Chapter 13. On elementary particles' spectra

1.0. Introduction

> We identify the area of interference, the diffraction halo, with the atom; we assert that the atom in reality is merely the diffraction phenomenon of an electron wave captured us it were by the nucleus of the atom. It is no longer a matter of chance that the size of the atom and the wavelength are of the same order of magnitude: it is a matter of course.
>
> **Erwin Schrödinger. The fundamental idea of wave mechanics.** ***Nobel Lecture, December 12, 1933***

According to modern representations all elementary particles are the bound states of a small set of more light particles. Among all these objects are now recognized as fully stable only electron, neutrino, proton and neutron in a bound state in the stable nuclei. All other particles are the spectra of particles, which decay into one another.

1.1. The spectra of the elementary particles

Generally, each elementary particle is defined by a set of various characteristics: mass, spin, electric charge, strong and weak "charges" (i.e. the characteristics, which define intensity of strong and weak interaction), numbers of "affinity" (numbers, due to which one family of particles differs from another: lepton number, baryon number and others), etc.

We say that the particles, which are characterized by identical characteristics, except for any one of them, compose a spectrum regarding this chosen characteristic. For example, if as such characteristic the mass of particles is accepted, we speak about a mass spectrum of elementary particles.

The first (Gottfried and Weisskopf, 1984) were the atomic and molecular photon radiation spectra; the second the nuclear gamma-quantum spectra, at which nature offers us a series of well-defined quantum states. Further were disclosed the lepton and hadron spectra.

The heavy charged leptons (muon and tauon) are heavier (i.e., more massive) replicas of the electron, and each has its own neutrino. Thus the electron is the ground state of a spectrum that we can call the spectrum of charged lepton; and the electron neutrino is the ground state of a spectrum of non-charged leptons – neutrinos. The proton is merely the ground state of a complex spectrum that we called the baryon spectrum. In an analogous manner, the $\pi's$ are the lowest-lying members of the meson spectrum.

According to the contemporary theory there are some limiting conditions of the composition of elementary particles, which can be named the conservation laws of this characteristic: e.g. the laws of conservation of energy, momentum, angular momentum, electric charge and charges of other interactions, laws of conservation of numbers of "affinity", etc. Some laws (principles) also exist - the uncertainty principle of Heisenberg, which restrict the transition from one family or spectrum to another.

If to speak, for example, about mass spectra of particles, there are following limitations for shaping of such spectra:

1) according to the energy-momentum conservation law the rest free light particles cannot decay to heavier particles, but heavy particles can decay to more light particles;

2) nevertheless, according to a uncertainty principle of Heisenberg, heavy particles cannot comprise the light particles as a ready particles (for example, the neutron cannot comprise electron as a free particle).

As is known, the existing theory cannot explain the occurrence of elementary particle characteristics and of their conservation laws: they are entered as consequences of experiments.

The conclusions of the quantum theory are undoubtedly correct and was confirmed by experiments. Thus we should show that they do not contradict to the results of nonlinear theory of elementary particles (NTEP).

Within the framework of NTEP the fundamental particles are the simple harmonic nonlinear waves. The purpose of this chapter is to show that the spectra of elementary particles appear due to the superposition of these nonlinear waves. Most of all we will be interested in mass spectra.

2.0. A hypothesis of formation of spectra of elementary particles in NTEP

The electromagnetic nonlinear waves have the same characteristics as elementary particles. The nonlinear harmonic waves have the masses, can have integer or half spin, can be charged or neutral, etc.

The mass of particles within the frameworks of NTEP is the "stopped" energy of the nonlinear standing wave, which are defined by frequency of this wave. Therefore to a heavy particle the nonlinear wave of relatively high frequency must correspond, and to light particle - the nonlinear wave of lower frequency. What another can change the mass of particle?

The nonlinear harmonic waves of NTEP correspond to the simple harmonic waves of classical electrodynamics (briefly CED). What possibility to change the characteristics of EM wave does exist in the classical electrodynamics?

As we know, the most simple possibility consists in the waves' superposition. In this case the harmonic waves of various frequencies can coexist in some composite formations.

