Chapter 15. The non-linear theory as string theory of Compton wavelength scale

1.0. Introduction. String theory review

Using the some of the many sources of modern string theory (in particular (Schwarz, 1987; Peat, 1988; Greene, 1999; etc) we briefly review the basics of string theory.

String theory is a model of fundamental physics whose building blocks are one-dimensional extended objects called strings, rather than the zero-dimensional point particles that form the basis for the Standard Model of particle physics.

1.1. Basic idea

The basic idea behind all string theories is that the constituents of reality are strings of extremely small size (possibly of the order of the Planck length, about 10^{-35} m) which vibrate at specific resonant frequencies. Thus, any particle should be thought of as a tiny vibrating object, rather than as a point. This object can vibrate in different modes (just as a guitar string can produce different notes), with every mode appearing as a different particle (electron, photon, etc.). Strings can split and combine, which would appear as particles emitting and absorbing other particles, presumably giving rise to the known interactions between particles.

String theory as a whole has not yet made falsifiable predictions that would allow it to be experimentally tested, though various planned observations and experiments, particularly, on the LHC, could confirm some essential aspects of the theory, such as supersymmetry and extra dimensions. In addition, the full theory is not yet understood. For example, the theory does not yet have a satisfactory definition outside of perturbation theory; the quantum mechanics of branes (higher dimensional objects than strings) is not understood; the behaviour of string theory in cosmological settings (time-dependent backgrounds) is still being worked out; finally, the principle by which string theory selects its vacuum state is a hotly contested topic (see string theory landscape).

1.2. Basic properties

String theory is formulated in terms of an action principle, either the Nambu-Goto action or the Polyakov action, which describes how strings move through space and time. Like springs with no external force applied, the strings tend to shrink, thus minimizing their potential energy, but conservation of energy prevents them from disappearing, and instead they oscillate. By applying the ideas of quantum mechanics to strings it is possible to deduce the different vibrational modes of strings, and that each vibrational state appears to be a different particle. The mass of each particle, and the fashion with which it can interact, are determined by the way the string vibrates - the string can vibrate in many different modes, just like a guitar string can produce different notes. The different modes, each corresponding to a different kind of particle, make up the "spectrum" of the theory.

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String theory includes both *open* strings, which have two distinct endpoints, and *closed* strings, where the endpoints are joined to make a complete loop (Fig. 16.1).

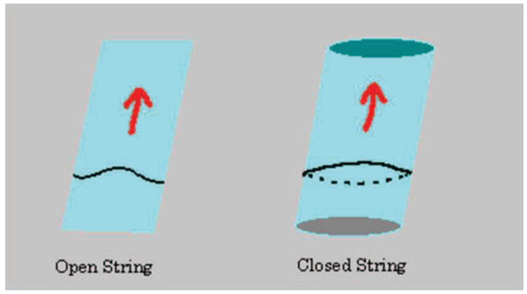

Fig. 16.1

The two types of string behave in slightly different ways, yielding two different spectra. For example, in most string theories, one of the closed string modes is the graviton, and one of the open string modes is the photon. Because the two ends of an open string can always meet and connect, forming a closed string, there are no string theories without closed strings.

The earliest string model - the bosonic string, which incorporated only bosons, describes - in low enough energies - a quantum gravity theory, which also includes (if open strings are incorporated as well) gauge fields such as the photon (or, more generally, any Yang-Mills theory). However, this model has problems. Most importantly, the theory has a fundamental instability, believed to result in the decay (at least partially) of space-time itself. Additionally, as the name implies, the spectrum of particles contains only bosons, particles which, like the photon, obey particular rules of behavior. Roughly speaking, bosons are the constituents of radiation, but not of matter, which is made of fermions.

Investigating how a string theory may include fermions in its spectrum led to the invention of supersymmetry, a mathematical relation between bosons and fermions. String theories which include fermionic vibrations are now known as superstring theories.

In some string theories (namely, closed strings and string in some version of the bosonic string), strings can split and reconnect in an opposite orientation (as in a Möbius strip or a Klein bottle).

