Exploration

On Squared Neutron Number in Reducing Nuclear Binding Energy

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Abstract

In this paper, we make an attempt to understand nuclear stability and binding energy in light of our proposed three atomic gravitational constants and nuclear elementary charge. Identifying the squared ratio of nuclear charge and ordinary electromagnetic charge with inverse of the strong coupling constant, from Z=26 onwards, (1/0.1152)=8.68 can be inferred as a representation of the maximum strength of nuclear binding energy. Considering 10.09 MeV as a unified strongly attracting binding energy coefficient and (1.162 MeV and 0.71 MeV) as repulsive energy coefficients, it is possible to have an effective binding energy coefficient of 9.18 MeV. Based on this energy coefficient, close to stable mass numbers, nuclear binding energy can be shown to increase with mass number and decrease with squared neutron number.

Keywords: Virtual atomic gravitational constant, nuclear elementary charge, nuclear stability, binding energy, squared neutron number.

1. Introduction

It is well established that, on large scales, stars, galaxies and universe are controlled by 'gravity' and on small scales, atoms and atomic nuclei are controlled by 'quantum mechanics'. It is also well established that, stars are made up of so many atoms, galaxies are made up of so many stars and universe is made up of so many galaxies. Very unfortunate thing is that, so far, either qualitatively or quantitatively, at atomic and nuclear scales, there exist no generally accepted unified theoretical models, no formulae or no numerical procedures for estimating the magnitude of the Newtonian gravitational constant, G_N.

As there is a large gap in between nuclear and Planck scales, with currently believed notion of unification paradigm, it seems impossible to implement gravity in atomic, nuclear and particle physics [1]. So far, many laboratory experiments had been carried out for estimating the magnitude of G_N. Its current recommended CODATA [2,3,4] value is 6.67408×10^{-11} $m^3kg^{-1}sec^{-2}$ and relative standard uncertainty is 4.7×10^{-5}. In a unified approach, one can see a great initiative taken by J. E. Brandenburg [5].

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Since 2007, scientists and engineers are trying to estimate the magnitude of G_N by 'Atomic interferometry' and gradiometers [6,7,8]. In this method, cold atoms are allowed to have free fall under gravity. Clearly speaking, an atomic gravity gradiometer is used to measure the differential acceleration experienced by two freely falling samples of laser-cooled rubidium atoms under the influence of nearby tungsten masses.

Point to be understood is that, even though materialistic atoms are having independent existence, they are not allowing scientists and engineers to explore the secrets of gravity at atomic scale. This may be due to incomplete unification paradigm, inadequacy of known physics and technological difficulties etc. In this challenging scenario, one fundamental question to be answered is: Is Newtonian gravitational constant having any physical existence? We would like to suggest that, it is a man created empirical constant and is having no physical existence. Clearly speaking, it is not real but virtual. For understanding the secrets of large scale gravitational effects, scientists consider it as a physical constant. In the same way, each atomic interaction can be allowed to have its own gravitational constant. With further study, their magnitudes can be refined for a better fit and understanding of the nature. The most desirable cases of any unified description are:

a) To implement gravity in microscopic physics and to estimate the magnitude of the Newtonian gravitational constant (G_N).
b) To simplify the complicated issues of known physics. (Understanding nuclear stability, nuclear binding energy, nuclear charge radii and neutron life time etc.)
c) To predict new effects, arising from a combination of the fields inherent in the unified description. (Understanding strong coupling constant, Fermi's weak coupling constant and radiation constants etc.)
d) To develop a model of microscopic quantum gravity.

With reference to our recent publications and conference presentations [9-13], we propose the following set of four semi empirical relations. Let,

Electromagnetic gravitational constant = G_e
Nuclear gravitational constant = G_s
Weak gravitational constant = G_w

$$\frac{m_p}{m_e} \cong 2\pi\sqrt{\frac{4\pi\varepsilon_0 G_e m_e^2}{e^2}} \cong \left(\frac{G_e m_e^2}{\hbar c}\right)\left(\frac{G_s m_p^2}{\hbar c}\right) \tag{1}$$

$$\hbar c \cong \left(\frac{m_p}{m_e}\right)^2 \left(G_e^2 G_N\right)^{1/3} m_p^2 \tag{2}$$

$$G_F \cong \left[\left(G_e m_p^2\right)^2 \left(G_N m_p^2\right)\right]^{\frac{1}{3}} \left(\frac{2G_s m_p}{c^2}\right)^2 \cong \frac{4G_w \hbar^2}{c^2} \tag{3}$$

$$\frac{G_w}{G_N} \cong \left(\frac{m_p}{m_e}\right)^{10} \tag{4}$$

Based on relation (1), magnitudes of (G_e, G_s) can be estimated. Based on relation (2), magnitude of G_N can be estimated. Based on relation (3), magnitudes of (G_F, G_w) can be estimated [14,15]. Again, based on relation (4), G_N can be estimated. Estimated values seem to be:

$$\begin{aligned}
&G_e \cong 2.374335\times10^{37}\ \text{m}^3\text{kg}^{-1}\text{sec}^{-2}\\
&G_s \cong 3.329561\times10^{28}\text{m}^3\text{kg}^{-1}\text{sec}^{-2}\\
&G_w \cong 2.909745\times10^{22}\ \text{m}^3\text{kg}^{-1}\text{sec}^{-2}\\
&G_N \cong 6.679855\times10^{-11}\ \text{m}^3\text{kg}^{-1}\text{sec}^{-2}\\
&G_F \cong 1.44021\times10^{-62}\ \text{J.m}^3
\end{aligned}$$

Even though our approach is speculative, role played by the four gravitational constants seems to be fairly natural. This kind of approach may help in producing a variety of such relations by using which in near future, an absolute set of relations can be developed. Proceeding further, estimated absolute theoretical value of G_N can be considered as a standard reference for future experiments. In a verifiable approach we have developed many interesting relations and we are working on deriving them [16] from basic principles.

By implementing the four such gravitational constants in String theory models, it may be possible to explore the hidden unified physics. With further study, a practical model of materialistic quantum gravity can be developed and magnitude of the Newtonian gravitational constant can be estimated in a theoretical approach bound to Fermi scale.