Analogically to the results of classical theory of EM waves, whose quantum non-linear generalization our theory is, we assume that in the framework of NTEP:

1) *the cause of formation of spectrum of composite particles is the superposition of simple (harmonic) nonlinear waves;*

2) *the cause of decay of particles is the disintegration of the composite nonlinear waves to the simple waves.*

We will recall the description of the superposition of waves in the classical theory, to attempt to use these ideas in NTEP.

2.1. Superposition of «linear» waves

As is known (Crawford, 1970), the composite system of waves can be represented by the superposition of the simpler waves, called "modes" (note that the terms: "simple harmonic oscillation", "harmonics", "normal oscillation", "own oscillation", "normal mode" or simply "mode" are identical; recall also that under "linear" wave we understand the wave, which is the solution of linear wave equation). Let's consider a simple case of such superposition.

In many physical phenomena the system represents a superposition of two harmonic oscillations, having various angular frequencies ω_1 and ω_2. These oscillations can, for example, correspond to two normal modes of the system, having two degrees of freedom. The known example of such system is the molecule of ammonia (Crawford, 1970).

It is possible to illustrate this fact based on example of energy spectrum of electron in hydrogen atom. Really, the electron energy spectrum in electron-proton system is from the general point of view a spectrum of electron masses. Then it is possible to speak about a basic mass (basic energy) of electron in the not excited state and about a lot of masses of electron in the excited states, when electron receives additional portions of energy (mass).

These portions are very small in comparison with the rest electron energy (mass). The increase of electron mass occurs due to absorption of photons, and the reduction of mass takes place due to emission of photons. On the other hand, we cannot tell that the electron contains a photon as a ready particle. In the case of particle composition and decay we cannot say the same about the energy portions. But nevertheless, it does not exclude that these are the same phenomenon.

It is easy to show (Crawford, 1970) that the change of electron energy as a result of its excitation corresponds to a hypothesis about the production of new particles owing to superposition of waves.

Let's consider the stable states of the electron in one-dimensional potential well with infinitely high walls, whose coordinates are $z=-\frac{L}{2}$ and $z=+\frac{L}{2}$. We will assume that the electron state is defined by superposition of the basic state and the first excited state:

$$\psi(z,t)=\psi_1(z,t)+\psi_2(z,t), \tag{13.2.1}$$

where $\psi_1(z,t)=A_1e^{-i\omega_1 t}\cos k_1 z$, $k_1 L=\pi$, $\psi_2(z,t)=A_2e^{-i\omega_2 t}\sin k_2 z$, $k_2 L=2\pi$.

The probability of electron state in the position z in the time moment t is equal to:

$$\begin{aligned}|\psi(z,t)|^2 &= \left|A_1e^{-i\omega_1 t}\cos k_1 z + A_2e^{-i\omega_2 t}\sin k_2 z\right|^2 = A_1^2\cos^2 k_1 z + \\ &+ A_2^2\sin^2 k_2 z + 2A_1A_2\cos k_1 z\cdot\sin k_2 z\cdot\cos(\omega_2-\omega_1)t\end{aligned}, \tag{13.2.2}$$

We can see that the expression (13.2.2) has a term, which makes harmonic oscillations with beats frequency between two Bohr frequencies ω_1 and ω_2. The average electron position in space between the wells can be found by means of the expression:

$$\bar{z}=\frac{\int z|\psi|^2 dz}{\int|\psi|^2 dz}=\frac{32L}{9\pi^2}\frac{A_1A_2}{A_1^2+A_2^2}\cos(\omega_2-\omega_1)t, \tag{13.2.3}$$

where the integration is from one wall $-\frac{L}{2}$ up to the other $+\frac{L}{2}$.

Obviously, the frequency of radiation is defined by beats frequency. Actually, electron is charged and, consequently, it will emit out the electromagnetic radiation of the same frequency, with which it oscillates. From the equation (13.2.3) we see that average position of charge oscillates with beats frequency $\omega_2-\omega_1$. Therefore the frequency of radiation is equal to beats frequency between two stationary states:

$$\omega_{rad}=\omega_2-\omega_1, \tag{13.2.4}$$

In the framework of NTEP, the non-normalized quantum wave function is simply the wave field. As a consequence of this fact, the square of this wave function (i.e. the possibility density in the framework of QED) is the energy density.

