

Exploration

The Analogs of CKM Mixing & Neutrino Oscillations for Particle & Its Dark Variants

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Abstract

The called 21-cm anomaly meaning that there is unexpected absorption of this line could be due to the transfer of energy from gas to dark matter leading to a cooling of the gas. This requires em interaction of the ordinary matter with dark matter but the allowed value of electric charge must be much smaller than elementary particle charges. In TGD Universe the interaction would be mediated by an ordinary photon transforming to dark photon having effective value $h_{eff}/h_0 = n$ larger than standard value h implying that em charge of dark matter particle is effectively reduced. Interaction vertices would involve only particles with the same value of $h_{eff}/h_0 = n$. In this article a simple model for the mixing of particle and its dark variants is proposed. Due to the transformations between different values of $h_{eff}/h_0 = n$ during propagation, mass squared eigenstates are mixtures of particle with various values of n . An the analog of CKM matrix describing the mixing follows from mass squared matrix containing non-diagonal terms. The model for neutrino oscillations is generalized so that it applies to any particle, also photon. The condition that "ordinary" photon is essentially massless during propagation forces to assume that during propagation photon is mixture of ordinary and dark photons, which would be both massive in absence of mixing. A reduction to ordinary photon would take place in the interaction vertices and therefore also in the absorption. The mixing provides a new contribution to particle mass besides that coming from p-adic thermodynamics and from the Kähler magnetic fields assignable to the string like object associated with the particle.

Keywords: CKM mixing, neutrino oscillation, particle, dark variant, analog, TGD framework.

1 Introduction

In TGD Universe dark matter in TGD sense corresponds to $h_{eff}/h_0 = n$, $h = 6h_0$ is a good guess [7, 12, 8] phases of ordinary matter associated with magnetic flux tubes. These flux tubes would be n -sheeted covering spaces, and n would correspond to the dimension of the extension of rationals in which Galois group acts. The evidence for this interpretation of dark matter is accumulating. I have already earlier discussed [6] one of the latest anomalies - so called 21-cm anomaly. This finding motivates a more detailed model for the interaction between different levels of dark matter hierarchy and in the sequel I will propose this kind of model.

1.1 21-cm anomaly

Sabine Hossenfelder (see <http://tinyurl.com/y7h5ys2r>) told about the article [1] discussing the possible interpretation (see <http://tinyurl.com/yasgfgq8>) of so called 21-cm anomaly associated with the hyperfine transition of hydrogen atom and observed by EDGES collaboration [2].

The EDGES Collaboration has recently reported the detection of a stronger-than-expected absorption feature in the global 21-cm spectrum, centered at a frequency corresponding to a redshift of $z \sim 17$. This observation has been interpreted as evidence that the gas was cooled during this era as a result of scattering with dark matter. In this study, we explore this possibility, applying constraints from the cosmic microwave background, light element abundances, Supernova 1987A, and a variety of laboratory experiments. After

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taking these constraints into account, we find that the vast majority of the parameter space capable of generating the observed 21-cm signal is ruled out. The only range of models that remains viable is that in which a small fraction, $\sim 0.3 - 2$ per cent, of the dark matter consists of particles with a mass of $\sim 10-80$ MeV and which couple to the photon through a small electric charge, $\epsilon \sim 10^{-6} - 10^{-4}$. Furthermore, in order to avoid being overproduced in the early universe, such models must be supplemented with an additional depletion mechanism, such as annihilations through a $L_\mu - L_\tau$ gauge boson or annihilations to a pair of rapidly decaying hidden sector scalars.

What has been found is an unexpectedly strong absorption feature in 21-cm spectrum: the redshift is about $z = \Delta f/f \simeq v/c \simeq 17$, which from Hubble law $v = HD$ corresponds to a distance $D \sim 2.3 \times 10^{11}$ ly. Dark matter interpretation would be in terms of scattering of the baryons of gas from dark matter at lower temperature. The anomalous absorption of 21 cm line could be explained with the cooling of gas caused by the flow of energy to a colder medium consisting of dark matter. If I understood correctly, this would generate a temperature difference between background radiation and gas and consequent energy flow to gas inducing the anomaly.

The article excludes large amount of parameter space able to generate the observed signal. The idea is that the interaction of baryons of the gas with dark matter. The interaction would be mediated by photons. The small em charge of the new particle is needed to make it "enough". My conviction is that tinkering with the quantization of electromagnetic charge is only a symptom about how desperate the situation is concerning interpretation of dark matter in terms of some exotic particles is. Something genuinely new physics is involved and the old recipes of particle physicists do not work.