Objective of this paper is to understand and fit nuclear stability and binding energy with the proposed three atomic gravitational constants.

2. Three simple assumptions pertaining to nuclear physics

With reference to our recent paper publications and conference proceedings [9-13], we propose the following three assumptions.

1) There exists a strong elementary charge in such a way that:

$$\frac{e_s}{e} \cong \left(\frac{G_s m_p^2}{\hbar c}\right) \cong \left[\frac{G_s m_e^2}{\left(G_e^2 G_N\right)^{1/3} m_p^2}\right] \tag{5}$$

$$\frac{e_s^2}{e^2} \cong \left(\frac{G_s m_p^2}{\hbar c}\right)^2 \cong \left(\frac{G_s m_p^3}{G_e m_e^3}\right) \tag{6}$$

$$\frac{e_s G_s}{e G_w} \cong \left(\frac{m_p}{m_e}\right)^2 \tag{7}$$

2) Strong coupling constant [1,15] can be expressed with

$$\alpha_s \cong \left(\frac{e}{e_s}\right)^2 \cong \left(\frac{\hbar c}{G_s m_p^2}\right)^2 \cong \left(\frac{G_e m_e^3}{G_s m_p^3}\right) \tag{8}$$

3) Nuclear charge radius can be addressed with

$$R_0 \cong \frac{2G_s m_p}{c^2} \tag{9}$$

3. Understanding proton-neutron stability with three atomic gravitational constants

Let,

$$\left.\begin{aligned} s &\cong \left\{\left(\frac{e_s}{m_p}\right) \div \left(\frac{e}{m_e}\right)\right\} \cong 0.001605 \\ &\cong \frac{G_s m_p m_e}{\hbar c} \cong \frac{\hbar c}{G_e m_e^2} \cong \frac{G_s^2}{G_e G_w} \end{aligned}\right\} \tag{10}$$

Nuclear beta stability line can be addressed with a relation of the form [relation 8 of ref.17],

$$\begin{aligned} A_s &\cong 2Z + s(2Z)^2 \cong 2Z + (4s)Z^2 \\ &\cong 2Z + kZ^2 \cong Z(2 + kZ) \end{aligned} \tag{11}$$

where $(4s) \cong k \cong 0.0064185$

By considering a factor like $\left[2 \pm \sqrt{k}\right]$, likely possible range of A_s can be addressed with,

$$\left.\begin{aligned} &(A_s)_{lower}^{upper} \cong Z\left[(2 \pm 0.08) + kZ\right] \\ &\rightarrow \begin{Bmatrix} (A_s)_{lower} \cong Z(1.92 + kZ) \\ (A_s)_{mean} \cong Z(2.0 + kZ) \\ (A_s)_{upper} \cong Z(2.08 + kZ) \end{Bmatrix} \end{aligned}\right\} \tag{12}$$

Interesting point to be noted is that, for Z=112,113 and 114, estimated lower stable mass numbers are 296,299 and 302respectively.Corresponding neutron numbers are 184,186 and 188. These neutron numbers are very close to the currently believed shell closure at N=184.It needs further study [18]. See table 1.

Table-1: Likely possible range of A_s for Z=5 to 118

Proton number	$(A_s)_{lower}$	$(A_s)_{mean}$	$(A_s)_{upper}$
5	10	10	11
6	12	12	13
7	14	14	15
8	16	16	17
9	18	19	19
10	20	21	21
11	22	23	24
12	24	25	26
13	26	27	28
14	28	29	30
15	30	31	33
16	32	34	35
17	34	36	37
18	37	38	40
19	39	40	42
20	41	43	44
21	43	45	47
22	45	47	49
23	48	49	51
24	50	52	54
25	52	54	56
26	54	56	58
27	57	59	61
28	59	61	63
29	61	63	66
30	63	66	68
31	66	68	71
32	68	71	73
33	70	73	76
34	73	75	78
35	75	78	81
36	77	80	83
37	80	83	86
38	82	85	88
39	85	88	91
40	87	90	93
41	90	93	96
42	92	95	99
43	94	98	101
44	97	100	104
45	99	103	107
46	102	106	109
47	104	108	112
48	107	111	115
49	109	113	117
50	112	116	120
51	115	119	123
52	117	121	126
53	120	124	128

54	122	127	131
55	125	129	134
56	128	132	137
57	130	135	139
58	133	138	142
59	136	140	145
60	138	143	148
61	141	146	151
62	144	149	154
63	146	151	157
64	149	154	159
65	152	157	162
66	155	160	165
67	157	163	168
68	160	166	171
69	163	169	174
70	166	171	177
71	169	174	180
72	172	177	183
73	174	180	186
74	177	183	189
75	180	186	192
76	183	189	195
77	186	192	198
78	189	195	201
79	192	198	204
80	195	201	207
81	198	204	211
82	201	207	214
83	204	210	217
84	207	213	220
85	210	216	223
86	213	219	226
87	216	223	230
88	219	226	233
89	222	229	236
90	225	232	239
91	228	235	242
92	231	238	246
93	234	242	249
94	237	245	252
95	240	248	256
96	243	251	259
97	247	254	262
98	250	258	265
99	253	261	269
100	256	264	272
101	259	267	276
102	263	271	279
103	266	274	282
104	269	277	286
105	272	281	289
106	276	284	293
107	279	287	296
108	282	291	300
109	286	294	303
110	289	298	306
111	292	301	310

112	296	305	313
113	299	308	317
114	302	311	321
115	306	315	324
116	309	318	328
117	312	322	331
118	316	325	335

4. Understanding nuclear binding energy with single unified energy coefficient

A. New integrated model

Based on the new integrated model proposed by N. Ghahramany et al [19,20],

$$B(Z,N) = \left\{ A - \left(\frac{\left(N^2 - Z^2\right) + \delta\left(N-Z\right)}{3Z} + 3 \right) \right\} \frac{m_n c^2}{\gamma} \tag{13}$$

where, $\gamma =$ Adjusting coefficient $\approx$(90 to 100).
if $N \neq Z,\ \delta\left(N-Z\right) = 0$ and if $N = Z,\ \delta\left(N-Z\right) = 1$.