As other example of such problem we will consider the calculation of more general case of the interference between waves of various frequencies.

We will assume that we have two EM waves 1 and 2, having electric fields $\vec{E}_1$ and. $\vec{E}_2$. The full field in the fixed point P of space will be the superposition of $\vec{E}_1$ and $\vec{E}_2$. Using complex representation of oscillations, we will write the expression for superposition of oscillations:

$$\vec{E}(t) = E_1 e^{-i(\omega_1 t+\varphi_1)} + E_2 e^{-i(\omega_2 t+\varphi_2)}, \qquad (13.2.5)$$

The energy flux is proportional to average value of $\vec{E}^2(t)$ for period T of the "fast" oscillations, occurring with average frequency:

$$\begin{aligned} 2 < E^2(T) >= \left|E(t)\right|^2 &= \left|E_1 e^{-i(\omega_1 t+\varphi_1)} + E_2 e^{-i(\omega_2 t+\varphi_2)}\right|^2 = \\ &= E_1^2 + E_2^2 + 2E_1 E_2 \cdot \cos\left[(\omega_2 - \omega_1)t + (\varphi_1 - \phi_{2)}\right] \end{aligned}, \qquad (13.2.6)$$

As we see, the energy flux varies with relatively slow beats frequency $\omega_2 - \omega_1$.

3.0. Superposition of the nonlinear electromagnetic waves

We should show at first that the superposition of the nonlinear electromagnetic waves exists, and secondly that due to this superposition it is possible to obtain all those results, which are known from the theory of “linear” electromagnetic waves. In other words, it is necessary to show, that in this case there are actually the series (spectra) of particles, each of which represents complication due to the superposition with other nonlinear waves.

As is known, all the phenomena of superposition of waves and their disintegration are described by Fourier theory. The Fourier theory of analysis-synthesis of functions show that any composite wave field consists from harmonic waves and can be analysed to harmonic waves.

We will show that Fourier theory is true not only in case of “linear” waves, but also in case of the nonlinear waves.

3.1. The real and complex form solutions of the wave equation as reflection of an objective reality

As we showed the wave equation can be described in two forms:

CED form:	NTEP form:
$\left(\frac{\partial^2}{\partial t^2} - c^2\vec{\nabla}^2\right)\vec{\Phi}(y) = 0$, where $\vec{\Phi}(y) = \{E_x, E_z, H_x, H_z\}$	$\left[\left(\hat{\alpha}_o\hat{\varepsilon}\right)^2 - c^2\left(\hat{\vec{\alpha}}\ \hat{\vec{p}}\right)^2\right]\Phi = 0$, where $\Phi = \begin{pmatrix} E_x \\ E_z \\ iH_x \\ iH_z \end{pmatrix}$, $\hat{\varepsilon} = i\hbar\frac{\partial}{\partial t}$, $\hat{\vec{p}} = -i\hbar\vec{\nabla}$ and $\hat{\alpha}_0$; $\hat{\vec{\alpha}}$; $\hat{\beta} \equiv \hat{\alpha}_4$ are Dirac's matrices

and has the solution, which can be written down in the form of real periodic (in particular, trigonometric) functions, as well as in the form of complex (exponential) functions:

CED form:	NTEP form:
$\vec{\Phi}(\vec{r},t)=\vec{\Phi}_0\cos(\omega\, t-\vec{k}\cdot\vec{r})$ $\vec{\Phi}(\vec{r},t)=\vec{\Phi}_0'\sin(\omega\, t-\vec{k}\cdot\vec{r})$	$\Phi=\Phi_o e^{-i(\omega t\pm ky)}$ or $\begin{cases}\vec{E}=\vec{E}_o e^{-i(\omega t\pm ky)},\\ \vec{H}=\vec{H}_o e^{-i(\omega t\pm ky)},\end{cases}$

Nowadays it is considered that the representation of the wave equation solution in complex form is only a formal mathematical method, since the final solutions should be real. It was also marked that the use of complex representation is dictated only by the reasons of convenience: in many cases the mathematical operations with exponential functions are much easier, than with trigonometric.