1.3. String Tension

While understanding the details of string and superstring theories requires considerable mathematical sophistication, some qualitative properties of quantum strings can be understood in a fairly intuitive fashion. For example, quantum strings have tension, much like regular strings made of twine; this tension is considered a fundamental parameter of the theory. The tension of a quantum string is closely related to its size. Consider a closed loop of string, left to move through space without external forces. Its tension will tend to contract it into a smaller and smaller loop. Classical intuition suggests that it might shrink to a single point, but this would violate Heisenberg's uncertainty principle. The characteristic size of the string loop will be a balance between the tension force, acting to make it small, and the uncertainty effect, which keeps it "stretched". Consequently, the minimum size of a string is related to the string tension.

The tension of ordinary strings - like guitar strings - is determined by plucking them. If we could pluck a superstring, we could determine the tension in the string. This intrinsic "stiffness" of strings has several significant consequences.

First, it is the huge tension of a string that causes its contraction to an infinitesimal size - the Planck length of 10^{-33} cm.

Second, the high tension results in the typical energy of a string being extremely high.

Strings' minimum energies are actually whole-number multiples of the Planck energy, which, translated into mass, yields the Planck mass (ten billion billion times that of a proton; roughly 1/100 of 1/000 of a gram; about the mass of a grain of sand).

Of course, the Planck mass is enormous by elementary-particle standards. How can strings, with average energies corresponding to the Planck mass, create the observed elementary particles with much smaller masses and lower energies?

1.4. String theory and quantum theory

A quantum mechanical theory of closed strings incorporates a lot of masseless particle like excitations. And again, there is one among them that is very special, and familiar. It is a massless spin 2 particle. Remember, that we didn't order this. But we get it anyway, and miraculously, it is exactly what we need to formulate the relativistic theory of gravitation. We struck lucky again. It is the crucial second characteristic of string theory.

By merely studying the theory of open and closed strings, or string theory, in its quantum mechanical formulation, we got everything we needed to formulate a unified theory that incorporates the Standard Model and relativistic theory of gravitation. That doesn't mean we already have the unified theory, but it says we're on the right track.

In string theory all the known fundamental particles - including the messenger particles - have properties resulting from types of vibrational patterns. This fact is one of the most attractive and unifying aspects of string theory - it postulates that all particles are made of the same "fabric," as opposed to the particle physics view that each elementary particle is in effect "cut from a different fabric" (Greene, 1999).

2.0. Mathematical basis of the string theory

The theory of strings is represented as the generalization of the theory of elementary particles. As is known, elementary particles are simultaneously waves, and all equations of the quantum field theory (i.e. of Standard Model and also of NTEP) are wave equations. This cannot be abolished by any new theory, since SM is very well checked experimentally.

This fact alone indicates that the theory of strings, if it attempts to describe elementary particles, must be reduced to the wave equations, or, in other words, it must have Lagrangian and action function, which correspond to wave equations.

In the simplest case the equation of motion of real wave field ψ will be written as:

$$\frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} + \Delta\psi = 0\,, \qquad (15.2.1)$$

The function of action, which corresponds to it, is:

$$S = -\frac{1}{2}c^2\int_t dt\int_V \partial_\nu\psi\,\partial^\nu\psi\,dx\,, \qquad (15.2.2)$$

and the Lorentz-invariant Lagrangian is

$$\overline{L} = -\frac{1}{2}c^2\sum_\nu \frac{\partial\psi}{\partial x_\nu}\frac{\partial\psi}{\partial x_\nu} \equiv \frac{1}{2}c^2\partial_\nu\psi\,\partial^\nu\psi\,, \qquad (15.2.3)$$

But that is for a nonrelativistic string, one with a wave velocity much smaller than the speed of light. How do we write the equation for a relativistic string?

In the nonrelativistic string, there was a clear difference between the space coordinate along the string, and the time coordinate. But in a relativistic string theory, we wind up having to consider the world sheet of the string as a two-dimensional spacetime of its own, where the division between space and time depends upon the observer.

