Essay

What if Familiar Properties Are Emergent?

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Abstract

We live in a local region that appears to have three dimensions of space and one of time, or 4-d spacetime. But this may not be the case for the universe as a whole, because when we include what lies beyond the Hubble distance and what lies within individual subatomic particles, a wider range of possible and actual dimensionalities must be assumed. If we consider the full range of dimensions however, it is seen there are specific maxima and minima, in terms of extent or compactness. We also observe that certain mathematical properties we take for granted may in fact be emergent. If we assume that associativity and commutativity are indeed emergent, this implies that interiority/exteriority and size/distance also emerge, which shapes the early evolution of the cosmos. This paper discusses how these considerations impact Physics as a whole, and give us insights into the Cosmology of the early universe.

Keywords: Emergence, early universe, cosmology, non-commutative geometry, non-associative geometry, constructivism, dimensionality.

Introduction

It is typical to imagine the present state of the universe is natural or ordinary, because that is what we know. In the larger scheme of things; the conditions we know are the natural consequences of a state more likely for the early universe, which created the exact conditions we observe today. Given a full spectrum of possible dimensions; there are both trends and special cases that stand out for special properties. We see specific maxima of (hyper-) volume and (hyper-) surface area for spheres (in 5-d and 8-d respectively). We also know the dimensional arrangement which is most compact (the Leech lattice in 24-d). But for Physics, we see that 3-d spaces have a special property where knots can be tied and not come loose. More specifically; continuous knotted figures can be constructed which cannot be untied. This is significant in both Mathematics and Physics. So if we presently live in a world with unchanging forms having duration in time (e.g. - sub-atomic particles); it should not surprise us that (Classically) it has three spatial dimensions. However; this does not exclude scenarios where the early universe had dimensions that are higher and/or lower than the specific geometric arrangement we now observe, nor does it imply that we can ignore the inherent properties of higher and lower dimensional spaces, in the present time. We already see that the picture from Classical Physics breaks down in regimes of high relative motion and regimes of extremely small size or high energy, such that

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the notion that reality has three dimensions of space plus time is not valid. Here we also examine what happens when the largest scale of space and the earliest moments of time are included.

If we reach beyond the Hubble distance; there may be regions of higher dimension where Physics behaves differently. We see this feature in DGP gravity [1], and in the 5-d black hole → white hole in 4-d spacetime models of Pourhasan, Afshordi, and Mann [2] and of Poplawski [3]. This is carried a step further in Cascading DGP [4,5], where dimensionality 'cascades' from 6-d at the greatest distances (or earliest period) to 5-d, and then to the 4-d spacetime we observe at this time, in our local universe. Peering back to the earliest moments of the cosmos; it is wise to include yet higher dimensions in our range of reflection, and to assume dimensionality may have a dual or even multiple definition, because it is relative not only to well-defined forms and their background conditions, but also to a specific sequence of defining events and/or observations. Rephrasing this; as we approach the Planck scale or the very early universe; geometry becomes first non-commutative and then non-associative, because conditions which define familiar properties are not present or have not yet emerged. This is demanded by the constructivist principle, which asserts quantities like dimensionality remain undefined until forms can be constructed sufficient to make a determination with. Relativity works only where particles are separable, down to $\sim 10^{-12}$ cm, and provides no useful information beyond this point. But if we imagine that the most distant regions or a precursor universe have higher dimensions; we need to consider how non-commutative and non-associative geometry alter that picture as well.