We have shown, that within the framework of NTEP the exponential solutions have the actual meaning, if we understand them in geometrical sense. For instance, the description of wave motion along the circular trajectory can be represented as the sum of two linear mutual-perpendicular oscillations.

Thus, it is possible to assume, that the existence of the real and complex descriptions indicates the existence of two types of real objects: the linear and nonlinear waves. In this case the real functions describe "linear" waves, and the complex functions describe the nonlinear waves

As is known, the Fourier analysis-synthesis theory allows equally to work both with real and complex functions.

From this two extremely important conclusions follows

1) *all tools of the Fourier analysis-synthesis theory in complex representation is the mathematical apparatus of the superposition and decomposition of complex nonlinear waves description (i.e. description of elementary particles).*

2) *the non-linear theory of the nonlinear waves is the theory, in which the principle of superposition takes place as well as in the linear theory*.

For this reason the classical (real) Maxwell-Lorenz theory can be written down in a complex form and it looks in such form simple and consistent. Transition from the nonlinear waves to "linear" (i.e. to one of components of the nonlinear wave) corresponds to transition from complex values to real.

Let us consider now some details of the Fourier analysis-synthesis theory in case of superposition of the nonlinear waves.

4.0. Elementary particles as stable wave packets of nonlinear waves

As is known, in case of superposition of more than two running harmonic waves the wave groups or *wave packets* are formed, which are the limited in space formations, which transfer energy and move with some group speed.

In the quantum mechanics a wave packet is the concept, described a field of matter, i.e. de Broglie particle waves, which is concentrated in the limited area. The probability to find a particle is distinct from zero only in the area, occupied by a wave packet.

This wave field is the result of superposition of the set of de Broglie plane waves, corresponding to the different wavelengths. The composition and decomposition of wave packets is described by Fourier theory.

It is meaningful to apply the concept of a wave packet when the used wave numbers $\vec{k}$ are grouped near to some $\vec{k}_0$ with small variation $\Delta\vec{k}$, $\Delta k << k_0$. In this case the wave field, i.e. wave packet will move during some time as a whole, with a little deformation. The group speed $k_0 u = \left(\frac{d\omega}{dk} \right)_{k=k_0}$ corresponds to a speed of a particle, described by this wave packet. As is known, the "smearing" of the wave packet does not take place if it can be decomposed on standing waves, i.e. if in the decomposition series for each vector $\vec{k}$ the vector $-\vec{k}$ with the same amplitude exists.

Since the superposition of "linear" waves leads to formation of the "linear" wave packets, it is consistently to conclude that superposition of the nonlinear waves will lead to formation of the nonlinear wave packets, i.e. to the composite elementary particles.

It is characteristic that the representation of wave function by the Fourier sum:

In the real form: $f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos n\omega\, t + b_n \sin(\omega\, t),$ where a_n, b_n are the Fourier coefficients.	In the complex form: $f(t) = \sum_{n=-\infty}^{\infty} c_n e^{-in\omega\, t}$ where c_n are the Fourier coefficients.

(i.e. the Fourier series or Fourier integral), contains the negative frequencies, which in the "linear" theory have no place. In the classical optics (Matveev, 1985) it is taken that $e^{i\omega\, t}$ describes the complex unit vector, which is started from the origin of coordinates. At increase of time *t* it rotates around this origin in a positive direction (by a rule of the right screw). In the same time the complex unit vector $e^{-i\omega\, t}$ rotates in the negative direction.

The above completely corresponds to our hypothesis on the correspondence of Fourier mathematical tools to the requirements of NTEP.

As a simple example of a wave packet formation, we will consider a packet, formed by the rectangular equidistant frequency spectrum of waves of equal amplitudes. The description of superposition of such waves can be made both in real (Crawford, 1970) and in a complex form (Matveev, 1985), which reflects the existence of the "linear" and non-linear world of particles.