Readers are encouraged to see references there in [19,20] for derivation part. Point to be noted is that, close to the beta stability line, $\left[\frac{N^2 - Z^2}{3Z}\right]$ takes care of the combined effects of coulombic and asymmetric effects. Point to be noted here is that, nuclear binding energy can be addressed with a single energy coefficient.

B. Our unified approach

Interesting points to be noted are:

1) $Z \cong 26$ seems to represent a characteristic reference number in understanding nuclear binding of light and heavy atomic nuclides [21,22].
2) With reference to electromagnetic interaction and based on proton number,

   a) For $Z \geq 26$, maximum strength of nuclear binding energy can be addressed with $\beta \cong \left(1/\alpha_s\right) \cong 8.68$.
   b) For $Z < 26$, strength of nuclear binding energy can be addressed with

$$\beta \cong \left(\frac{Z}{26}\right)^{\sqrt{k}} \left(\frac{1}{\alpha_s}\right) \cong \left(\frac{Z}{26}\right)^{0.08} \times 8.68. \tag{14}$$

3) Close to stable mass numbers, mass number helps in increasing binding energy and squared neutron number aids in reducing the binding energy.
4) There exists a single and unified binding energy coefficient and it can be chosen to fall in between

$$\left.\begin{array}{l}\dfrac{e_s^2}{8\pi\varepsilon_0\left(G_s m_p/c^2\right)}-\dfrac{3}{5}\dfrac{e^2}{8\pi\varepsilon_0\left(G_s m_p/c^2\right)}\cong 8.93\text{ MeV}\\ \text{and } \dfrac{e_s^2}{8\pi\varepsilon_0\left(G_s m_p/c^2\right)}-\dfrac{e^2}{8\pi\varepsilon_0\left(G_s m_p/c^2\right)}\cong 9.40\text{ MeV}\end{array}\right\} \tag{15}$$

where $\dfrac{e_s^2}{8\pi\varepsilon_0\left(G_s m_p/c^2\right)}\cong 10.09\text{ MeV}$ can be considered as the attractive nuclear binding energy coefficient and $\dfrac{3}{5}\dfrac{e^2}{8\pi\varepsilon_0\left(G_s m_p/c^2\right)}\cong 0.71\text{ MeV}$ or $\dfrac{e^2}{8\pi\varepsilon_0\left(G_s m_p/c^2\right)}\cong 1.162\text{ MeV}$ can be considered as the repulsive nuclear binding energy coefficient. To fit the data we consider,

$$B_0 \cong \frac{8.93+9.4}{2}\cong 9.16\approx 9.18\text{ MeV} \tag{16}$$

Based on the above relations and close to the stable mass numbers of $(Z\approx 2\text{ to }118)$, with a common energy coefficient of 9.18 MeV, we present the following three terms for fitting and understanding nuclear binding energy.

First term helps in **increasing** the binding energy and can be considered as,

$$\begin{array}{l}T_1 \cong \eta\times A\times 9.18\text{ MeV}\\ \text{where }\begin{cases}\eta\cong\left(\dfrac{Z}{26}\right)^{0.08} & \text{for } Z<26\\ \eta\cong 1 & \text{for } Z\geq 26\end{cases}\end{array} \tag{17}$$

Second term helps in **decreasing** the binding energy and can be considered as,

$$\left.\begin{array}{l}T_2\cong\eta\left[1+\left(\left(\dfrac{A}{2Z}\right)^{\frac{2}{3}}\left(\dfrac{G_s^2}{G_e G_w}\right)N^2\right)\right]\times 9.18\text{ MeV}\\ \cong\eta\left[1+\left(0.001605\left(\dfrac{A}{2Z}\right)^{\frac{2}{3}}N^2\right)\right]\times 9.18\text{ MeV}\end{array}\right\} \tag{18}$$

Third term also helps in **decreasing** the binding energy for $(N<N_s)$ or $(A<A_s)$ and can be considered as,

$$\left.\begin{array}{l} T_3 \cong \eta\left[\dfrac{(A_s - A)^2}{Z}\right]\times 9.18\text{ MeV} \\ \cong \eta\left[\dfrac{(N_s - N)^2}{Z}\right]\times 9.18\text{ MeV} \end{array}\right\} \quad (19)$$

Thus, binding energy can be fitted with,

$$\left.\begin{array}{ll} (B_A) \cong T_1 - T_2 - T_3 & (\text{for } A < A_s) \\ (B_A) \cong T_1 - T_2 & (\text{for } A \ge A_s) \end{array}\right\} \quad (20)$$

See the following figure 1. Dashed red curve plotted with relations (11), (17), (18) and (20) can be compared with the green curve plotted with the standard semi empirical mass formula (SEMF). For medium and heavy atomic nuclides, fit is excellent. It seems that some correction is required for light and super heavy atoms. See table 2 for the estimated data close to stable mass numbers.

We are working on applying the relations (17), (18), (19) and (20) to mass numbers below and above the stable mass numbers. See tables 3 to 7 for the estimated isotopic binding energy of Z=28, 40, 50, 66 and 82. It needs further study.