The classical equation can be written here as

$$\frac{1}{c^2}\frac{\partial^2 X^\mu(\sigma,\tau)}{\partial\tau^2}-\frac{\partial^2 X^\mu(\sigma,\tau)}{\partial\sigma^2}=0, \qquad (15.2.4)$$

where σ and τ are coordinates on the string world sheet representing space and time along the string, and the parameter c^2 is the ratio of the string tension to the string mass per unit length.

These equations of motion can be derived from Euler-Lagrange equations from an action based on the string world sheet

$$S=-T\int d\sigma\int d\tau\sqrt{-h}h^{ab}\partial_a X^\mu\partial_b X^\nu, \qquad (15.2.5)$$

where $T_{string}=\frac{1}{2\pi a'}$ is string tension; the spacetime coordinates X^μ of the string in this picture are also fields X^μ in a two-dimension field theory defined on the surface that a string sweeps out as it travels in space. The partial derivatives are with respect to the coordinates σ and τ on the world sheet and h^{ab} is the two-dimensional metric defined on the string world sheet.

The general solution to the relativistic string equations of motion looks very similar to the classical nonrelativistic case above. The transverse space coordinates can be expanded in normal harmonic modes. This string solution is unlike a guitar string in that it isn't tied down at either end and so travels freely through spacetime as it oscillates. The string (15.2.4) is an open string, with ends that are floppy.

Relativistic Lagrangian of the motion of point particle is used as initial Lagrangian of the theory of strings. On the basis of the last the action, function of the Nambu-Goto of non-pointlike particle is introduced into the theory:

$$S(x)=-\frac{1}{2}\int d\sigma\int d\tau\sqrt{\dot{X}^2\cdot X'^2-\left(\dot{X}\cdot X'\right)^2}, \qquad (15.2.6)$$

The Nambu-Goto action square root can be recorded in the linear form (this makes possible more simply to pass to quantum representation). The equivalent action of Polyakov, introduced on this basis, is the initial Lagrangian of the string theory:

$$S(X,\gamma)=-T\int d\sigma\int d\tau\sqrt{-\gamma}\gamma^{ab}\partial_a X^\mu\partial_b X^\nu\eta_{\mu\nu}, \qquad (15.2.7)$$

For a closed string, the boundary conditions are periodic, and the resulting oscillating solution looks like two open string oscillations moving in the opposite direction around the string. These two types of closed string modes are called right-movers and left-movers, and this difference will be important later in the supersymmetric heterotic string theory.

This is classical string. When we add quantum mechanics by making the string momentum and position obey quantum commutation relations, the oscillator mode amplitudes have the known commutation relations.

The quantized string oscillator modes wind up giving representations of the Poincaré group, through which quantum states of mass and spin are classified in a relativistic quantum field theory.

So this is where the elementary particle arise in string theory. Particles in a string theory are like the harmonic notes played on a string with a fixed tension T. The parameter a' is called the string parameter and the square root of this number represents the approximate distance scale at which string effects should become observable.

By looking at the quantum mechanics of the relativistic string normal modes, one can deduce that the quantum modes of the string look just like the particles we see in spacetime, with mass.

Remember that boundary conditions are important for string behavior. Strings can be open, with ends that travel at the speed of light, or closed, with their ends joined in a ring.

One of the particle states of a closed string has zero mass and two units of spin, the same mass and spin as a graviton, the particle that is supposed to be the carrier of the gravitational force.

3.0. The NTEP as theory of strings

Let's compare the NTEP with modern theory of strings, as it is described by one of the founders of this theory (Schwarz, 1987):

"*Strings can have two various topology, which refer as opened and closed. The open strings are pieces of lines with free ends, while the closed strings represent loops with topology of a circle and have no free ends…*

… Various quantum-mechanical excitation (normal modes) of string for each solution of the given theory of strings are interpreted as a spectrum of elementary particles. The excitations can include the rotary and oscillatory degrees of freedom of a string and also the excitations of various "internal" degrees of freedom, which it possesses. The internal degrees of freedom are caused by symmetry of the Lie groups, supersymmetry, etc.

The theory of strings gives the uniform approach to the rich world of the elementary particles, considered as a various modes of excitations of a unique fundamental string ".

Let us note the essential features of the string theory (and its modern variant - the superstring theory) compared with NTEP.