The remainder of this paper will discuss how cosmological events might serve to define or fix geometrical properties, giving rise to our familiar 3-dimensional expanse and a directional arrow of time. But this is seen to go hand in hand with how the natural evolution of geometric spaces tends to cause physical conditions to evolve that are favorable for universes like ours to exist. The geometric properties of higher-dimensional spaces are seen to give rise to the lower dimensional spaces and their properties. The Octonions include or contain the Quaternions, which in turn contain the Complex numbers, of which the Reals are a subset. The Octonions are also the source of all the exceptional Lie groups, E8, E7, E6, F4, and G2 – where G2 is the smallest. In the rolling ball analogy of Élie Cartan; the symmetries of G2 are seen to be well-modeled by a ball rolling without slipping on another ball of 3 times the radius, and the analogy works best in 5 dimensions. So if the notion of a 5-d universe giving rise through a black hole to 4-d spacetime holds true; we could be living in a literal embodiment of Cartan's rolling-ball analogy – where our 4-d spacetime universe is a bubble riding upon a larger 5-dimensional bulk. But if the 'cascading' hypothesis is correct; a still higher-dimensional space might be a precursor to that 5-d cosmological state and the ensuing 5-d → 4-d transition – which then gave birth to the universe we now inhabit. However; a rather large number of proposals are now on the table, for the Physics community, in which higher dimensions play a part. To treat this topic properly requires knowing what happens when geometry becomes non-commutative, making size and distance relative, or even non-associative, where interiority/exteriority is relative. So we will examine how associativity and commutativity might arise cosmologically.

How do we get to Here?

When exploring early universe Cosmology or attempting to craft a working theory of everything (TOE); we must consider how and whether a theory can deliver us to conditions we see in the universe today. While String Theory made impressive strides early on, and remains a top TOE candidate to this day; some critics think it provides too many possibilities, without a clear procedure to select for physically realistic options, or to assure that we can arrive at a universe exactly like ours – starting from that basis. This has been called the 'Landscape Problem' referring to a broad range of possible realities extended like a landscape to the horizon and no easy way to find realities that are not only possible, but actually real or physically-realistic. This does not mean String Theory is wrong, but it is limited in its power to make firm predictions unless a more precise way to select a universe like ours can be found. Two papers have recently appeared by String theorist Cumrun Vafa [6,7] (who coined the term swampland) and other authors, one of which is co-authored by Paul Steinhardt – who has been one of the sternest critics of this issue. Luckily; this work presents a way past the impasse which has plagued String Theory. If we use astrophysical observations as a guide, to delineate the kind of universe we inhabit; it shows us ways to resolve this issue, but only if we upend some of the prior assumptions about what possible theoretical universes are valid. Instead of the range of solutions theorists have been looking to, perhaps we need to see others as physically-realistic. This is likely the case if our present-day cosmos is dominated by entropy or exhibits accelerating expansion, and both appear true.

The author has argued [8] that in the real universe; there is interplay between global and local symmetries with a global asymmetry, similar to what one can observe in the Mandelbrot Set. Since Emmy Noether's discovery of a connection between conserved quantities and preserved symmetries [9]; scientists have been somewhat preoccupied with the Maths and Physics where symmetry applies, and ignoring what lies beyond. But if any of various theories of Emergent, Thermodynamic, or Entropic theories of gravitation [10,11,12] is true; this implies the universe we inhabit is asymmetric with respect to time. It is unclear whether this is due to a dissipative mechanism, a property or type of energy, or purely geometric considerations – but it is clear that this tendency cannot be ignored. The 2^{nd} law of Thermodynamics is inexorable in its influence, and it is one of the few things which all physicists think is valid. But there are several possibilities on the table by which a state of maximum entropy, or greatest extent, could lead to a new beginning. So this leaves open the question of whether the universe will meet a cold dark end, or if the lowest energy phase of the cosmos gives rise to a new cycle. However; we see a trend in the present day cosmos of continuing expansion, which ultimately leads to things being more spread out. We think that the early universe inflated or expanded in size from a small volume, which grew into the universe we see today – plus an extent beyond the Hubble distance. We don't really know what will happen next, in the distant future of the cosmos. Is it so radical to imagine that much of what lies beyond, or came before, resides in a higher dimension?