Figure 1. Binding energy per nucleon close to stable mass numbers of Z = 2 to 118

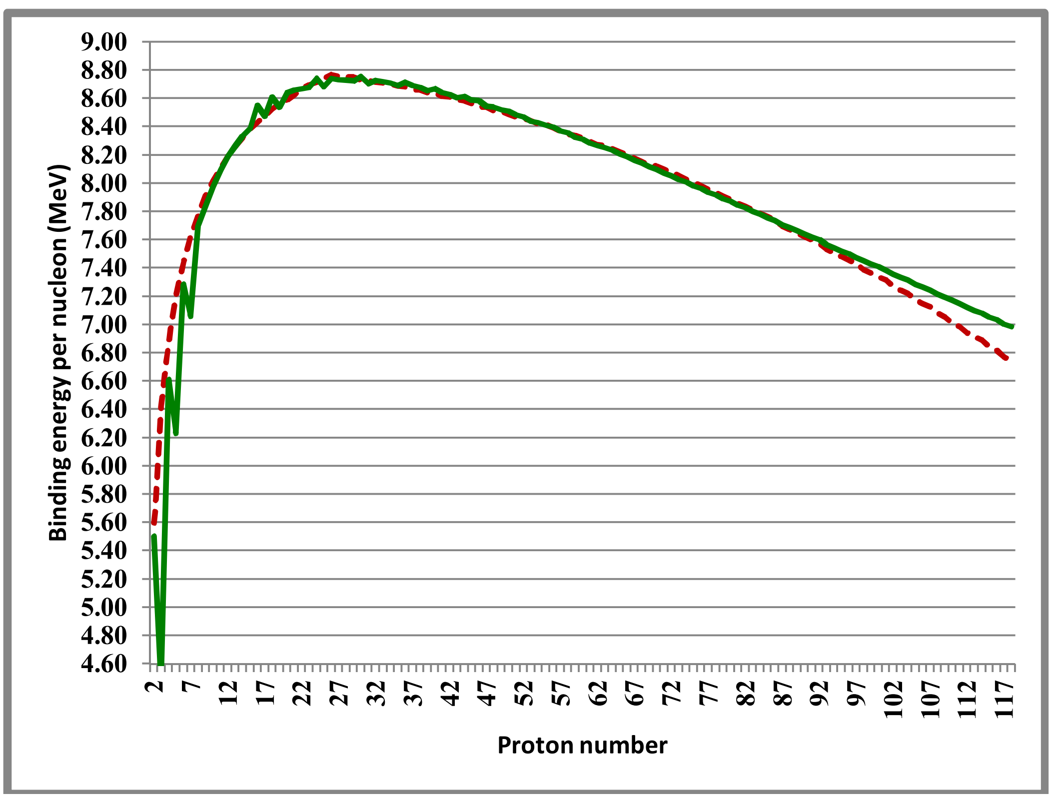

Table 2. Estimated nuclear binding energy close to stable mass numbers of Z=2 to 118

Proton number	Estimated mass number close to stable mass number	Neutron number	Estimated Binding energy (MeV)	SEMF binding energy	Error(MeV)
2	4	2	22.38	22.01	-0.37
3	6	3	38.51	26.88	-11.63
4	8	4	55.12	52.86	-2.26
5	10	5	72.09	62.29	-9.80
6	12	6	89.33	87.39	-1.94
7	14	7	106.80	98.81	-7.99
8	16	8	124.45	123.25	-1.20
9	19	10	150.39	148.85	-1.54
10	21	11	168.38	167.52	-0.87
11	23	12	186.49	186.14	-0.35
12	25	13	204.70	204.72	0.02
13	27	14	223.00	223.22	0.22
14	29	15	241.39	241.65	0.26
15	31	16	259.85	259.98	0.13
16	34	18	286.62	290.77	4.15
17	36	19	305.22	305.06	-0.16
18	38	20	323.88	327.23	3.35
19	40	21	342.59	341.47	-1.12
20	43	23	369.54	371.57	2.03
21	45	24	388.34	389.59	1.25
22	47	25	407.18	407.47	0.29
23	49	26	426.05	425.20	-0.85
24	52	28	453.08	454.57	1.49
25	54	29	472.01	468.89	-3.13
26	56	30	490.97	489.58	-1.39
27	59	32	516.43	515.20	-1.23
28	61	33	533.81	532.52	-1.30
29	63	34	551.16	549.67	-1.50
30	66	36	576.35	577.93	1.57
31	68	37	593.61	591.98	-1.63
32	71	39	618.58	619.81	1.22
33	73	40	635.75	636.62	0.87
34	75	41	652.88	653.27	0.39
35	78	43	677.58	677.88	0.30
36	80	44	694.62	697.05	2.43
37	83	46	719.10	721.32	2.21
38	85	47	736.05	737.59	1.54
39	88	49	760.32	761.58	1.26
40	90	50	777.18	780.20	3.02
41	93	52	801.23	803.88	2.64
42	95	53	817.99	819.75	1.76
43	98	55	841.83	843.16	1.33
44	100	56	858.50	861.24	2.74
45	103	58	882.13	884.37	2.24
46	106	60	905.60	909.61	4.00
47	108	61	922.12	922.70	0.59

48	111	63	945.38	947.65	2.27
49	113	64	961.80	962.85	1.05
50	116	66	984.84	987.48	2.63
51	119	68	1007.74	1009.66	1.92
52	121	69	1024.00	1024.59	0.59
53	127	74	1065.66	1070.80	5.14
54	127	73	1069.21	1070.45	1.24
55	129	74	1085.32	1085.10	-0.21
56	132	76	1107.63	1108.72	1.10
57	135	78	1129.78	1130.06	0.27
58	138	80	1151.79	1153.27	1.49
59	140	81	1167.68	1165.55	-2.13
60	143	83	1189.47	1188.52	-0.95
61	146	85	1211.11	1209.31	-1.80
62	149	87	1232.59	1231.90	-0.69
63	151	88	1248.27	1245.86	-2.41
64	154	90	1269.54	1268.20	-1.33
65	157	92	1290.65	1288.44	-2.21
66	160	94	1311.62	1310.44	-1.18
67	163	96	1332.43	1330.38	-2.05
68	166	98	1353.08	1352.04	-1.04
69	169	100	1373.59	1371.69	-1.90
70	171	101	1388.86	1385.09	-3.77
71	174	103	1409.15	1404.54	-4.61
72	177	105	1429.28	1425.66	-3.62
73	180	107	1449.27	1444.84	-4.43
74	183	109	1469.10	1465.66	-3.43
75	186	111	1488.77	1484.57	-4.20
76	189	113	1508.29	1505.10	-3.20
77	192	115	1527.66	1523.74	-3.92
78	195	117	1546.88	1543.98	-2.89
79	198	119	1565.94	1562.37	-3.57
80	201	121	1584.85	1582.34	-2.51
81	204	123	1603.60	1600.48	-3.12
82	207	125	1622.20	1620.17	-2.03
83	210	127	1640.65	1638.07	-2.58
84	213	129	1658.94	1657.49	-1.45
85	216	131	1677.08	1675.15	-1.93
86	219	133	1695.07	1694.32	-0.75
87	223	136	1716.42	1718.61	2.18
88	226	138	1734.01	1737.47	3.46
89	229	140	1751.44	1754.62	3.18
90	232	142	1768.72	1773.24	4.52
91	235	144	1785.84	1790.16	4.32
92	238	146	1802.81	1808.53	5.72
93	242	149	1822.54	1830.19	7.65
94	245	151	1839.11	1848.29	9.19
95	248	153	1855.52	1864.75	9.23
96	251	155	1871.78	1882.62	10.84
97	254	157	1887.89	1898.87	10.97
98	258	160	1906.22	1922.72	16.50
99	261	162	1921.93	1938.72	16.79
100	264	164	1937.48	1956.10	18.62