We will find the expression for a packet $\psi(t)$ formed by superposition of *N* various harmonic components, which have equal amplitude *A*, an identical initial phase, equal to zero, and which frequencies distributed by regular intervals between the lowest frequency ω_1 and the highest frequency ω_2. Generally we have:

in the real form $\psi(t) = A\cos\omega_1 t + $ $+ A\sum_{n=1}^{N-1} \cos(\omega_1 + n\delta\omega)\; t + A\cos\omega_2 t$	in the complex form $\psi(t) = A\sum_{n=0}^{N-1} e^{i(\omega\, t + n\delta\omega t)}$

where $\delta\omega$ is the difference of frequencies of two next components, $n = 1,2,3,..., N-1$ and $\omega_2 = \omega_1 + N\delta\omega$.

These formulas represent the composite wave function $\psi(t)$ in the form of linear superposition of the lot of harmonic components. It appears that this sum can be expressed in the form, which are the generalization of the above case of two oscillations:

$$\psi(t) = A(t)\cos\omega_m t\,, \tag{13.4.1}$$

where $A(t) = A\dfrac{\sin(0,5N\delta\omega\cdot t)}{\sin(0,5\delta\omega\cdot t)}$ is the variable amplitude, ω_m is the average frequency of a wave packet. The amplitude $A(t)$ describes a wave packet envelope. It is possible to show (Crawford, 1970) that for a wave packet the Heisenberg uncertainty principles are true, what proves their origin in wave origin of matter.

Since the nonlinear waves already represent the limited objects, the elementary particles can be combined not from infinite Fourier series of waves, but they can be presented by the sum of the limited number of the nonlinear waves.

Let us show that any nonlinear wave packet can be presented as the sum of wave sub-packets. In this case, obviously, superposition of several big packets can be considered not as superposition of their separate harmonic components, but as superposition of their sub-packets.

Let's consider the splitting of a big packet into two sub-packets. We will present a composite wave $\psi(t)$ (see above (13.4.1)) in the following form:

$$\begin{aligned}\psi(t) &= A\cos\omega_1 t + A\sum_{n=1}^{N-1}\cos(\omega_1 + n\delta\omega)t + A\cos\omega_2 t = \\ &= (A\cos\omega_1 t + A\sum_{m=1}^{N_1-1}\cos(\omega_1 + m\delta\omega)t + A\cos\omega_2^{'} t) + \quad , \\ &+ (A\cos\omega_1^{'} t + A\sum_{l=1}^{N_2-1}\cos(\omega_1^{'} + l\delta\omega)t + A\cos\omega_2 t)\end{aligned} \tag{13.4.2}$$

where $N = N_1 + N_2\,, \omega_2^{'} = \omega_1 + N_1\delta\omega\,,\ \omega_1^{'} = \omega_1 + (N_1+1)\delta\omega = \omega_2^{'} + \delta\omega\,.$

Thus, we can represent the wave packet $\psi(t)$ as two sub-packets:

$$\psi(t) = \psi_1(t) + \psi_2(t)\,, \tag{13.4.3}$$

where $\psi_1(t) = A\cos\omega_1 t + A\sum_{m=1}^{N_1-1}\cos(\omega_1 + m\delta\omega)t + A\cos\omega_2^{'} t$ and

$\psi_2(t) = A\cos\omega_1^{'} t + A\sum_{l=1}^{N_2-1}\cos(\omega_1^{'} + l\delta\omega)\ t + A\cos\omega_2 t\,.$

It is convenient to enter for a normal harmonic the symbol $\psi(t)$, for a packet of wave packets - the symbol ${}^{\Sigma}\psi(t)$, for a sub-packet the symbol ${}^{\Sigma}\psi_i(t)$, where sigma means the sum . Then in the general case any packet can be written down in the form of the sum of sub-packets:

$${}^{\Sigma}\psi(t) = {}^{\Sigma}\psi_1(t) + ... + {}^{\Sigma}\psi_2(t) = \sum_i {}^{\Sigma}\psi_i(t)\,, \tag{13.4.4}$$

From the above follows that decomposition of packets is not single-valued, since harmonic waves can be grouped in the sub-packets in various ways. It is allows to explain the possibility of the disintegration of particle along the different channels.