The more difficult piece is to show that a universe like ours would spontaneously arise out of a higher-dimensional reality. Already back in 1938; Einstein and Bergmann wrote in response to Kaluza and Klein about a theoretical structure where a 5-d origin gave rise to the current 4-d spacetime [13], but then they retreated from this formulation when they saw it led to the prediction of a scalar field, which they thought was undesirable or unphysical [14]. Since that time; there have been numerous attempts to show how a higher-dimensional reality could give rise to the current universe. However; the most salient properties of non-commutative and non-associative geometries are often left out of the picture. Alain Connes has emphatically pointed out "Noncommutative measure spaces evolve with time!" [15] and explained that this principle extends into other areas involving non-commutative geometry. But this feature is even more prominent in the non-associative octonions, as was explained by P.C. Kainen [16], and he suggests that evolutive properties can be a blessing instead of a curse. Under the octonions; the onset of inflation is almost automatic, and it proceeds until certain geometric maxima are reached, because surface area for spheres is maximal in 8-d space [17], but volume is maximal in 5-d [18]. Almost every scenario for early universe cosmology involves the cosmos being dense and compact in its early phase, and then inflating and/or expanding to the volume of a vast expanse which we can observe only a portion of today. So it is important to note that the maxima of compactness and expansiveness occur in a specific dimensionality.

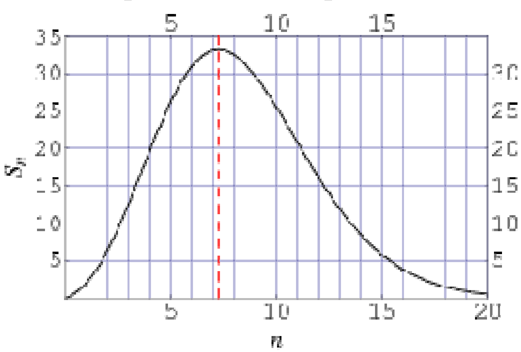

Figure 1- hypersurface area of the n-sphere

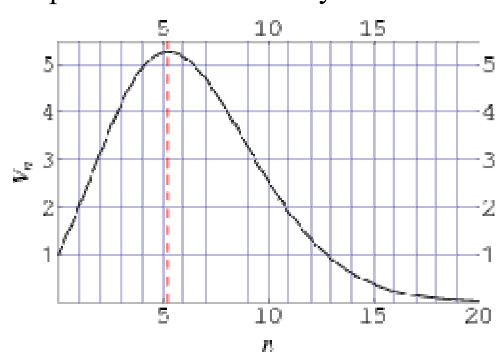

Figure 2 – hypervolume of the n-ball

There are clearly difficulties to visualizing all of this. Computers can help us grasp the dynamics occurring in higher dimensional spaces, but ultimately one must learn to leave behind the perspective of 2 and 3-dimensional reality, to fully understand what is going on in higher dimensions. Alain Connes suggests [19] alternating study of the technicalities with periods of reverie – perhaps reclining on a couch to let the mind reflect on what was read – as a helpful way to learn topics like non-commutative geometry, which demand such perspective. This is better than racking the brain or hitting oneself on the head, trying to wrap ones mind around the concepts. One must be careful of course, to lead the mind back when it wanders too far. It is best to remember that complications introduced by the considerations above are unavoidable in a cosmological setting, even if this is not well-treated in the literature. When I spoke with Tevian Dray at GR21 [20]; I mentioned that as one approaches the Planck scale, or near the rim of a black hole, geometry becomes first non-commutative and then non-associative, then asked why physicists don't deal with these issues. He agreed emphatically that these things are a factor, and answered that most either don't know they are a problem or don't know how to deal with them. So it may be that the Physics people simply haven't quite caught up with the Math folks. But it is wise to understand that they might be important, even if this is not made explicit. Non-commuting variables are common in Physics, especially in Quantum Mechanics, but non-commutative geometry is sometimes present even where one might not expect it to play a part. Aviators must learn that the order and sequence of maneuvers is as important as their direction or extent, to stay aloft, for example.