1.2 TGD based explanation of 21-cm anomaly

In TGD framework the dark matter at lower temperature would be $h_{eff}/h = n$ phases of ordinary matter residing at magnetic flux tubes. This picture follows from what I call adelic physics [10, 11]. This kind of energy transfer between ordinary and dark matter is a general signature of dark matter in TGD sense, and there are indications from some experiments relating to primordial life forms for this kind of energy flow in lab scale [9] (see <http://tinyurl.com/yassnhzb>).

The ordinary photon line appearing in the Feynman diagram describing the exchange of photon would be replaced with a photon line containing a vertex in which the photon transforms to dark photon. The coupling in the vertex - call it m^2 - would have dimensions of mass squared. This would transform the coupling e^2 associated with the photon exchange effectively to $e^2 m^2/p^2$, where p^2 is photon's virtual mass squared. The slow rate for the transformation of ordinary photon to dark photon could be seen as an effective reduction of electromagnetic charge for dark matter particle from its quantized value.

Remark: In biological systems dark cyclotron photons would transform to ordinary photons and would be interpreted as bio-photons with energies in visible and UV.

The importance of this finding is that it supports the view about dark matter as ordinary particles in a new phase. There are electromagnetic interactions but the transformation of ordinary photons to dark photons slows down the process and makes these exotic phases effectively dark.

The above picture motivates the attempt to construct a model for the mixing of not only ordinary photons but any particle with its dark variants with various values of $h_{eff}/h_0 = n$ by generalizing the formalism developed for the mixing of neutrinos and their oscillations. Also now oscillations are predicted and they could serve as a test for TGD based model of dark matter. Also the description at the level of Feynman diagrams is briefly summarize. This picture in principle allows the modelling of the energy transfer between ordinary and dark sectors.

2 Mixing of dark photons

In TGD framework dark matter corresponds to phases of ordinary matter with non-standard value of Planck constant $h_{eff}/h_0 = n$ [4]. Here $h = 6h_0$ is a good guess [7, 12]. It has been assumed that only the

reaction vertices would be between particles with same value of $h_{eff}/h = n$, whereas the transformation changing the value of n during propagation is assumed to be possible. For instance, biophotons would be ordinary photons emerging when dark photons transform to ordinary photons. Therefore the mixing of ordinary particles with their dark variants can be considered.

This allows to deduce the general form of propagator which is simple for the mixed mass squared eigenstates in terms of mass squared matrix. There is however a problem associated with photons. They must have extremely small mass although p-adic mass calculations suggests that photon has very small p-adic thermal mass squared [3]. Are they exactly massless and what conditions masslessness poses on mixing? It turns out that the eigenstates of n most naturally have same mass and the mixing makes other state massless so that ordinary photon would not have minimal value of n - presumably $n = 6$ - during propagation but in absorption the state would be projected to $n = 6$.

2.1 Mixing and oscillations of ordinary and dark particles

Could the analog of CKM mixing take place for ordinary and dark photons? Is the analog of neutrino oscillations possible for photon and dark photon? Could these oscillations occur also for neutrinos besides ordinary neutrino oscillations? The model for the analog of ordinary-dark oscillations could be essentially the same as that for neutrino oscillations and consist of the following pieces.

In the case of neutrino mixing involving 3 neutrinos the calculation gives the result given in Wikipedia article (see <http://tinyurl.com/ooV344k>). Since the formula does not depend on the number of flavors, it easily generalize to the case that one has arbitrary number N of values of $h_{eff}/h_0 = n$, which mix. The analog of CKM matrix describing the mixing of neutrinos, the mass squared differences, and the distance L between source and receiver determines the oscillation dynamics and generalizes as such to the description of mixing and oscillation of particles with different values of h_{eff} . For N values of n including $n = n_0 = 6$ assigned with ordinary matter, the analog of CKM matrix is $N \times N$ unitary matrix.

This matrix, call it C , is completely determined by the mass squared matrix with non-diagonal components. Mass squared eigenstates are superpositions of states with well-defined value of n_{eff} having the rows of this matrix as coefficients. Therefore the non-diagonal component of mass squared matrix, to called K^2 , describing the mixing of different values of n determines both mixing and oscillations.