Using the above representations it is easy to prove also that superposition of sub-packets leads to the same consequences as superposition of separate harmonic waves. In other words, it leads to beats and to change of the energy level, as in the case the particles' interaction.

Besides the nonlinearity in NTEP there is one more serious difference from linear electrodynamics. In NTEP together with the full periodic nonlinear waves (bosons), the half-period nonlinear waves - fermions - exist also. This creates a great number of additional variants of the wave superposition, which are not present in linear electrodynamics. Furthermore, the nonlinearity is the origin of one more characteristic of particles - the currents.

It is clear also that the superposition of the nonlinear waves in comparison with the superposition of ”linear” waves has more variants of the spatial arrangement of waves, and, hence, more complex mathematical description. Actually we can see this in the case of description of hadrons.

It is easy to see that the principle of superposition does not provide stability of composite particles. Thus, we should additionally find out the conditions of stability of the nonlinear waves.

5.0. Conditions of stability and quantization of elementary particles in NTEP

In framework of NTEP the particles are the spatial formations or spatial packets. As an example of three-dimensional packets we can consider also the superposition of the usual waves of different direction in the space.

As is known (Shpolskii, 1951), at such superposition of harmonic waves can be formed the Lissajous figures of two various types. At the waves with commensurable frequencies (i.e. when frequencies are correlated as the rational fractions) are formed the standing waves and Lissajous figures are stable. At incommensurable frequencies the motion of waves is referred to as quasi-periodic, and Lissajous figures are not stable (i.e. they constantly change their form).

In the physics of waves and oscillations two sorts of the tasks exist, which lead to the formation of the composite waves and oscillations.

An example of first type is oscillation of the body surface or volume (sphere, cylinder, torus, etc.), by which we represent a particle. Here the suitable mechanical example is the oscillation of the drop of liquid in zero gravity. In a nuclear physics the similar model is the drop model of a nucleus.

The same types problems the oscillation of vortex rings in a perfect liquid or gas is that, which studied by Kelvin (we will conditionally name these as “Kelvin's problems”). In case of the oscillations of the linear vortex (Kelvin, 1867) he obtained the exact solution. Here Kelvin has compared the radiation spectra of the atoms (obtained little time before by Bunsen) to possible spectra of oscillation of vortex (note that in Kelvin’s articles the term “atom” is used in sense of “elementary particle”).

Comparison of such type of oscillations with observable results is also available in contemporary works, e.g. in (Paper collection, 1975; Kopiev and Chernyshev, 2000) and others. Certain of the Kelvin significant conclusions from the paper “On Vortex Atom” (Kelvin, 1867) we cite below:

“*As the experiments illustrate, the vortex atom has perfectly definite fundamental modes of vibration, depending solely on that motion the existence of which constitutes it. The discovery of these fundamental modes forms an intensely interesting problem of pure mathematics…*

One very simple result… is the following. Let such a vortex be given with its section differing from exact circular figure by an infinitesimal harmonic deviation of order ***i****. This form will travel as waves round the axis of the cylinder in the same direction as the vortex rotation, with an angular velocity equal to (****i****-1)/****i*** *of the angular velocity of this rotation. Hence, as the number of crests in a whole circumference is equal to* ***i****, for an harmonic deviation of order* ***i*** *there are* ***i****-1 periods of vibration in* *the period of revolution of the vortex. For the case* ***i****=1 there is no vibration, and the solution expresses merely an infinitesimally displaced vortex with its circular form unchanged. The case* ***i****=2 corresponds to elliptic deformation of the circular section; and for it the period of vibration is, therefore, simply the period of revolution*".

As examples of other type of problems are oscillations of sound and electromagnetic waves into various types of the closed cavities, whose surface is motionless. Such cavities refer to as closed wave-guides or resonators and consequently we will conditionally name this type of problems as "resonator problems". In the classical physics a set of researches is devoted to such type of problems.

