**Article**

# Inflationary Scenario in Bianchi Type-V Spacetime with Variable Bulk Viscosity & Dark Energy in Radiation Dominated Phase

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## Abstract

Inflationary scenario in Bianchi Type V space-time with variable bulk viscosity and dark energy in radiation dominated phase is discussed. We consider exact cosmological solutions of the Einstein gravitational equations with non-interacting combination of a classical scalar field and isotropic radiation as a source. The model isotropizes in special case and at late time i.e. the model isotropizes asymptotically and the presence of bulk viscosity accelerates the isotropization. The spatial volume increases exponentially representing inflationary scenario and this expansion continues for long enough, thus solves horizon problem.. The deceleration parameter (q) < 0 indicates accelerating universe. Thus, the model represents not only expanding but accelerating universe. (Riess et al.[3], Perlmutter et al.[4]) The slow roll parameters ($\in,\delta,S$) are in agreement with Planck (2013) results. In earlier paper Bali and Singh discussed inflationary scenario in Bianchi-Type V space time with variable bulk viscosity and vacuum energy density for barotropic build distribution where analytical solution was not possible.

**Keywords:** Inflationary, Bianchi V, variable bulk viscosity, dark energy, radiation phase.

## 1. Introduction

In the early stage of the big-bang, most of the energy was in the form of radiation and radiation was the dominant influence on the expansion of the universe. The roles of matter and radiation changed after the cooling with the expansion and the universe entered a matter dominated era. The study of radiation dominated era helps us to learn about the origin and large scale structure of the universe and to know how the universe was formed. Recent results suggest that we have entered an era dominated by Dark Energy (Frieman et al.[1]). The cosmological constant is given the symbol $\Lambda$ and considered as a source term in the Einstein field equation and has been viewed as equivalent to 'mass' of empty space or dark energy. A wide range of observations suggest that cosmological constant ($\Lambda$) is the most favoured candidate of dark energy representing energy density of vacuum. Barrow and Shaw[2] suggested that cosmological constant term ($\Lambda$) corresponds to a very small value of the order $10^{-122}$ when applied to Friedmann universe. Before 1998, there was no direct Astronomical evidence for Dark Energy ($\Lambda$). In 1998, 1999, two independent groups led by Riess et al.[3], Perlmutter et al.[4] used Type Ia Supernovae to show that

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universe is not only expanding but accelerating. Edmund et al.[5] have reviewed the observational evidence for the current accelerated expansion of the universe and presented a number of dark energy models. A number of Dark Energy cosmological models have been investigated by several authors viz. Berman[6], Beesham[7], Abdussattar and Vishwakarma[8], Saha[9], Singh et al.[10], Bali and Singh[11,12] in different contexts.

Bianchi Type V models create more interest in the study because these models contain isotropic special cases and allow small anisotropy levels at any instant of cosmic time. These models have been studied by number of authors viz. Collins[13], Wainwright et al.[14], Roy and Singh[15], Banerjee and Sanyal[16], Ram[17], Bali and Kumawat[18], Bali et al.[19].

The isotropization of the cosmic fluid induced by viscosity is an important physical effect as discussed by Brevik and Petterson[20,21]. The effect of bulk viscosity on the evolution of cosmological models has been studied by several authors viz. Gron[22], Padmanabhan and Chitre[23], Lima et al.[24], Maartens and Mendez[25], Brevik et al.[26].

Inflation is the rapid exponential expansion of the early universe by a factor of $10^{78}$ in volume driven by a negative pressure vacuum energy density. The outstanding problems of cosmology are of explaining the observed isotropy, homogeneity, flatness and specific entropy of universe. Guth[27] has discussed the inflationary universe as a possible natural explanation for the observed large scale homogeneity and near critical density (flatness) of the universal expansion. Barrow and Turner[28] have pointed out that large amount of anisotropy will not undergo the inflationary phase. A universe with only moderate anisotropy will undergo inflation and will be rapidly isotropized. The inflationary models incorporate all the predictions of the standard model for the observable universe because the inflationary universes have the same behaviour after $t = 10^{-32}$ seconds or so for observable universe. Several versions of inflationary scenario have been studied by number of authors viz. Linde[29], Wald[30], Gron[31], Barrow[32], La and Steinhardt[33], Ellis and Madsen[34], Bali and Jain[35]. Bali[36] discussed the significance of inflation for isotropization of universe. Rothman and Ellis[37] have pointed that we can have solution of the isotropy problem if we work with anisotropic metric and the model isotropizes in special case.