101	267	166	1952.88	1971.90	19.01
102	271	169	1970.07	1993.59	23.53
103	274	171	1985.07	2009.17	24.10
104	277	173	1999.91	2026.08	26.17
105	281	176	2016.20	2047.28	31.08
106	284	178	2030.64	2063.96	33.32
107	287	180	2044.93	2079.10	34.17
108	291	183	2060.32	2099.83	39.51
109	294	185	2074.20	2114.76	40.56
110	298	188	2088.94	2136.55	47.61
111	301	190	2102.42	2151.27	48.85
112	305	193	2116.50	2171.33	54.83
113	308	195	2129.58	2185.85	56.26
114	311	197	2142.51	2201.63	59.12
115	315	200	2155.69	2221.27	65.58
116	318	202	2168.21	2236.83	68.61
117	322	205	2180.74	2254.82	74.08
118	325	207	2192.86	2270.17	77.31

Table 3. Isotopic binding energy of Z=28

Proton number	Mass number	Neutron number	Estimated mass number close to stable mass number	Estimated Binding energy (MeV)	SEMF binding energy	Error (MeV)
28	56	28	61	485.15	476.54	-8.61
28	57	29	61	496.30	487.95	-8.35
28	58	30	61	506.73	501.75	-4.99
28	59	31	61	516.47	511.65	-4.82
28	60	32	61	525.49	523.97	-1.52
28	61	33	61	533.81	532.52	-1.30
28	62	34	61	541.75	543.50	1.74
28	63	35	61	549.64	550.82	1.18
28	64	36	61	557.47	560.58	3.11
28	65	37	61	565.24	566.79	1.55
28	66	38	61	572.96	575.45	2.49
28	67	39	61	580.62	580.65	0.02
28	68	40	61	588.23	588.30	0.07
28	69	41	61	595.77	592.57	-3.20
28	70	42	61	603.26	599.30	-3.96
28	71	43	61	610.69	602.74	-7.95
28	72	44	61	618.05	608.62	-9.44
28	73	45	61	625.36	611.28	-14.08
28	74	46	61	632.60	616.38	-16.22
28	75	47	61	639.77	618.34	-21.44
28	76	48	61	646.89	622.72	-24.17

Table 4. Isotopic binding energy of Z=40

Proton number	Mass number	Neutron number	Estimated mass number close to stable mass number	Estimated Binding energy (MeV)	SEMF binding energy	Error (MeV)
40	80	40	90	678.70	666.18	-12.51
40	81	41	90	690.84	678.57	-12.27
40	82	42	90	702.47	693.05	-9.42
40	83	43	90	713.59	704.34	-9.25
40	84	44	90	724.21	717.74	-6.47
40	85	45	90	734.32	728.01	-6.30
40	86	46	90	743.91	740.40	-3.51
40	87	47	90	753.00	749.73	-3.26
40	88	48	90	761.57	761.18	-0.39
40	89	49	90	769.63	769.63	0.00
40	90	50	90	777.18	780.20	3.02
40	91	51	90	784.44	787.83	3.39
40	92	52	90	791.65	797.57	5.92
40	93	53	90	798.80	804.43	5.63
40	94	54	90	805.90	813.41	7.51
40	95	55	90	812.94	819.55	6.61
40	96	56	90	819.92	827.80	7.88
40	97	57	90	826.85	833.27	6.42
40	98	58	90	833.71	840.84	7.13
40	99	59	90	840.52	845.67	5.15
40	100	60	90	847.27	852.61	5.34
40	101	61	90	853.96	856.85	2.89
40	102	62	90	860.59	863.18	2.60
40	103	63	90	867.15	866.86	-0.29
40	104	64	90	873.65	872.63	-1.02
40	105	65	90	880.10	875.78	-4.31
40	106	66	90	886.47	881.02	-5.46
40	107	67	90	892.79	883.67	-9.12
40	108	68	90	899.04	888.40	-10.64
40	109	69	90	905.23	890.58	-14.64
40	110	70	90	911.35	894.84	-16.51

Table 5. Isotopic binding energy of Z=50

Proton number	Mass number	Neutron number	Estimated mass number close to stable mass number	Estimated Binding energy (MeV)	SEMF binding energy	Error (MeV)
50	100	50	116	824.98	809.31	-15.67
50	101	51	116	838.11	822.27	-15.84
50	102	52	116	850.82	837.17	-13.66
50	103	53	116	863.12	849.24	-13.88
50	104	54	116	875.00	863.24	-11.76
50	105	55	116	886.46	874.47	-11.99
50	106	56	116	897.50	887.63	-9.87
50	107	57	116	908.13	898.07	-10.06