The above first and second type of problems leads to the solution in the form of the standing waves. Thus, it is possible to assume that stability of elementary particles is connected with a formation of standing waves.

As we noted, the condition of standing wave formation is the commensurability of wavelength with the size of body, in which the wave propagates. Therefore, the possible solution of these sorts of problems must be defined by the boundary conditions, which specify the value or the normal derivative of the function on a surface.

Below we will show that from the boundary states follow the quantization conditions for elementary particles.

5.1. Photon wave equation of classical electrodynamics

From Maxwell-Lorentz equations it is easy to obtain (Matveev, 1989) wave equation for the electric and magnetic field vectors:

$$\left(\frac{\partial^2}{\partial t^2} - c^2\vec{\nabla}^2\right)F(\vec{r},t) = 0\,, \qquad (13.5.1)$$

where $\vec{F}$ is whichever of the EM wave functions.

The general harmonic solution of this wave equation has the complex

$$F(\vec{r},t) = F(\vec{r})e^{-i\omega t} = F_0 e^{i(\vec{k}\vec{r}-\omega t)}\,, \qquad (13.5.2')$$

or trigonometric forms

$$F(\vec{r},t) = F_0 \cos(\vec{k}\vec{r} - \omega\, t)\,, \qquad (13.5.2'')$$

where $\omega = 2\pi\nu$ is the angular frequency, $\vec{k} = \frac{2\pi}{\lambda}\frac{\vec{p}}{|\vec{p}|}$ is the wave vector (here ν is the linear frequency, $k = |\vec{k}|$ called the *wave number)*. Using these solutions it is also easy to obtain the dispersion law for EM waves:

$$\frac{\omega^2}{c^2} = k_x^2 + k_y^2 + k_z^2$$

Putting this solution in (13.5.1) we find for $F(\vec{r})$ the following equation for stationary waves:

$$\left(\vec{\nabla}^2 + k^2\right)F(\vec{r}) = 0\,, \qquad (13.5.3)$$

where $k = \omega/\upsilon = 2\pi/\upsilon\; T = 2\pi/\lambda$, T is the period and λ - wavelength.

The equation (13.5.3) refers to as Helmholtz equation and is universal for the description of space dependence of characteristics of harmonic waves.

Using this equation, Kirchhoff developed the theory of the diffraction and interference of light, which was excellently confirmed by the numerous experiments.

5.2. Wave equation solution for resonator

To analyse the electromagnetic wave equation solution for resonator we will take (Wainstein, 1957) an orthogonal box from metal with *a*, *b* and *d* sites as our model of resonator. We will show that this solution is the standing electromagnetic waves.

According to (13.5.3) the electric field must satisfy the equations $\left(\vec{\nabla}^2 + k^2\right)\vec{E}(\vec{r}) = 0$ and $\vec{\nabla}\vec{E} = 0$ with the boundary state $\vec{E}_{II} = 0$ at the walls of the cavity (because inside the walls the electric energy will be rapidly dissipated because of polarization currents, the electric field intensity drops rapidly to zero into the walls). However, the perpendicular to the walls electric field can exists, which is caused by surface charge. These define the possible solution:

$$\begin{cases} \vec{E}_x = E_{0x}k_x \cos k_x x \ \sin k_y y \ \sin k_z z \\ \vec{E}_y = E_{0y}k_y \sin k_x x \ \cos k_y y \ \sin k_z z, \\ \vec{E}_z = E_{0z}k_z \sin k_x x \ \sin k_y y \ \cos k_z z \end{cases} \tag{13.5.4}$$

For example, taking any x, for which $\sin k_x x = 0$, we will obtain that the second and third terms of system (13.5.4) are identically zero, but the first term certainly isn't.