One conceptual difficulty is seeing that a single object can be large or growing from one perspective, while being small or shrinking from another. We've learned that objects can appear to grow or shrink, if in motion toward or away from us, but also that objects have a certain or definite size. We treat distance similarly, but it is not absolute. We have gotten used to a perspective where the distance from point A to point B is the same as that from point B to point A. But if you are navigating across an estuary body like the Hudson near my home, where one must deal with both the river current and the tides, the simple rule does not apply because the reality is a lot more complicated. In a 5-d black hole → white hole in 4-d spacetime scenario, one should not expect things in the precursor universe to have the same size and distinctness they do in ours. It is perhaps more reasonable to assume that the precursor universe is inside out with respect to our present-day cosmos and that size and distance measures depend upon which side of the dimensional boundary you are measuring things from. Such an event exhibits the Hartle-Hawking condition, making it difficult to determine a precise ratio of before and after measurements, if indeed it is clearly-defined. A precursor universe inside-out with respect to ours allows a violation of Coleman-Mandula as well. This calls into question some of the experiments [21] and studies [22] purporting to limit the size of possible extra dimensions, or to exclude their possibility entirely by citing such violations. It must be remembered that what we see as tiny curled-up dimensions could be vast, and that from that space our spacetime might appear tiny, regardless of how paradoxical it might seem.

We note that familiar forms arise or emerge from pure energy in the early universe. To simplify things I will state that; energy is bound into droplets of form with specific properties we call sub-atomic particles, which are the basis for more complex forms – that can only come to be once the universe has cooled sufficiently. Because creation and annihilation of particles is rapid and continuous in the early cosmos, it is like an energetic or quantum-mechanical soup. It's most salient property is that matter and energy are coupled, with particle mass being converted into pure energy and then energetic emanations turning into particles with mass. But the cosmos is seen to grow in size, during this epoch, likely very rapidly at the start. It is typically seen to begin in a very small volume of space, but this goes hand in hand with the fact that photons sufficient in energy to exist at the outset approach the Planck length in size – and that is exceedingly small. We see that as a photon's energy gets bigger, its size becomes smaller, but there it encounters a natural limit. So this sets the minimum size the early cosmos could possibly be.

And yet; we know that no sub-atomic particles can exist below this size limit, as well. Only with the appearance of fermionic particles at baryogenesis can Pauli exclusion take hold, and only after this point is interiority/exteriority seen to be well-defined by constructivists. In this sense; we also mark the emergence of associativity, within a framework of what must be considered non-associative geometric and algebraic spaces. But if we employ constructivism again to the background (space itself); we must wait until at least the end of inflation before its dimensionality is firmly set, or assume *D* is continuously variable until a regime of measurement repeatability is established. So the actual number of dimensions possessed by space cannot be known for all eras of the cosmos, simply because it is not well-defined until defining events take place. This is true regardless which cosmological model we adopt. While it may be seen as an unheralded pathology of the Big Bang model, it is a defining feature in other theories. This is why Quantum Gravity theorists often strive to formulate background-independent theories. It is understood dimensionality is a geometric attribute we cannot assume to have a particular value, *a priori*, without a way to take measurements sufficient to triangulate within that space and thus to make a determination of its value. In braneworld theories; a higher dimensional background (often 10-d) is assumed for the bulk, because this is convenient to applying the tools of String Theory and M-Theory. But here too; we are starting from spaces with non-associative geometry, and working toward conditions where associativity and commutativity are well-defined.

Conclusions

In the time of Ptolemy; we had an understanding that the Earth was at the center of the Cosmos, and that the heavens revolved around us. But this slowly changed due to pioneering thinkers and careful observers like Copernicus and Galileo; who put the Sun at the center of the planets, and held that the Earth revolved around it. This greatly simplified our understanding of Astronomy, and helped to usher in a new era of observational Science. But at the time; it was seen as a bold and heretical conjecture. In the current era; people still have the understanding that we live in three dimensions of space plus time, which became the prevailing view in the time of Newton, despite advances and discoveries made more than a century ago – which call that understanding into question. The advent of Relativity and Quantum Mechanics almost put age-old beliefs to rout, but it was much the same back then. To accept a heliocentric view; people of that era had to turn their understanding of the universe inside-out, placing something in the heavens in the privileged place we had once occupied. The Earth was no longer the center of the entire Cosmos. And gradually; it came to be understood that neither is our Sun the most prominent in the galaxy, nor is our home the Milky Way the largest in the local universe, so our own prominence is not as great as we had imagined our place in the Cosmos to be. Instead; our Earth is a far-flung product of the universe and its cosmology – which created the conditions for us to be.