First of all we pay attention to the fact that the string have the Planck scale lengths (about 10^{-35} m), which do not has place in the real world.

Note also that here the formalism of multi-dimensional Riemann space is used and therefore the theory of strings contains very complex mathematical apparatus. To explain what has here the real sense and what is mathematical fiction is very complicated.

Further: in the theory of strings the meaning of strings itself are mysterious. They are some energy formations, which can be named "energetic line", which do not attached to any concrete physical objects. But in nature there is no energy without matter. In the quantum theory of elementary particles, the objects, which carry an energy, are de Broglie's waves, described by ψ - function.

Difficulties of initial string theory have led to the need to incorporate in the theory of the so-called supersymmetric particles, which have never been observed in experiments. That is why the modern string theory is called superstring theory. In NTEP there is no need for such a complication of the theory.

Thus, applying the theory of strings to the description of elementary particles, we, first of all, must connect the strings with the known theoretical objects. Let us look how this it is possible to make sequentially, not disrupting principles both the one and the other theory.

Obviously, in order to switch over to real elementary particles within the framework of NTEP (where the particles are not pointlike) we must examine the functions X^{μ} as the wave functions (then at least an energy of strings finds the carrier, and also the vibrations of the strings, which generate different string modifications, become understandable). As it is not difficult to see, in this case the above action is similar to the action of the wave equation.

As we see, there is the big number of contact points between two theories. It is possible to assume that NTEP is the theory of strings, which describes the elementary particles of real (not of Planck) scale. Below we will try to substantiate this claim, without detailed mathematical proofs, since they are fully described in previous chapters .

In the NTEP it is shown that at twirling of these strings within the strong electromagnetic field the closed strings, corresponding to the massive non-linear waves – solitons, are formed. It is also shown here that above twirling is similar to the transformations of the gauge type. The peculiar solitons, which are the constituents of this theory, are identical with the objects of Standard Model theory. In particular they have masses, can be only in two states – bosonic and fermionic, can have positive and negative charges, etc.

It is shown that the equations of this theory fully coincide with quantum field theory equations. The theory initiates the question, whether between the modern string theory and NTEP some connection exists?

A solitary stable wave is defined here as a spatially confined, non-dispersive and non-singular solution of a non-linear wave theory. For any non-linear wave theory the solitons are the same fundamental solutions, as the usual waves are the fundamental solutions of the linear equations. As it is known the newer fundamental non-Abelian gauge theories are non-linear and have the soliton solutions. In the framework of the quantum field theory it is not difficult to find the relations between solitons and elementary particles that go very deep and are entirely unexpected from a classical viewpoint.

According to modern theory the observed substance of the Universe consists of photons, leptons and quarks, among them besides electromagnetic interactions, act also strong and weak interactions. All these interactions are described by the unified theory – the Standard Model theory, which deeply generalizes Maxwell's theory. Instead of the vectors of the usual electrical and magnetic fields $\vec{E},\vec{H}$, several similar vector fields $\vec{E}_i$ and $\vec{H}_i$ act in it, waves of which by their nature are strongly non-linear. First C.N. Yang and R. Mills made such generalization of Maxwell's theory in 1954 (so-called Yang-Mills theories). Let us emphasize that the nonlinearity is so deeply placed in the nature of Yang-Mills fields as well as in the nature of the solitary waves. This nonlinearity unavoidably leads to the fact (Ryder, 1985) that the solitons must play the significant role in the structure of the Universe.

3.1. Bosonic open string in the NTEP

Let us show that in the nonlinear theory the quanta of electromagnetic waves can be represented as massless boson strings of Compton wavelength scale.

In accordance with the Planck- Einstein theory each photon has zero mass, energy ε , momentum $\vec{p}$, frequency ω and wavelength λ , whose values are mutually unambiguously connected among themselves: $\varepsilon = \hbar\omega$, $\vec{p} = \hbar\vec{k}$, $\varepsilon = cp$, ($\vec{k} = \vec{k}^0/\lambdabar$ is wave vector, $\lambdabar = \lambda/2\pi$ is reduced wavelength).