50	108	58	116	918.34	910.44	-7.89
50	109	59	116	928.12	920.13	-7.99
50	110	60	116	937.49	931.76	-5.73
50	111	61	116	946.43	940.74	-5.70
50	112	62	116	954.96	951.65	-3.31
50	113	63	116	963.06	959.96	-3.11
50	114	64	116	970.75	970.20	-0.55
50	115	65	116	978.01	977.88	-0.13
50	116	66	116	984.84	987.48	2.63
50	117	67	116	991.44	994.55	3.11
50	118	68	116	997.98	1003.55	5.57
50	119	69	116	1004.47	1010.05	5.58
50	120	70	116	1010.89	1018.47	7.58
50	121	71	116	1017.26	1024.43	7.17
50	122	72	116	1023.57	1032.30	8.73
50	123	73	116	1029.82	1037.75	7.92
50	124	74	116	1036.02	1045.10	9.08
50	125	75	116	1042.15	1050.05	7.90
50	126	76	116	1048.22	1056.91	8.69
50	127	77	116	1054.23	1061.39	7.16
50	128	78	116	1060.18	1067.77	7.59
50	129	79	116	1066.07	1071.81	5.74
50	130	80	116	1071.90	1077.74	5.84
50	131	81	116	1077.66	1081.35	3.69
50	132	82	116	1083.37	1086.85	3.49
50	133	83	116	1089.00	1090.06	1.05
50	134	84	116	1094.58	1095.15	0.57
50	135	85	116	1100.09	1097.96	-2.13
50	136	86	116	1105.54	1102.66	-2.88
50	137	87	116	1110.92	1105.11	-5.81
50	138	88	116	1116.23	1109.42	-6.81
50	139	89	116	1121.48	1111.52	-9.97
50	140	90	116	1126.66	1115.47	-11.19

Table 6. Isotopic binding energy of Z=66

Proton number	Mass number	Neutron number	Estimated mass number close to stable mass number	Estimated Binding energy (MeV)	SEMF binding energy	Error (MeV)
66	132	66	160	1029.35	1010.33	-19.02
66	133	67	160	1043.89	1023.99	-19.90
66	134	68	160	1058.10	1039.39	-18.71
66	135	69	160	1071.98	1052.36	-19.62
66	136	70	160	1085.54	1067.07	-18.47
66	137	71	160	1098.76	1079.38	-19.38
66	138	72	160	1111.66	1093.43	-18.23
66	139	73	160	1124.23	1105.11	-19.12
66	140	74	160	1136.47	1118.52	-17.95
66	141	75	160	1148.38	1129.61	-18.78
66	142	76	160	1159.97	1142.41	-17.55
66	143	77	160	1171.22	1152.91	-18.30

66	144	78	160	1182.14	1165.14	-17.00
66	145	79	160	1192.73	1175.09	-17.64
66	146	80	160	1202.99	1186.75	-16.23
66	147	81	160	1212.91	1196.17	-16.74
66	148	82	160	1222.51	1207.30	-15.21
66	149	83	160	1231.77	1216.21	-15.56
66	150	84	160	1240.70	1226.83	-13.87
66	151	85	160	1249.30	1235.24	-14.05
66	152	86	160	1257.56	1245.37	-12.19
66	153	87	160	1265.49	1253.31	-12.18
66	154	88	160	1273.08	1262.97	-10.12
66	155	89	160	1280.34	1270.46	-9.89
66	156	90	160	1287.27	1279.65	-7.62
66	157	91	160	1293.86	1286.71	-7.15
66	158	92	160	1300.12	1295.47	-4.65
66	159	93	160	1306.03	1302.10	-3.93
66	160	94	160	1311.62	1310.44	-1.18
66	161	95	160	1317.00	1316.67	-0.33
66	162	96	160	1322.33	1324.60	2.27
66	163	97	160	1327.60	1330.45	2.85
66	164	98	160	1332.80	1337.98	5.18
66	165	99	160	1337.95	1343.46	5.51
66	166	100	160	1343.04	1350.61	7.57
66	167	101	160	1348.07	1355.73	7.66
66	168	102	160	1353.03	1362.52	9.49
66	169	103	160	1357.94	1367.28	9.35
66	170	104	160	1362.78	1373.72	10.94
66	171	105	160	1367.56	1378.15	10.59
66	172	106	160	1372.28	1384.25	11.97
66	173	107	160	1376.94	1388.36	11.42
66	174	108	160	1381.53	1394.13	12.60
66	175	109	160	1386.06	1397.92	11.86
66	176	110	160	1390.53	1403.38	12.85
66	177	111	160	1394.93	1406.87	11.94
66	178	112	160	1399.27	1412.02	12.75
66	179	113	160	1403.55	1415.22	11.68
66	180	114	160	1407.75	1420.07	12.32

Table 7. Isotopic binding energy of Z=82						
Proton number	Mass number	Neutron number	Estimated mass number close to stable mass number	Estimated Binding energy (MeV)	SEMF binding energy	Error (MeV)
82	164	82	207	1190.27	1177.82	-12.45
82	165	83	207	1206.12	1192.04	-14.09
82	166	84	207	1221.70	1207.83	-13.87
82	167	85	207	1237.01	1221.48	-15.53
82	168	86	207	1252.05	1236.72	-15.33
82	169	87	207	1266.81	1249.82	-16.98
82	170	88	207	1281.29	1264.51	-16.79