A non-trivial modification of the formula for the neutrino oscillations comes from the fact that plane wave factor $s \exp([iE_i - p]L/\hbar_{eff}(\alpha))$ depend on the value of $\hbar_{eff}(\alpha) = n^\alpha \hbar_0$.

The following model applies to any particle species.

1. The mixing of ordinary and dark particles would be an analog of CKM mixing for quarks and leptons. Now ordinary particle and its dark variants would mix with each other. Note that given value of n can correspond to several extensions of rationals. In principle also this degeneracy must be also be taken into account.
2. The analog of neutrino oscillations would mean that ordinary particles disappear from beam by transforming to dark particles and can be regenerated. The formalism for neutrino oscillations seems to generalize almost as such to ordinary-dark particle oscillations. Oscillations could be used as test for TGD view about dark matter.
3. In the initial and final state the particle would be either ordinary or dark with some value of n being analogous to a flavor eigenstate for neutrino. These states are not eigenstates of mass and energy and it convenient to express them as mass squared eigenstates related by CKM matrix to eigenstates of n . During propagation states can be regarded as superpositions of eigenstates of mass squared operator M^2 . This hermitian operator is sum of ordinary mass squared operators for the sectors labelled by n but there are non-diagonal term is causing the mixing.
4. One has on mass shell condition in momentum space which can be written as

$$(p^2 - M_{op}^2)\Psi = 0 \quad . \quad (2.1)$$

p^2 represents four momentum square in various sectors labelled by n^α and can be regarded as direct sum $p^2 = \oplus p^2(n^\alpha)$.

For given value of 3-momentum the situation is identical for a system consisting of N coupled harmonic oscillators and the situation is mathematically equivalent to the diagonalization of the system by finding the eigenmodes and eigenfrequencies.

5. Mass squared operator is direct sum

$$M_{op}^2 = \oplus_{n^\alpha} m^2(n^\alpha) + K^2 \quad . \quad (2.2)$$

$K_{\alpha\beta}^2$ is non-diagonal coupling different sectors n^α and thus mixing of partial waves with different values of n . The assumption has been that $m^2(n^\alpha)$ does not depend on n^α . The presence of the non-diagonal mixing term $K_{\alpha\beta}^2$ causes mass squared eigenstates to have different masses.

M_{op}^2 would have for $N = 2$ (ordinary particle and its dark variant with single value of n) the form

$$M_{op}^2 = \begin{bmatrix} m^2 & K^2 \\ K^2 & m^2 \end{bmatrix} \quad . \quad (2.3)$$

Note that one K^2 can be also complex.

6. In this form the value of \hbar_{eff} is not visible at all in p^2 . At the space-time level $p^2 = E^2 - p_3^2$ must be however expressed as d'Alembert operator via the usual rules $E \rightarrow i\hbar_{eff}\partial_t$ and $p \rightarrow i\hbar_{eff}\partial_z$ so that one has

$$\begin{aligned} (-\square - M_{op}^2)\Psi &= 0 \quad , \quad \square = \oplus_\alpha \square_{n^\alpha} \quad , \\ \square_n &= n^2 \hbar_0^2 \square \quad , \quad \square = \partial_t^2 - \partial_x^2 - \partial_y^2 - \partial_z^2 \quad . \end{aligned} \quad (2.4)$$

Plane wave solutions are of form $exp(i(E - p)z/n\hbar_0)$ and differ by a scaling of the argument. This applies also to general solutions. One has fractally scaled variants of the solution and K^2 matrix defines coupling between them.

7. This formulation generalizes trivially to general 4-D case solutions and to general solutions of d'Alembert type field equations. In QFT language one has an analog of N -component scalar field for which mass squared matrix M_{op}^2 containing quadratic couplings between field components. The generalization seems obvious also for more general fields such as spinor fields and gauge fields. For instance, for gauge fields one would have N copies of gauge fields with non-diagonal couplings. The invariants $F_{n^\alpha}^{\mu\nu} F_{n^\beta, \mu\nu}$ are suggestive for gauge invariant couplings.

The new element is that these N fields have different value of \hbar_{eff} and the solutions are fractally scaled variants of each other.