Keeping in view of the suggestion made by Rothman and Ellis[37], we consider spatially homogeneous and anisotropic Bianchi Type V space-time for the study of inflationary scenario with bulk viscosity and dark energy for a radiation dominated phase. In earlier paper Bali and Singh[38] discussed inflationary scenario in Bianchi-Type V space time with variable bulk viscosity and vacuum energy density for barotropic build distribution where analytical solution was not possible. In this paper, we have investigated analytical solution for inflationary scenario in Bianchi Type V space-time with variable bulk viscosity and dark energy for radiation dominated phase and discussed physical results in terms of cosmic time. To get the deterministic model of the universe, we assume the condition $\Lambda \sim \frac{1}{R^2}$ as used by Chen and Wu[39], R being average scale factor of our expanding universe. It has been shown that such a time varying (Λ) leads to no conflict with existing observations.

## 2. Metric & Field Equations

We consider Bianchi Type V line-element in orthogonal form as

$$ds^2 = -dt^2 + A^2\, dx^2 + e^{2x}(B^2 dy^2 + C^2\, dz^2) \tag{1}$$

where A, B, C are metric potentials and are function of t-alone.

We assume the coordinates to be comoving so that $v^1 = v^2 = v^3 = 0, v^4 = 1$.

The Lagrangian in which gravity is minimally coupled to the scalar field (ϕ) is given by (Stein-Schabes[40]) as

$$L = \int \sqrt{-g}\left[R - \frac{1}{2} g^{ij}\partial_{\iota}\phi\partial_{j}\phi - V(\phi)\right]d^4x \tag{2}$$

Modified Einstein's field equation (in gravitational units 8πG = c = 1) with cosmological term (Λ) are given by Synge[41] for (+,+,+,-) signature as

$$R_{ij} - \frac{1}{2}R\, g_{ij} + \Lambda(t) = -\, T_{ij} \tag{3}$$

The cosmic fluid is assumed to be viscous fluid. The energy momentum tensor with scalar field (ϕ) is given as

$$T_{ij} = (\rho + p)v_i v_j + pg_{ij} - \zeta\theta(g_{ij} + v_i v_j) + \partial_i\phi\partial_j\phi - \left\{\frac{1}{2}\partial_k\phi\partial^k\phi + V(\phi)\right\}g_{ij} \tag{4}$$

where ρ is the matter density, p the isotropic pressure, ζ is the coefficient of bulk viscosity, θ the expansion in the model and V(ϕ) is the potential.

The energy conservation law coincides with the equation of motion for ϕ and we have

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}\,\partial^\mu\phi) = -\frac{dV}{d\phi} \tag{5}$$

which leads to

$$\phi_{44} + \phi_4\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) = \frac{dV(\phi)}{d\phi} \tag{6}$$

The homogeneous scalar field $\phi$ which is identified with the inflation is only function of cosmic time t and $\phi_4 = \frac{d\phi}{dt}$.

To get the deterministic inflationary scenario for radiation dominated phase ($\rho$=3p), we also assume $\zeta = \sqrt{\rho/3}$, $\rho = 3H^2$ as considered by Barrow[32] and potential $V(\phi)$ = Constant = K (say).

The Einstein's field equation (3) for the space-time (1) with equation (4) and above consideration leads to

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4C_4}{BC} - \frac{1}{A^2} + \Lambda = \left[\frac{2\rho}{3} - \frac{1}{2}\phi_4^2 + K\right] \tag{7}$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4C_4}{AC} - \frac{1}{A^2} + \Lambda = \left[\frac{2\rho}{3} - \frac{1}{2}\phi_4^2 + K\right] \tag{8}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4B_4}{AB} - \frac{1}{A^2} + \Lambda = \left[\frac{2\rho}{3} - \frac{1}{2}\phi_4^2 + K\right] \tag{9}$$

$$\frac{A_4B_4}{AB} + \frac{A_4C_4}{AC} + \frac{B_4C_4}{BC} - \frac{3}{A^2} + \Lambda = \left[\rho + \frac{1}{2}\phi_4^2 + K\right] \tag{10}$$

$$\frac{2A_4}{A} - \frac{B_4}{B} - \frac{C_4}{C} = 0 \tag{11}$$

Equation (11) leads to

$$A^2 = \ell BC$$

where $\ell$ is constant of integration.

For simplicity, we assume constant of integration unity i.e.

$$A^2 = BC \tag{12}$$

## 3. Solution of Field Equations

Equations (7), (8) and (10) lead to

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{A_4B_4}{AB} + \frac{A_4C_4}{AC} + \frac{2B_4C_4}{BC} - \frac{4}{A^2} + 2\Lambda = \frac{5\rho}{3} + 2K \tag{13}$$

which leads to

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{A_4B_4}{AB} + \frac{A_4C_4}{AC} + \frac{2B_4C_4}{BC} - \frac{4}{A^2} + 2\Lambda = 2K + \frac{5}{4}\frac{(BC)_4^2}{(BC)^2} \tag{14}$$

as $\rho = 3H^2$, $H = \theta/3$.