82	171	89	207	1295.51	1277.09	-18.42
82	172	90	207	1309.44	1291.25	-18.20
82	173	91	207	1323.11	1303.32	-19.79
82	174	92	207	1336.50	1316.97	-19.53
82	175	93	207	1349.61	1328.56	-21.05
82	176	94	207	1362.45	1341.72	-20.73
82	177	95	207	1375.01	1352.83	-22.18
82	178	96	207	1387.30	1365.51	-21.79
82	179	97	207	1399.31	1376.17	-23.14
82	180	98	207	1411.04	1388.39	-22.65
82	181	99	207	1422.50	1398.61	-23.89
82	182	100	207	1433.68	1410.39	-23.29
82	183	101	207	1444.58	1420.17	-24.41
82	184	102	207	1455.20	1431.52	-23.68
82	185	103	207	1465.55	1440.90	-24.65
82	186	104	207	1475.62	1451.83	-23.79
82	187	105	207	1485.41	1460.80	-24.60
82	188	106	207	1494.91	1471.33	-23.58
82	189	107	207	1504.14	1479.92	-24.23
82	190	108	207	1513.10	1490.06	-23.04
82	191	109	207	1521.77	1498.27	-23.50
82	192	110	207	1530.16	1508.03	-22.13
82	193	111	207	1538.27	1515.88	-22.39
82	194	112	207	1546.10	1525.27	-20.82
82	195	113	207	1553.64	1532.77	-20.88
82	196	114	207	1560.91	1541.81	-19.10
82	197	115	207	1567.90	1548.96	-18.94
82	198	116	207	1574.60	1557.65	-16.95
82	199	117	207	1581.02	1564.48	-16.55
82	200	118	207	1587.16	1572.83	-14.33
82	201	119	207	1593.02	1579.34	-13.68
82	202	120	207	1598.59	1587.37	-11.22
82	203	121	207	1603.88	1593.56	-10.32
82	204	122	207	1608.89	1601.28	-7.61
82	205	123	207	1613.61	1607.16	-6.44
82	206	124	207	1618.05	1614.58	-3.47
82	207	125	207	1622.20	1620.17	-2.03
82	208	126	207	1626.18	1627.29	1.10
82	209	127	207	1630.10	1632.59	2.49
82	210	128	207	1633.96	1639.42	5.46
82	211	129	207	1637.76	1644.45	6.69
82	212	130	207	1641.50	1650.99	9.50
82	213	131	207	1645.17	1655.75	10.58
82	214	132	207	1648.78	1662.03	13.24
82	215	133	207	1652.33	1666.53	14.19
82	216	134	207	1655.82	1672.53	16.71
82	217	135	207	1659.24	1676.78	17.54
82	218	136	207	1662.60	1682.52	19.93
82	219	137	207	1665.89	1686.52	20.63
82	220	138	207	1669.13	1692.02	22.89
82	221	139	207	1672.29	1695.77	23.48
82	222	140	207	1675.40	1701.03	25.63
82	223	141	207	1678.43	1704.55	26.11

82	224	142	207	1681.41	1709.56	28.15
82	225	143	207	1684.31	1712.86	28.54
82	226	144	207	1687.16	1717.64	30.48
82	227	145	207	1689.93	1720.71	30.78
82	228	146	207	1692.64	1725.26	32.62
82	229	147	207	1695.29	1728.12	32.83
82	230	148	207	1697.86	1732.46	34.59

5. Understanding neutron life time with four gravitational constants

One of the key objectives of any unified description is to simplify or eliminate the complicated issues of known physics. In this context, in a quantitative approach, we noticed that, the four gravitational constants play a crucial role in understanding and estimating neutron life time [23]. The following three strange relations can be given some consideration.

$$\left(t_n\right)_x \approx \left(\frac{G_e}{G_w}\right)\left(\frac{G_e m_n^2}{\left(m_n - m_p\right)c^3}\right) \approx 874.94 \text{ sec} \tag{21}$$

$$\left(t_n\right)_y \approx \sqrt{\frac{G_e}{G_N}}\left(\frac{G_s m_n^2}{\left(m_n - m_p\right)c^3}\right) \approx 896.45 \text{ sec} \tag{22}$$

Considering the geometric mean of relations (21) and (22), it is possible to show that,

$$\left(t_n\right)_{xy} \approx \sqrt{\frac{G_s}{G_w}\sqrt{\frac{G_e}{G_N}}}\left(\frac{G_e m_n^2}{\left(m_n - m_p\right)c^3}\right) \approx 885.63 \text{ sec} \tag{23}$$

Relation (23) seems to constitute all the four gravitational constants. It needs further study. Considering $\left(t_n\right)_x^{2/3}\left(t_n\right)_y^{1/3} \approx 882.05$ sec or $\left(t_n\right)_x^{1/3}\left(t_n\right)_y^{2/3} \approx 889.2$ sec, it may be possible to fit the experimental values of neutron life. Plausible point to be noted is that, Relativistic mass of neutron seems to play a crucial role in understanding the increasing neutron life time. It can be understood with,

$$t_n \propto \frac{m_n^2}{\left[1-\left(v^2/c^2\right)\right]} \tag{24}$$

6. Nuclear charge radii

As per the current literature [24], nuclear charge radii can be expressed with the following formulae.

$$R_c \cong \left\{1+\left[0.015\left(\frac{N-\left(N/Z\right)}{Z}\right)\right]\right\} Z^{1/3} \times 1.245 \text{ fm} \tag{25}$$

$$R_c \cong \left\{1-0.349\left(\frac{N-Z}{N}\right)\right\} N^{1/3} \times 1.262 \text{ fm} \tag{26}$$

$$R_c \cong \left\{1-\left[0.182\left(\frac{N-Z}{A}\right)\right]+\frac{1.652}{A}\right\} A^{1/3} \times 0.966 \text{ fm} \tag{27}$$

Our earlier proposed relation is,

$$R_{(Z,A)} \cong \left\{Z^{1/3}+\left(\sqrt{Z(A-Z)}\right)^{1/3}\right\}\left(\frac{G_s m_p}{c^2}\right) \tag{28}$$

Based on these relations and by considering the charge radii of stable atomic nuclides, R_0 and G_s can be fitted.

7. Discussion

Based on the data presented in tables 1 to 7, we would like to suggest that,

(1) Semi empirical mass formula is having 5 energy terms and 5 different energy coefficients. Those 5 energy coefficients are no way connected with unification paradigm. This can be considered as a major drawback of traditional nuclear binding energy estimation scheme.
(2) From the proposed relations, our earlier works [25-28] and from Ghahramany's integrated nuclear model [19,20], it is very to clear say that, nuclear binding energy can be understood with a single unified energy coefficient.
(3) Squared neutron number plays a crucial role in understanding nuclear binding energy at stable mass numbers. It can be confirmed from table-2.
(4) The ratio, $\frac{G_s m_p m_e}{\hbar c} \cong \frac{\hbar c}{G_e m_e^2} \cong \frac{G_s^2}{G_e G_w} \cong 0.001605$ seems to play a very interesting role in estimating neutron-proton stability and estimating the major reduction part of nuclear binding energy. Hence it can be validated as a characteristic result oriented number.
(5) We are working on understanding the physics of the very small ratio $\left(\frac{A}{2Z}\right)^{\frac{2}{3}} \cong (1.0 \text{ to } 1.24)$ of relation (18) in term-2.
(6) Compared to Semi empirical mass formula, term-3 seems to work well in estimating the total binding energy for $(A < A_s)$.
(7) With further study, for $(A > A_s)$, by introducing a third term, accuracy can be improved.