8. The eigenstates $|i\rangle$ of the d'Alembert type operator are eigenstates of M_{op}^2 and eigenvalues are mass squared eigenvalues m_i^2 . $|i\rangle$ are superpositions states with fixed value of n with coefficients, which are the components of the analog C of CKM matrix:

$$|i, x\rangle = C_{i\alpha} e^{i \frac{p_4 \cdot x}{n^\alpha \hbar_0}} |n^\alpha\rangle . \tag{2.5}$$

Here one has summation of the repeated index α appearing as both upper and lower index. This holds quite generally for Fourier basis. Therefore the non-diagonal part of mass squared operator determines the C as a prediction.

The S-matrix for the effectively 2-D system considered is needed to deduce oscillation probabilities. One has a beam of particles with momentum p independent of value of n travelling distance L along line $z = t$. The mass parameter $m^2(n)$ is independent of n .

1. To deduce S-matrix start from the expression of the identity operator Id as

$$Id = |i, t = 0\rangle \langle i, t = 0|$$

acting at the end $z = 0$. The states $|i, t = 0\rangle$ correspond to the starting point $z = 0$ of propagation. The notation $|n^\alpha, t = 0\rangle = |n^\alpha\rangle$ will be used. Time evolution shifts the states $|i, t = 0\rangle = C_{i\alpha} |n^\alpha\rangle$ to $t = z = L$ by the above time evolution.

2. S-matrix is obtained by translating the states $|i, t = 0\rangle$ appearing in the identity operator to $(t = L, z = L)$.

$$S = \sum_i |i, t = L\rangle \langle i, t = 0| . \tag{2.6}$$

3. One can find the expression of S in the basis $|i, t = L\rangle$ by writing $|i, t = L\rangle$ as a superposition of states $|n^\alpha\rangle$:

$$|i, t = 0\rangle = C_{i\alpha} |n^\alpha\rangle \rightarrow |i, t = L\rangle = C_{i\alpha} U_\alpha^i |n^\alpha\rangle , \tag{2.7}$$

$$U_\alpha^i = e^{i \frac{(E_i - p)L}{n^\alpha \hbar_0}} , \quad E_i = \sqrt{p^2 + m_i^2} .$$

Using this formula one can express S using basis $|n^\alpha\rangle$.

$$S = S_{\alpha\beta} |n^\alpha\rangle \langle n^\beta| ,$$

$$S_{\alpha\beta} = \overline{C}_\alpha^i C_{i\beta} U_\alpha^i . \tag{2.8}$$

Here the summation convention for the repeated index i applies.

What are needed are the oscillation probabilities $P_{\alpha\beta}$.

1. The probabilities that an eigenstate $|n^\alpha\rangle$ transforms to eigenstate $|n^\beta\rangle$ during the travel are given by

$$\begin{aligned}
 P_{\alpha\beta} &= |S_{\alpha\beta}|^2 = Y_{\alpha\beta ij} U_{\alpha\beta}^{ij} \quad , \quad Y_{\alpha\beta ij} = \bar{C}_{i\alpha} C_{i\beta} C_{j\alpha} \bar{C}_{j\beta} \\
 U_{\alpha\beta}^{ij} &= U_{\alpha}^i \bar{U}_{\beta}^j = \cos(X_{\alpha\beta}^{ij}) + i \sin(X_{\alpha\beta}^{ij}) \quad , \quad X_{\alpha\beta}^{ij} = \frac{(E_i - p)L}{n^\alpha \hbar_0} - \frac{(E_j - p)L}{n^\beta \hbar_0} \quad .
 \end{aligned}
 \tag{2.9}$$

2. One can decompose $P_{\alpha\beta}$ as

$$P_{\alpha\beta} = \text{Re}[Y_{\alpha\beta ij}] \cos(X_{\alpha\beta}^{ij}) - \text{Im}[Y_{\alpha\beta ij}] \sin(X_{\alpha\beta}^{ij}) \quad , \tag{2.10}$$

and apply trigonometric formula $\cos(2x) = 1 - 2\sin^2(x)$, and decompose the summation to indices to 3 groups with $i < j$, $j < i$ and $i = j$ to get

$$P_{\alpha\beta} = \delta_{\alpha\beta} - 4 \sum_{i < j} \text{Re}[Y_{\alpha\beta ij}] \sin^2\left(\frac{X_{\alpha\beta}^{ij}}{2}\right) - 2 \sum_{i < j} \text{Im}[Y_{\alpha\beta ij}] \sin(X_{\alpha\beta}^{ij}) \quad . \tag{2.11}$$

Note that $\sum_{\beta} P_{\alpha\beta} = 1$ holds true since in the summation second term vanishes due to unitary condition $U^\dagger U = 1$ and $i > j$ condition in the formula.