Now we assume that $\Lambda = \frac{m}{R^2}$ as considered by Chen and Wu[39]. For the sake of simplicity, we take m = 2 i.e. $\Lambda = \frac{2}{R^2}$.

Using equation (12) and value of Λ in equation (14), we have

$$\frac{(BC)_{44}}{BC} + \frac{1}{2}\frac{(BC)_4^2}{(BC)^2} = 2K + \frac{5}{4}\frac{(BC)_4^2}{(BC)^2} \tag{15}$$

Thus, equation (15) leads to

$$2\mu_{44} - \frac{3}{2}\frac{\mu_4^2}{\mu} = 4K\mu \tag{16}$$

where BC = μ

To solve this equation, we assume

$$\mu_4 = f(\mu), \mu_{44} = f'f \text{ where } f' = \frac{\partial f}{\partial \mu}$$

Thus, we have

$$\frac{df^2}{d\mu} - \frac{3}{2\mu}f^2 = 4K\mu$$

which leads to

$$\mu_4 = f = \sqrt{8K\mu^2 + \alpha\mu^{3/2}} \tag{17}$$

where α is constant of integration.

This leads to

$$\mu = \beta^4 \sin h^4(at + b) \tag{18}$$

where

Equation (17) leads to

$$\beta = \sqrt{\frac{\alpha}{8K}} \text{ and } a = \sqrt{\frac{K}{2}}$$

Equations (8) and (9) lead to

$$\frac{\nu_4}{\nu} = \frac{L}{\mu^{3/2}}$$

where L is constant of integration and BC = μ, B/C = ν.

Using (18) and solving, we get

$$\nu = \exp\left\{\frac{L}{15a\beta^6}(4\text{cosech}^2T\coth T - 3\text{cosech}^4T\coth T - 8\coth T)\right\} \tag{19}$$

where at+b = T

we have

$$A^2 = \mu = \beta^4 \sin h^4 T \tag{20}$$

$$B^2 = \mu\nu = \beta^4 \sinh^4 T \exp\left\{\frac{L}{15a\beta^6}(4\text{cosech}^2 T\coth T - 3\text{cosech}^4 T\coth T - 8\coth T)\right\} \tag{21}$$

$$C^2 = \frac{\mu}{\nu} = \beta^4 \sinh^4 T \exp\left\{-\frac{L}{15a\beta^6}(4\text{cosech}^2 T\coth T - 3\text{cosech}^4 T\coth T - 8\coth T)\right\} \tag{22}$$

Therefore, the metric (1) leads to

$$ds^2 = -\frac{dT^2}{a^2} + \sinh^4 T dX^2 + \sinh^4 T(\nu\, dY^2 + \nu^{-1} dZ^2) \tag{23}$$

where $\beta^2 x = X, \beta^2 y = Y, \beta^2 z = Z$ and $\nu$ is determined by equation (19).

## 4. Physical & Geometrical Aspects

The expansion ($\theta$), Hubble parameter (H), matter density ($\rho$), pressure (p), spatial volume ($R^3$), shear ($\sigma$), deceleration (q), dark energy ($\Lambda$), anisotropy parameter ($\widehat{A}$), slow roll parameters ($\in,\delta$) and the third slow parameter (S) for the model (23) are given by

$$\theta = \frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C} = 6a \coth T \tag{24}$$

$$\text{Hubble parameter } H = \frac{\theta}{3} = 2a\coth T \tag{25}$$

$$\text{Matter density } (\rho) = 3H^2 = 12a^2 \coth^2 T \tag{26}$$

$$\rho^{1/2} = \sqrt{3H} = 2a\sqrt{3}\coth T \tag{27}$$

$$\text{Isotropic Pressure } (p) = \rho/3 = 4a^2 \cot h^2 T$$

$$\text{Spatial Volume } R^3 = ABC = \beta^6 \sinh^6 T \tag{28}$$

$$\text{Shear } (\sigma) = \frac{1}{2}\left(\frac{B_4}{B} - \frac{C_4}{C}\right) = \frac{L}{2\beta^6 \sinh^6 T} \tag{29}$$

$$\text{Dark energy } (\Lambda) = \frac{2}{R^2} = \frac{2}{\beta^4 \sinh^4 T} \tag{30}$$

$$\text{Deceleration parameter } q = -\frac{R_{44}/R}{R_4^2/R^2} = -\frac{1}{2}\{1+\tanh^2 T\} \tag{31}$$

