Article

Friedmann Cosmology with Bulk Viscosity and Decaying Λ

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Abstract

We present an isotropic and homogeneous flat cosmological model for bulk viscous fluid distribution. Cosmological model obtained by assuming a dynamical law for the decay of Λ. Physical and kinematical parameters of the model are discussed. The model is found to be compatible with the results of recent cosmological observations.

Keywords: FRW, viscosity, variable cosmological term.

1 Introduction

The Friedmann cosmological models are arrived by applying homogeneity and isotropy to the Einstein field equations. These models yield to the fact that our universe at present is expanding with acceleration [1-4]. In Einstein's theory of general relativity, to account for such an expansion, one needs to introduce some new energy density with a large negative pressure in the present universe, in addition to the usual relativistic or non-relativistic matter. This exotic matter causing cosmic acceleration is known as dark energy. The nature of dark energy is unknown and many radically different models related to this dark energy have been proposed [5, 6]. The simplest and most theoretically appealing candidate of dark energy is the vacuum energy or the cosmological constant Λ with a equation of state parameter $\omega = -1$. The extremely small value of Λ provide a very good fit to current data, however, it has difficulties to reconcile the small observational value with the estimates from quantum field theories, which exceed observational limits by 120 orders of magnitude. To explain the decay of the density, a number of dynamical models have been suggested in which cosmological term Λ varies with cosmic time t. These models give rise to an effective cosmological term which as long as the universe expands, decays from a huge value at initial times to the small value observed at present. Cosmological models with different decay laws for the variation of cosmological term were investigated during last two decades [7-11].

In early stages of evolution of universe, the dissipative processes play a significant role for the high degree of isotropy we observe today. The inclusion of dissipative term in the energy-momentum of cosmic fluid seems to be the best generalization of matter term of the gravitational field equations. Eckart [12] developed the first relativistic theory of non-equilibrium thermodynamics to the effect of bulk viscosity. Padmanabhan and Chitre [13] investigated that the presence of bulk viscosity leads to inflationary like solutions in general relativity. Johri and Sudarshan [14] have investigated the effect of bulk viscosity on the evolution of Friedmann models. The effect of bulk viscosity on cosmological evolution has also been studied by Sahni and Starobinski [6], Singh et al. [15], Peebles [16], Bali and Sharma [17], Bali and Pradhan [18], Bali and Kumawat [19], Fabris et al. [20], Datta Chaudhary and Sil [21].

In this article, we examine the possibility of the phenomenological decay of Λ within the framework of FRW cosmological model filled with bulk viscous matter. The dynamical laws for decay of Λ have been widely studied by Singh and Baghel [22], Arbab [23], Carvalho et al. [24], Chen and Wu [25], Schutzhold [26], Vishwakarma [27] to name only a few.

2 Metric and Field Equations

We consider homogeneous and isotropic spatially flat FRW universe represented by the line-element

$$ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2), \tag{2.1}$$

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where $a(t)$ is the scale factor. We assume the cosmic matter consisting of bulk viscous fluid represented by the energy-momentum tensor

$$T_{ij} = (\rho + \bar{p})v_i v_j + \bar{p} g_{ij}, \tag{2.2}$$

where ρ is energy density and $\bar{p}$ is the effective pressure given by

$$\bar{p} = p - \zeta v^i{}_{;i}, \tag{2.3}$$

satisfying linear equation of state

$$p = (\omega - 1)\rho \ ; \quad 1 \leq \omega \leq 2. \tag{2.4}$$

Here p is the isotropic pressure and v^i, the flow vector of the fluid satisfying $v_i v^i = -1$. The Einstein field equations (in gravitational units $8\pi G = c = 1$) and time-dependent cosmological term $\Lambda(t)$, in comoving system of coordinates lead to

$$\bar{p} - \Lambda = (2q - 1)H^2, \tag{2.5}$$

$$\rho + \Lambda = 3H^2. \tag{2.6}$$

where H is the Hubble parameter and q is the deceleration parameter defined as

$$H = \frac{\dot{a}}{a} \tag{2.7}$$

and

$$q = -1 - \frac{\dot{H}}{H^2}, \tag{2.8}$$

where an overhead dot $(\cdot)$ denotes ordinary derivative with respect to cosmic time t. The vanishing divergence of Einstein tensor gives rise to

$$\dot{\rho} + 3(\rho + \bar{p})H + \dot{\Lambda} = 0. \tag{2.9}$$

The system of Eqs. (5), (6) and (9) are three equations in four unknowns $a(t)$, ρ, $\bar{p}$ and $\Lambda(t)$. We require one more equation to close the system of equations. We consider a dynamical law for decay of Λ of the form [5]

$$\Lambda = \frac{\sigma}{a^m}, \tag{2.10}$$

where σ and m are non-negative constants. Thus, from Eqs. (5), (6), (9) and (10), we obtain

$$2\dot{H} + 3(\omega - 3\zeta_0)H^2 = \sigma\omega a^{-m}, \tag{2.11}$$

where coefficient of bulk viscosity ζ is taken as

$$\zeta = 3\zeta_0 H \ ; \quad \zeta_0 \geq 0. \tag{2.12}$$

On integration, (11) give

$$a(t) = \left(\sqrt{\frac{\sigma\omega}{3\omega - m - 9\zeta_0}}\frac{mt}{2}\right)^{2/m}, \tag{2.13}$$

constant of integration being related to the choice of the origin.
Matter density ρ and cosmological term Λ for the model are given by

$$\rho = \frac{4(m + 9\zeta_0)}{\omega m^2 t^2}, \tag{2.14}$$

$$\Lambda = \frac{4(3\omega - m - 9\zeta_0)}{\omega m^2 t^2}. \tag{2.15}$$

The expansion scaler θ, deceleration parameter q and ratio $\Omega = \Lambda/\rho$ are obtained as

$$\theta = \frac{6}{mt}, \tag{2.16}$$

$$q = \frac{m}{2} - 1, \tag{2.17}$$

$$\Omega = \frac{3\omega}{m + 9\zeta_0} - 1. \tag{2.18}$$

The model has point singularity at $t = 0$. We observe that the model starts with ρ, p, Λ, θ all infinite and become zero at $t = \infty$. It is to note that ρ/θ^2 and Ω are constant. Thus matter density is comparable with vacuum density and expansion throughout the evolution.

3 Conclusion

In this paper spatially flat FRW cosmological model for bulk viscous fluid distribution with decaying cosmological term Λ is investigated, where $\Lambda \propto a^{-m}$; $a(t)$ is the scale factor of the universe and $m(\geq 0)$ is a constant. The coefficient of bulk viscosity is taken as $\zeta = 3\zeta_0 H$, where ζ_0 is a constant. We observe that the presence of bulk viscosity is to increase the value of matter density ρ and to decrease the value of vacuum energy density Λ. We also observe that the model obtained starts with a big-bang at $t = 0$, the expansion in the model decreases as time increases and drops to zero as $t \to \infty$. From Eq. (17), one concludes that for $m > 2$, the model represents a decelerating universe whereas for $0 \leq m < 2$, it gives rise to an accelerating universe. When $m = 2$, we obtain $H = 1/t$ and $q = 0$, so that every galaxy moves with a constant speed.

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