As it is known, in framework of QED (Akhiezer and Berestetskii, 1965) for construction of the theory of the photons and their interaction with other particles the Maxwell equations along with the relationship $\varepsilon = \hbar\omega$ are sufficient. To obtain the photon wave function the second order wave equations for EM field vectors $\vec{\mathrm{E}}$ and $\vec{\mathrm{H}}$ are used.

Factorizing the wave equation to the equations for retarded and advanced waves, we receive two equations of first degree regarding to some function f_k , which correspond to a wave vector k and is some generalization of the EM field vectors. The equation for this function is equivalent to the

Maxwell-Lorentz equations. The function f_k is interpreted as local wave function of a photon in the momentum representation, which doesn't give possibility of the space description of photon interactions.

The attempt to enter the photon function in the coordinate representation has strike on an insuperable difficulty. According to analysis of Landau, L.D. and Peierls, R. (Landau and Peierls, 1930) and later of Cook, R.J. (Cook, 1982a,b) and Inagaki, T. (Inagaki, 1994) the photon wave function is nonlocal object. The result means that the localization of a photon in a area smaller of the photon wavelength is impossible.

Note that since the photon characteristics are mutually unambiguously connected among themselves, we can insist that photon has only one own independent characteristic (e.g., the wavelength). Then, keeping in mind the wavelength of photon, it is possible to say that photon is conditionally one-dimensional formation.

The one-dimensional object, which, on the one hand, obeys the wave equation, and on the other hand is not localized, in physics is referred to as a *string* (not forgetting of course that this supposition can have no relationship to the real structure of a photon) (see Fig. 16.2).

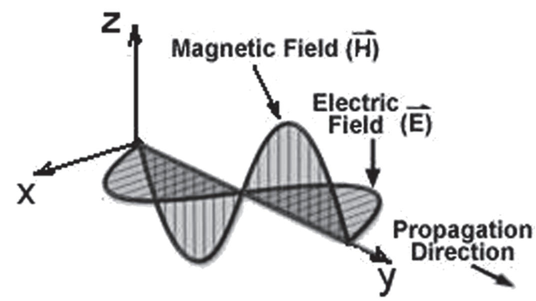

Fig. 16.2

These allows us to ***introduce that the fundamental particle of an EM field - photon - is the open relativistic EM string with one wavelength size, which corresponds to the Compton electron wavelength scale.***

The main proof of validity of this assertion is the opportunity to construct on its basis the theory, which mathematically coincides completely with the results of quantum field theory (QFT).

Since a photon is a boson, we can expect that the photon string theory will be cognate to the initial modern string theory, in which the boson strings are the source material, from which by twirling, the close strings, i.e. the elementary particles, are formed. Obviously, in the case of the photon string the introduction of such postulate is not needed. Actually, the bending of a trajectory of an EM wave in the strong EM field follows already from the Maxwell-Lorentz theory.

3.2. Production of the closed strings of electron and positron

Thus, we can suppose that ***under certain external conditions the EM-string can start to move along the closed nonlinear trajectory, forming the closed strings (or in other words, solitons), which can be considered as EM elementary particles.***

It is obvious, that due to the quantum nature of a photon string the formed closed strings should possess, at least, a rest mass and the angular momentum (spin). Moreover, the detail analysis (see previous chapters) shows that such EM elementary particles can have electric charge, helicity and all other characteristics and parameters of real elementary particles.

Then we can consider the reaction of electron-positron pair production from EM-string as production of two closed string (see Fig.16.3)

a) b)

Fig. 16.3 Electron-positron pair production:
a) in QFT as Feynman diagram; b) in string theory

In the NTEP this process is illustrated by Fig. 16.4:

