)^{6/3-m}}\right]. \tag{3.28}$$

Clearly $\gamma \neq 4/3$, which means that we cannot derive cosmological model for the matter disordered radiation from these solutions.

We observe that the spatial volume is zero as time $t = -\frac{c_2}{c_1}$ provided $m<3$. At this epoch ρ_m, p_m,

ρ_r, θ and σ are all infinite. Therefore, the model starts evolving with a big-bank singularity at $t = -\frac{c_2}{c_1}$ and the expansion decreases as the time increases. At t tends to infinity, the physical and kinematical parameters all tend to zero but the spatial volume becomes infinite, which indicate that the model essentially gives an empty space for large time. The anisotropy parameter tends to a constant as $t \to \infty$, and therefore the anisotropy in the universe is maintained throughout the passage of time. For $2 < m < 3$, the model corresponds to an accelerating universe, whereas for $0 < m < 2$ it represents a standard decelerating universe. The scalar function ϕ is infinite at the initial singularity and is a monotonically decreasing function of time and ultimately dies out for large time.

3.2 Model for $m = 3$

In this section, we derive an inflationary model for $m = 3$ by using the average scale factor obtained in (3.10). From (3.10) and (3.11), we can write the cosmic scale factor A in the form

$$A = \exp\left(\frac{kt}{b(n+1)}\right). \tag{3.29}$$

Substituting (3.10) in (3.11) and integrating, we obtain

$$D = \exp\left(\frac{-Kt}{k}e^{-kt/b}\right). \tag{3.30}$$

Therefore, from (3.6) and (3.29), the solutions for factors B and C are

$$B = e^{\frac{knt}{2b(n+1)}} \exp\left(\frac{-Kt}{k}e^{-kt/b}\right), \tag{3.31}$$

$$C = e^{\frac{knt}{2b(n+1)}} \exp\left(\frac{Kt}{k}e^{-kt/b}\right). \tag{3.32}$$

Hence, the model (2.1) with scale factors A, B and C given in (3.29), (3.31) and (3.32) represents an exponentially expanding universe.

For this model the physical and kinematical parameters are obtained as

$$H_1 = \frac{k}{b(n+1)}, \tag{3.33}$$

$$H_2 = \frac{k}{b(n+1)} - Ke^{\frac{-kt}{b}}, \tag{3.34}$$

$$H_2 = \frac{k}{b(n+1)} + Ke^{\frac{-kt}{b}}, \tag{3.35}$$

$$H = \frac{k}{3b}. \tag{3.36}$$

The expansion scalar, shear scalar, anisotropy parameter and deceleration parameter are obtained as

$$\theta = \frac{k}{b}, \tag{3.37}$$

$$\sigma = ke^{-\frac{kt}{b}}, \tag{3.38}$$

$$A_m = \frac{1}{2}\left(\frac{n-2}{n+1}\right)^2 + \frac{9K^2b^2}{k^2}e^{\frac{-2kt}{b}}, \tag{3.39}$$

$$q = -1. \tag{3.40}$$

Equation (11) has the general solution

$$\phi = -\frac{b\phi_0}{k}e^{-kt/b} \tag{3.41}$$

where ϕ_0 is an arbitrary constant of integration.

We assume that the pressure and energy density of the matter field obey the gamma-law of equation of state (3.25). Then from (2.7)-(2.10), we obtain matter density, matter pressure and radiation density as follows:

$$\rho_m = \frac{1}{(4-3\gamma)}\left[\frac{10k^2n^2+4nk^2}{4b^2(n+1)^2} + (2\phi_0^2 - 2K^2)e^{-2kt/b}\right], \tag{3.42}$$

$$p_m = \frac{(\gamma-1)}{(4-3\gamma)}\left[\frac{10k^2n^2+4nk^2}{4b^2(n+1)^2} + (2\phi_0^2 - 2K^2)e^{-2kt/b}\right], \tag{3.43}$$

$$\rho_r = \frac{3}{4-3\gamma}\left[\frac{3k^2n^2-(1-\gamma)(k^2n^2+4nk^2)}{4b^2(n+1)^2} + \frac{\gamma(2K^2+\phi_0^2)-4K^2}{2}e^{-2kt/b}\right]. \tag{3.44}$$

We observe that this model has no finite singularity. We set the coordinates at $t = 0$. A this epoch the expansion scalar, shear scalar and anisotropy parameter are constant. The spatial volume increases exponentially as time increases. The model has constant expansion and decreasing function of time tending to zero at late time. The matter pressure , matter energy density and radiation density tend to constant values as $t \to \infty$. The anisotropy parameter tends to a constant for large time, which means that the anisotropy in the universe is maintained throughout. Since the deceleration parameter $q = -1$, the model exhibits early inflation and late time acceleration which is in accordance with the present scenario of modern cosmology (Reiss et al. [14]; Perlmutter et al. [15]; Schmidt et al. [16] etc.).

4 Conclusion

The theories of gravitation involving scalar fields are very important in modern cosmology since scalar fields play a vital role in the study of early stages of evolution of the universe. in this paper, we have presented exact solution of Einstein's field equations in two-fluid cosmology in the presence of a zero-rest-mass scalar field for a Bianchi type-I space-time by assuming that the anisotropy in the model is inversely proportional to m^{th} power of the average scale factor. We have obtained models of the universe in two types of cosmologies, one with power-law expansion and the other one with exponential expansion. The universe with power-law expansion has a finite singularity and approaches to an empty space for large time. The universe with exponential expansion has no singularity and enters into the de Sitter phase at late times. We have also discussed the physical and geometric behaviours of the cosmological models separately . The models discussed here will be useful in the investigation of early stages of evolution of the universe and structure formation of galaxies.

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