8. Conclusion

In a unified approach, understanding nuclear binding energy with a single energy coefficient is a very challenging task. In this context, our proposed views and relations can be given some consideration.

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References

[1] G. Rosi, F. Sorrentino, L. Cacciapuoti, M. Prevedelli, and G. M. Tino. Precision measurement of the Newtonian gravitational constant using cold atoms, Nature **510** (7506), 518–521 (2014).
[2] P. J. Mohr, D. B. Newell, and B. N. Taylor, CODATA recommended values of the fundamental constants: 2014, Rev. Mod. Phys. **88**, 035009 (2016).
[3] C. Rothleitner and S. Schlamminger. Measurements of the Newtonian constant of gravitation, G. Rev. Sci. Instrum. 88, 111101 (2017)
[4] Li, Qing et al. Measurements of the gravitational constant using two independent methods. Nature **560**, 582–588 (2018)
[5] John E. Brandenburg. The GEMS (Gravity-EM Super) Unification Theory: the Unification of the Four Forces of Nature, Prediction of New 21 MeV and 22 MeV Particles, and Correspondence with ElectroWeak Theory. Journal of Multidisciplinary Engineering Science Studies. Vol. 2 Issue 10, 968-984 (2016)
[6] G. Lamporesi et al. Determination of the Newtonian Gravitational Constant Using Atom Interferometry. Phys. Rev. Lett. 100, 050801 (2008)
[7] G. Rosi, Challenging the big *G* measurement with atoms and light. J. Phys. B: At., Mol. Opt. Phys. **49**(20), 202002 (2016)
[8] B. Canuel et al. Exploring gravity with the MIGA large scale atom interferometer. Science reports, 8:14064 (2018)
[9] Seshavatharam U.V.S and Lakshminarayana S. Towards a workable model of final unification. International Journal of Mathematics and Physics 7, No1,117-130. (2016)
[10] Seshavatharam U.V.S and Lakshminarayana S. Understanding the basics of final unification with three gravitational constants associated with nuclear, electromagnetic and gravitational interactions. Journal of Nuclear Physics, Material Sciences, Radiation and Applications Vol-4, No-1, 1-19. (2017)
[11] Seshavatharam U.V.S and Lakshminarayana S. A Virtual Model of Microscopic Quantum Gravity. Prespacetime Journal, Vol 9, Issue 1, pp. 58-82 (2018).
[12] Seshavatharam U. V. S et al. Understanding the constructional features of materialistic atoms in the light of strong nuclear gravitational coupling. Materials Today: 3/10PB, Proceedings 3 pp. 3976-3981 (2016)
[13] Seshavatharam U.V.S and Lakshminarayana S. Fermi scale applications of strong (nuclear) gravity-1. Proceedings of the DAE Symp. on Nucl. Phys. 63 72-73. (2018)
[14] Roberto Onofrio. On weak interactions as short-distance manifestations of gravity. Modern Physics Letters A 28, 1350022 (2013)
[15] C. Patrignani et al. (Particle Data Group), Chin. Phys. C, 40, 100001, 2016 and 2017 update
[16] Seshavatharam U.V.S and Lakshminarayana S. Gravity in the Shadow of Stable Atoms and Their Three Interactions. To be presented in Nucleus2019, Dubana, Russia (preprints.org/manuscript/201901.0271/v1)

[17] P. Roy Chowdhury et al. Modified Bethe-Weizsacker mass formula with isotonic shift and new driplines. Mod.Phys.Lett. A20 1605-1618. (2005)
[18] A. V. Karpov et al. Decay Properties and Stability of Heaviest Elements. International Journal of Modern Physics E Vol. 21, No. 2 1250013 (2012)
[19] Ghahramany et al. New approach to nuclear binding energy in integrated nuclear model. Journal of Theoretical and Applied Physics 2012, 6:3
[20] N. Ghahramany et al. Stability and Mass Parabola in Integrated Nuclear Model. Universal Journal of Physics and Application 1(1): 18-25, (2013).
[21] Weizsäcker, Carl Friedrich von, On the theory of nuclear masses; Journal of Physics **96** pages 431-458 (1935)
[22] W. D. Myers et al. Table of Nuclear Masses according to the 1994 Thomas-Fermi Model.(from nsdssd.lbl.gov)
[23] RW Pattie Jr et al.Measurement of the neutron lifetime using a magneto-gravitational trap and in situ detection. Science 11,
Vol. 360, Issue 6389, pp. 627-632 (2018)
[24] T. Bayram, S. Akkoyun, S. O. Kara and A. Sinan, New Parameters for Nuclear Charge Radius Formulas, ActaPhysicaPolonica B. 44(8), 1791-1799 (2013)
[25] Seshavatharam U.V.S and Lakshminarayana S. A Review on Nuclear Binding Energy Connected with Strong Interaction. Prespacetime Journal, Volume 8, Issue 10, pp. 1255-1271 (2018)
[26] Seshavatharam U.V.S and Lakshminarayana S. On the role of 'reciprocal' of the strong coupling constant in nuclear structure. Journal of Nuclear Sciences, Vol. 4, No.2, 31-44. (2017)
[27] Seshavatharam U.V.S and Lakshminarayana S. On the role of strong coupling constant and nucleons in understanding nuclear stability and binding energy. Journal of Nuclear Sciences, Vol. 4, No.1, 7-18, (2017)
[28] Seshavatharam U.V.S and Lakshminarayana S. A new approach to understand nuclear stability and binding energy. Proceedings of the DAE-BRNS Symp. On Nucl. Phys. 62, 106-107 (2017)