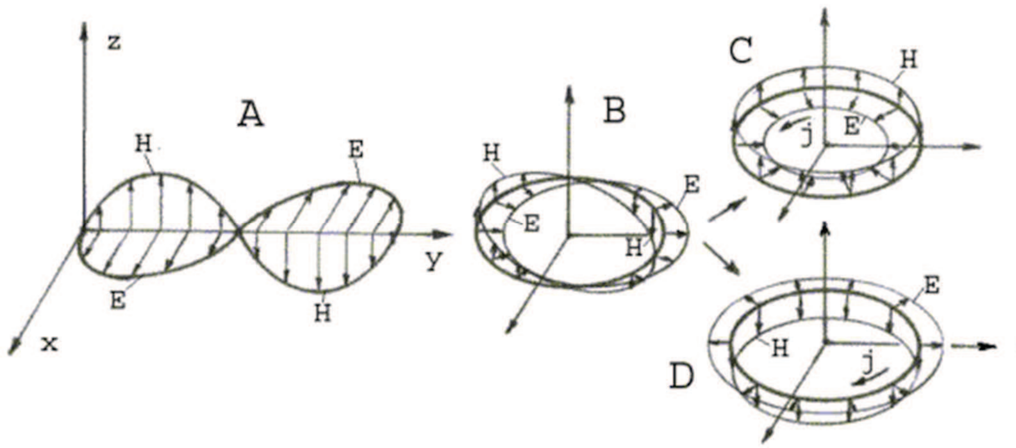

Fig. 16.4

Thus, conditionally speaking, from one vector particle (i.e., open string) we receive two antisymmetrical semi-vector particles, (two spinors), which according to Fig. 16.4 and solution of Dirac electron equation are close strings.

3.3. The neutrino closed string

In the previous chapters it is also shown that at plane twirling and division of the **circularly polarized** initial photon are produced the neutral massive leptons – the same type as neutrino and antineutrino, which are also described by Dirac equations. Figure 16.5 shows the distribution of the electric field connected with the circularly polarized wave of the positive (right) and negative (left) helicity:

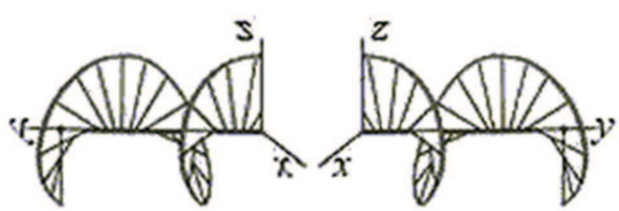

Fig. 16.5

The twirled half-periods of such photons give the EM particles with inner helicity (Fig. 16.6)

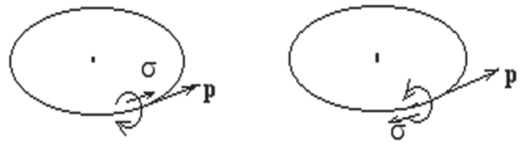

Fig.16.6)

In this case neutrino as twirled helicoids represents Moebius's strip: its field vector at end of one coil has the opposite direction in relation to the initial vector, and only at two coils, comes back to the starting position (Fig. 16.7). This property of the EM-lepton vector corresponds to the same property of wave function of Dirac lepton theory.

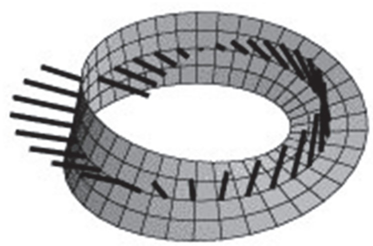

Fig.16.7

3.4. The hadrons' strings

Further in this research it is described the occurrence of spatial particles, as the superposition of the twirled semi-vector particles (spinors). The equations of such particles' superpositions coincide with Young-Mills equations for hadrons (mesons and baryons). In this case the spatial superposition of two twirled semi-photons generates the mesons, and spatial superposition of three twirled semi-vectors leads to occurrence of baryons, e.g. proton (Fig.16.8):