Anisotropic parameter ( $\widehat{A}$ ) is defined as

$$\widehat{A} = \frac{1}{3}\left[\left(\frac{H_1}{H}-1\right)^2 + \left(\frac{H_2}{H}-1\right)^2 + \left(\frac{H_3}{H}-1\right)^2\right] \tag{32}$$

where

$$H_1 = \frac{A_4}{A}, H_2 = \frac{B_4}{B}, H_3 = \frac{C_4}{C}$$

Thus

$$\widehat{A} = \frac{L^2}{24\, a^2 \beta^{12} \sinh^{12} T \coth^2 T} \tag{33}$$

The slow-roll parameter $\in$ and $\delta$ are defined by Unnikrishnan and Sahni[42] as

$$\in = -H_4/H^2 = \frac{1}{2\cosh^2 T}$$

$$\in \sim 1/2$$

and

$$\delta = \in - \frac{\in_4}{2H\in}$$

$$\sim \in$$

$$\delta \sim \frac{1}{2}$$

Thus the slow roll parameter corresponds to

$$\in << 1,\ \delta << 1$$

Now we discuss third slow role parameter (S) as given by canonical scalar field ($\phi$) with the Lagrangian

$$L(\phi, X) = X - V(\phi) \tag{34}$$

where

$$X = \frac{\phi_4^2}{2}$$

For a generic $L(\phi,X)$, the third slow roll parameter (S) is defined as given by Unnikrishnan and Sahni[42]

$$S = \frac{(C_s)_4}{HC_s} \tag{35}$$

where $C_s$ is the speed of sound of the scalar field as is given by

$$C_S^2 = \frac{\partial L/\partial X}{\partial L/\partial X + 2X\partial^2 L/\partial X^2} = 1 \tag{36}$$

Thus S = 0.

Here for a canonical scalar field, the value of S is identically zero.

Equation (6) for V(ϕ) = constant, leads to

$$\phi_{44} + \phi_4\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) = 0$$

Thus, we have

$$\phi_4 = \frac{\ell}{ABC} \tag{37}$$

where $\ell$ is constant of integration, which leads to

$$\phi = \frac{\ell}{15a\beta^6}[4\coth T \cosh^2 T - 3\operatorname{cosech}^4 T \coth T - 8\coth T] + M \tag{38}$$

M being constant of integration.

**Entropy**

Combining first and second laws of thermodynamics for the system of comoving volume V as

$$T\,dS = d(\rho V) + pdV = d[(\rho + p)V] - Vdp \tag{39}$$

which leads to a relation between ρ and T as

$$dp = (\rho + p)\frac{dT}{T} \tag{40}$$

T being absolute temperature.

Now using the radiation dominating condition p = ρ/3, we get

$$\rho^{1/4} = T \tag{41}$$

Equations (39) and (41) lead to

$$dS = d\left[\frac{(\rho + p)V}{T} + K\right]$$

This leads to

$$S = \frac{4\rho V}{3T} \tag{42}$$

The entropy density s is

$$s = \frac{S}{V} = \frac{4}{3}\frac{\rho}{T} \tag{43}$$

Using (41) this leads to

$$s = \frac{4}{3}T^3 \tag{44}$$

Thus we observe that the entropy per unit volume is proportional to absolute temperature T.

## 5. Conclusion

The spatial volume ($R^3$) increase exponentially with time representing inflationary scenario in the universe and this expansion continues for long enough, thus solves horizon problem. We observe that the matter density ρ, the expansion θ, the coefficient of bulk viscosity and the Hubble parameter H all diverge at T = 0 for the model (23). Since $\frac{\sigma}{\theta} = 0$ and Anisotropy parameter ($\widehat{A}$)= 0 when L = 0 or at late time. Thus the model isotropizes in special case or at late time which matches with the result as mentioned by Rothman and Ellis[37]. The dark energy (Λ) decreases with time and $\Lambda \sim \frac{1}{T^4}$ to the first approximation. The deceleration parameter q < 0 represents a accelerating universe. Thus, the model not only represents expanding universe but accelerating universe which matches with the result by Riess et al.[3] and Perlmutter et al.[4] The scalar field ϕ decreases with time. This model exists during the span of time $0 < T < \infty$. Thus inflationary scenario exists with decaying dark energy in Bianchi type V space-time. The entropy per unit volume is proportional to the absolute temperature. The slow roll parameters ($\in,\delta$) and S are in agreement with Planck[43] results. The model has Point Type singularity at T = 0 (MacCallum[44]). The analytical solution obtained for radiation dominated phase has been very helpful to discuss inflationary scenario of the Universe. The presence of bulk viscosity accelerates the isotropization.

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