3. In the completely relativistic situation $p \gg m_i$ one can make the analog of non-relativistic approximation as $E_i = p + m_i^2/2p$. In this case one has

$$X_{\alpha\beta}^{ij} = \frac{(E_i - p)L}{n^\alpha \hbar_0} - \frac{(E_j - p)L}{n^\beta \hbar_0} \simeq \frac{m_i^2 L}{pn^\alpha \hbar_0} - \frac{m_j^2 L}{pn^\beta \hbar_0} \quad . \tag{2.12}$$

4. For given 3-momentum p $P_{\alpha\beta}$ is a sum over $N \times (N - 1)$ periodic functions of L with periods

$$\lambda_{\alpha\beta}^{ij} = \frac{2\pi}{X_{\alpha\beta}^{ij}} \quad . \tag{2.13}$$

5. At the limit of large L the trigonometric factors oscillate rapidly and in the averaging over sources region. The term proportional to $\sin(x)$ gives zero whereas $\sin^2(x)$ gives average 1/2. The probabilities for various transitions induced by the oscillations depend on the analog of CKM matrix only. If the distance L is very large and the dependence on the mass squared differences and distance disappears in the averaging over the source region and one obtains

$$P_{\alpha\beta} = \delta_{\alpha\beta} - 2\text{Re}[Y_{\alpha\beta ij}] \quad . \tag{2.14}$$

Some general comments are in order.

1. The oscillation is detectable if the size of the non-diagonal part K^2 of the mass matrix is large enough as compared to the diagonal part. It is not clear whether this condition holds true for say fermions. The absence of tachyons requires that the value of m^2 (no dependence on n) is positive. m^2 could be interpreted as thermal mass squared in terms of p-adic mass calculations [5, 3]. In the case of massless particles the mixing during propagation can however make the mass arbitrarily small as will be found.
2. What can be measured is the diagonal probability P_{11} , where $\alpha = 1$ corresponds to $h_{eff} = h$. The formula reduces to that for neutrino oscillations or its generalization to N flavors since $h_{eff} = h$ holds true now:

$$X_{11}^{ij} = \frac{(E_i - p)L}{\hbar} - \frac{(E_j - p)L}{\hbar} \simeq \frac{(m_i^2 - m_j^2)L}{p\hbar} . \quad (2.15)$$

Remark: The part of P_{11} proportional to sine function has sine opposite to that in the formula of Wikipedia article (see <http://tinyurl.com/ooV344k>): the reason is that the definition of $Y_{\alpha\beta ij}$ used here is complex conjugate of that used in Wikipedia formula.

3. Mass squared matrix and mixing matrix are not uniquely determined by the mass squared eigenvalues. Any unitary transform $M_D^2 \rightarrow UM_D^2U^\dagger$ of the mass matrix M_D^2 has the same eigenvalues. If the states with well-defined h_{eff} have the same mass in absence of mixing, $UM_D^2U^\dagger$ must have diagonal part equal to $m^2 Id$.

This gives N conditions on U in both real and complex case. The conditions are however not dependent since the trace of M_D^2 equal to Nm^2 is preserved in the transformation so that there are only $N - 1$ conditions in both real and complex case.

Since the number of the independent elements of a unitary matrix with unit determinant is $N^2 - 1$, this leaves in complex case $(N - 1)^2$ parameter set of mass matrices with the same eigenvalues. Orthogonal matrix has $(N-1)N/2$ independent elements so that one has $(N-2)(N-1)/2$ parameters in the real case. For $N = 2$ complex case one has 1-parameter set of solutions corresponding to the phase of K^2 , in the real $N = 2$ case one has two solutions corresponding to two signs for K^2 . For $N = 3$ one has 4 parameters in complex case and 1 parameter in real case.

2.2 Mass squared matrix for photons

What can one say about mass squared matrix for photons? Consider a situation in which only two photons are mixed.

1. The most general form of mass matrix is in the case of single value of n given by $M_{op}^2 = [m^2, K^2; \overline{K^2}, m^2]$. Note that the diagonal element is assumed to be nonvanishing: this allows to avoid tachyonic mass squared eigenstate. The eigen values of M_{op}^2 are given by

$$M_{\pm}^2 = m^2 \pm |K|^2 . \quad (2.16)$$

2. The condition $M_{\pm}^2 \geq 0$ gives $m^2 \geq |K|^2$. For the general mass squared matrix $M_{op}^2 = [m_1^2, K^2; \overline{K^2}, m_2^2]$ the condition reads $m_1 m_2 \geq |K|^2$. If m_1 is very small, m_2 must be large in the scale defined by $|K|$.