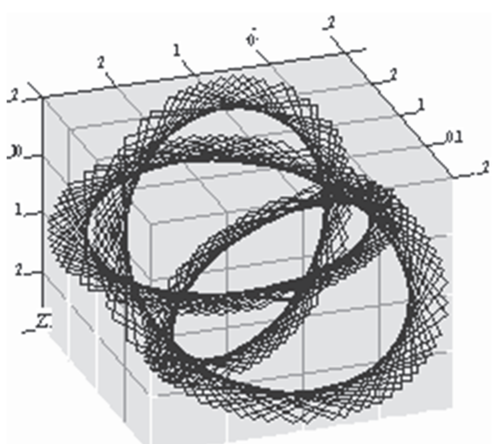

Fig.16.8

4.0. The vibrational modes of strings as the mass spectra of elementary particles

To the question of origin of the mass spectra of elementary particles, we have devoted the chapters 13 and 14 . We have shown that the appearance of mass spectra requires that the particles had a size. In this case, as we have shown, the particles can serve as a potential well for the resonance absorption of an additional portion of electromagnetic energy in the form of the corresponding de Broglie wave. Both the bases of the mass spectrum appearance are adequate to the hypothesis about the origin of the mass spectra of elementary particles in string theory. A major difference lies only in the fact that the energy (mass) of excitations in this case have energies which correspond to the scale of the Compton wavelength of an electron and not of the length scale of Planck.

Why is not modern string theory confirmed?

It was assumed that the experimental discovery of supersymmetric particles on the Large Hadron Collider (LHC) will be a substantial confirmation of the theory of superstrings.

The resent observations and experiments on the LHC do not confirm some essential aspects of the string theory. The last LHC results cast doubt on supersymmetry theory. Data was presented at the Lepton-Photon science meeting “Lepton Photon 2011” in Mumbai. (ANI) 22–27 August 2011 (http://www.tifr.res.in/~lp11/, http://truthdive.com/2011/08/28/LHC-results-cast-doubt-on-supersymmetry-theory.html):

Results from the Large Hadron Collider (LHC) have disappointed theorists on the lookout for Higgs boson and has them rethinking that the basic idea of supersymmetry might be wrong. The theory of supersymmetry in its simplest form is that as well as the subatomic particles we know about, there are “super-particles” that are similar, but have slightly different characteristics. The theory, which was developed 20 years ago, can help to explain why there is more material in the Universe than we can detect – so-called “dark matter”.

Researchers failed to find evidence of so-called “supersymmetric” particles, which many physicists had hoped would plug holes in the current theory. According to Professor Jordan Nash of Imperial College London, who is working on one of the LHC’s experiments, researchers could have seen some evidence of supersymmetry by now. “The fact that we haven’t seen any evidence of it tells us that either our understanding of it is incomplete, or it’s a little different to what we thought – or maybe it doesn’t exist at all,” he said.

According to Dr. Tara Shears of Liverpool University, a spokesman for the LHCb experiment: “It does rather put supersymmetry on the spot”.

Dr Joseph Lykken of Fermilab, who is among the conference organisers, says that he and others working in the field are “disappointed” by the results – or rather, the lack of them: “The worry is that the basic idea of supersymmetry might be wrong. There’s a certain amount of worry that’s creeping into our discussions… It’s a beautiful idea. It explains dark matter, it explains the Higgs boson, it explains some aspects of cosmology; but that doesn’t mean it’s right… It could be that this whole framework has some fundamental flaws and we have to start over again and figure out a new direction”.

As we know, any theory can be written down and formulated by means of many different methods. For example, in the classical mechanics it is equally possible to use the approach of Newton, Lagrange, Jacobi, Hamilton, etc. In quantum mechanics the matrix mechanics of Heisenberg, wave mechanics of Schrödinger, integrals along the paths of Feynman, etc are used for the description of elementary particle motion.

The equations of electrodynamics can also be written down in the forms of eight scalar equations, four vector equations, two tensor equations, quaternion and octanion equations and others. But all these approaches (as Feynman remarked in his lectures "Electrodynamics", volume 28 (Feynman, Leighton and Sands, 1964)) give the same results: in order to calculate something in the electrodynamics, it is necessary to turn to eight scalar equations. In other words, all methods of description, enumerated above, are almost useless.

Furthermore: it is possible to write them down in 11 different orthogonal systems, and also in the space of many dimensions, including a Riemannian space of an infinite number of measurements.

Thus, many things in abstract theories have only mathematical sense. From the above analysis it is possible to conclude that the modern theory of strings is one of the methods of super-abstract mathematical description of modern elementary particle theory. Maybe, namely this description of elementary particle is the only destination of the string theory. And the rest of its content is only a useless formal abstract construction.