Perhaps its time to turn our understanding of the universe inside-out once again, by reexamining assumptions about dimensionality and the origin of space and time. We are tempted to invoke the principle of Occam's razor here, because it specifically warns against multiplying entities unnecessarily, but perhaps we must avoid making assumptions about the nature of space. When walking across a grassy plane; it is easy to imagine the world as flat. But from a great enough altitude, or peering out over the ocean; one can easily see it is curved. Only from a great height, or distant from the Earth, can one see that it is round. And yet; the Earth was that way all along, and it did not become a round ball only when we could travel to the heavens. Similarly; we might find ourselves in a region of 3-d space mainly because it supports the phenomenology for the physical processes that brought our world and ourselves into existence. Ehrenfest [23] wrote eloquently about this, back in 1918. Knot theory provides compelling reasons why a 3-d space is so special, and explains why it provides the context for Physics as we know it. But this does not imply that the universe as a whole, or reality on every level of scale, is created identically in this mold. We know instead that we must adjust our assumptions in the microcosmos of the sub-atomic realm, and most likely in regimes of considerably less extreme small size. There may be a need to make similar adjustments to our understanding of what happens at the opposite end of the spectrum – the realm of extreme size and distance – where our observable universe is only a portion of a much larger Cosmos. Here too we must relax our assumption that we reside in a 3-d space or in 4-d spacetime, to include higher dimensions as possible.

It is a necessary condition that variables locally commute, for the laws of Classical Physics to apply, because this is what allows verifiability or repeatability of measurement. But in Modern Physics; we encounter complications both in the microcausal realm – which we treat with Quantum Mechanics – and in the cosmological realm – for which we apply Relativity. However; to imagine similar complications are not a part of everyday life is to ignore truths known to the Ancients, because it remains a fact that one cannot step into the same stream twice, because the water which was there before has now already moved downstream. To cross the Hudson near my home; one must deal with both the current of the river flowing downstream and the tides, which make the river flow both out and in. One must steer a different course to cross from shore to the opposite shore, depending on the strength and direction of the tide, and on whether one has chosen to embark before or after the tide changes direction. In such instances; things depend upon both the order of natural events and the timing of our choices, so there are complications similar to those arising in non-commutative geometry, and somewhat like those appearing in relativity and quantum mechanics. Thus we should be aware that we have already been dealing with the world as higher-dimensional, even while not acknowledging or recognizing it as such. What I propose here is the more radical possibility that we are living in a universe that is, on the whole, higher-dimensional – if we include its entirety – and that the observable cosmos is only a ball or bubble within or riding upon that realm.

In this context; independent objects with distinct surfaces, and being able to move objects a certain distance and return them by going the same distance in the opposite direction, depend on the prior emergence of properties to support those possibilities. If not for associativity, keeping objects from overlapping would be compromised; so living in a regime where the associative rule applies turns out to be essential to having distinct and separate objects which can be moved relative to each other. Likewise; having the commutative rule in place is essential for measurability and the exact reversibility of measurements of size and distance. If there was a higher-dimensional start to the cosmos or if the universe/multiverse is in the bulk higher-dimensional; the familiar regime of the local universe must have evolved cosmologically from radically different conditions in the early universe. While we can take certain properties for granted; there is no reason to believe that nature started that way, and some compelling evidence that nature might have done things differently. We may live in a bubble of 4-d spacetime embedded in, or perhaps riding upon, a larger bulk of higher dimension. In the standard Big Bang scenario, an expanding fireball is contained by its own opacity, until decoupling or recombination lets the light shine through, which is the CMB. But if inflation occurred in a higher-dimensional space, we could be seeing the surface of a white hole out there instead. If we assume this is true; it explains some of the troubling things that would otherwise plague Physics with a conundrum, and may help to reconcile the divergent tales of Relativity and Quantum Mechanics.

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