One can argue that this form of mass squared matrix is the only reasonable option. If $n = 6$ photon is massless one obtains photons with masses $m^2 = \pm K^2$ and tachyonic photon is physically very problematic. It must be remembered that for TGD all particles are massless in 8-D sense and can be massive in 4-D sense. Therefore the assumption that "free" photon is massive need not lead to problems.

3. The mass of what we identify as ordinary photon and identified now as a mixed photon with lowest mass is extremely small: the recent upper bound is 7×10^{17} eV, which corresponds to Compton length of 10^{11} meters, which is of the order one astronomical unit AU: this probably relates to the measurement method. Photons thus behave like massless particles in the scale of Sun-Earth system. Therefore the approximation would $m^2 = |K|^2$ is excellent. The masses would be $M_-^2 = 0$ and $M_+^2 = 2m^2$.

Dark photons in TGD sense play a key role in TGD inspired model of living matter. Bio-photons would result in the transformation of dark photons to ordinary photons. Mass squared eigenstates of photons have mass spectrum and a natural question is whether dark photon mass relevant to biology corresponds to a Compton length scale relevant to biology. In p-adic physics Compton lengths correspond to p-adic length scales which by p-adic length scale hypothesis correspond to primes $p \simeq 2^k$ near power of 2 (slightly below it).

Mersenne primes and their Gaussian analogs are especially interesting physically and in the length scale range 10 nm (neural membrane thickness) and 2.5μ (size scale of nucleus) there are as many as 4 Gaussian Mersennes $M_{G,k} = (1+i)^k - 1$ corresponding to $k \in \{151, 157, 163, 167\}$. Could the p-adic mass scales $m/m_e = 2^{(k-127)/2}$ associated with these length scales be especially important in biology. More generally all p-adic mass scales assignable to these two kinds of Mersenne primes could be important as mass scales of mixed photons.

2.3 Could the mixing with dark photons provide an additional contribution to particle masses?

p-Adic thermodynamics [3] provides an excellent description of particle massivation in the fermionic sector. It assumes only p-adic thermodynamics and superconformal invariance with partition functions determined by it, p-adic length scale hypothesis, and canonical identification $x = \sum x_n p^n \rightarrow \sum x_n p^{-n}$ mapping p-adic thermodynamical mass squared expectations to their real counterparts.

This need not however be the entire story. It is not clear whether one can really understand most of the hadron mass in this manner and whether gauge boson masses involving in the usual approach Higgs mechanism can be completely understood in this manner. Therefore one can ask whether the mixing of particles with their dark variants could contribute to the particle masses. In case of gauge bosons this contribution could be significant.

2.4 Description of ordinary-dark scattering diagrams

One would like also to develop a model for the scattering of ordinary and dark particles via exchange of ordinary photons transforming to dark photons or vice versa. Here one must be satisfied to phenomenological description although it is clear that there are non-trivial issues related to the gauge invariance in presence of massivation. The general TGD picture strongly suggests that these problems can be solved. In twistor lift of TGD particles become massless in 8-D sense and can be massive in 4-D sense.

The simplest assumption is that the massless photon propagator $D = P/p^2 - i\epsilon$, where P is a projector to the space of physical polarizations, is replaced with matrix propagator

$$D = \left[\frac{P}{p^2 Id - M^2(op)} \right]_{ij} = \frac{P}{p^2} \sum_{n \geq 0} \left[\frac{M^2(op)}{p^2} \right]_{ij}^n . \tag{2.17}$$

For the mass squared eigenstates this gives diagonal matrix with poles corresponding to mass squared eigenvalues. What looks problematic is that the projector P for massive states projects to a 3-D space of polarization and for massless states to 2-D space of polarization. If also ordinary photon has very small mass as p-adic mass calculations strongly suggest, also it has longitudinal polarization and all projectors are 3-D.

The reaction vertices are possible only between particles with same value of n so that the propagator must be replaced in this basis by $C^\dagger DC$, where C is the analog of CKM mixing matrix mediating transition to mass eigenstates.

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