Also if we will choose $\vec{k}$ so that $\vec{k} \cdot \vec{E}_0 = 0$, from $div\vec{E} = 0$ using (13.5.4) we find:

$$\vec{\nabla}\vec{E} = (E_{0x}k_x + E_{0y}k_y + E_{0z}k_z)\sin k_x x \cdot \sin k_y y \cdot \sin k_z z = 0 .$$

In this case wave equation requires fulfilling of the relationships: $k_x = m\pi/a$, $k_y = n\pi/b$, $k_z = l\pi/d$, $\omega^2 = c^2\left(k_x^2 + k_y^2 + k_z^2\right)$ or $\omega = c\sqrt{k_x^2 + k_y^2 + k_z^2}$, where (l, m, n) are positive integers, e.g. (1, 1, 0) or (3, 2, 4). In other words, each possible standing electromagnetic wave in the box corresponds to a point in the (k_x, k_y, k_z) space, labelled by three positive integers.

Since the magnetic field satisfies the same equations and the boundary states as the electric field, the solution for magnetic field will look exactly the same as for the electric field (an alternative way is to use the relationship $\vec{B} = \vec{\nabla} \times \vec{E}/i\omega$, which can be easily obtained from Maxwell theory).

Thus, the character of the general solution for EM wave in the cavity is the standing electromagnetic wave.

It is easy to see that the above description of a resonance of the “linear” electromagnetic waves, if we make it in the complex form, will correspond to the description of the resonance of the nonlinear waves.

Let us show now that solutions of quantum wave equations for the steady states give identical results.

6.0. The quantum wave equations and their solutions for stationary waves

6.1. De Broglie waves as nonlinear EM waves

De Broglie has assumed that mater particles together with corpuscular properties have the wave properties and can be described by the same formula of a plane wave as electromagnetic wave:

$$\psi(\vec{r},t) = \psi(\vec{r})e^{-i\omega t} = \psi_0 e^{i(\vec{k}\vec{r}-\omega t)} = \psi_0 e^{\frac{i}{\hbar}(\vec{p}\vec{r}-\varepsilon t)}$$

He shows that to the energy and momentum of a particle in corpuscular picture the wave frequency and wavelength in a wave picture correspond as follows:

$$\varepsilon = \hbar\omega\,, \quad \vec{p} = \frac{\hbar}{2\pi\lambda}\frac{\vec{p}}{|\vec{p}|} = \hbar\vec{k}$$

Thus the dispersion law for de Broglie wave it is easy to find from the energy-momentum conservation law for a particle:

$$\frac{\varepsilon^2}{c^2} = m_0^2 c^2 + \vec{p}^{\,2}$$

Really, replacing the energy and momentum by the wave characteristics, we will obtain the dispersion correlation for waves of matter:

$$\frac{\omega^2}{c^2} = \frac{m_0 c^2}{\hbar^2} + \vec{k}^{\,2}$$

It is easy to see, that within the framework of NTEP this dispersion correlation satisfies the equation of the nonlinear photon.

We will consider now, to what wave equation this dispersion correlation corresponds.

6.1. Helmholtz equation for de Broglie waves

The Helmholtz equation (13.5.3) describes the waves of various nature in homogeneous mediums with constant frequency ($\omega = const$) and vacuum. The constancy of wavelength is here not supposed.

Planck's correlation $\varepsilon = \hbar\omega$ shows that the condition $\omega = const$ entails the equality $\varepsilon = const$. Hence, Helmholtz equation can be applied to de Broglie waves at the description of motion of corpuscles in potential fields when their full energy is constant:

$$\varepsilon = \varepsilon_k + \varepsilon_p = p^2/2m + \varepsilon_p = const\,, \qquad (13.6.1)$$

where $\varepsilon_k = p^2/2m$ is a kinetic energy, $\varepsilon_p(\vec{r}) \equiv V(\vec{r})$ is potential energy of a corpuscle in a field. From de Broglie's correlation $\vec{p} = \hbar\vec{k}$ in view of (13.6.1) the equality follows:

$$k^2 = \frac{2m}{\hbar^2}(\varepsilon - \varepsilon_p)\,, \qquad (13.6.2)$$

Substituting the expression (13.6.2) in (13.5.3) we receive the equation:

$$\left(\vec{\nabla}^2 + \frac{2m}{\hbar^2}(\varepsilon - \varepsilon_p)\right)F(\vec{r}) = 0\,, \qquad (13.6.3),$$

named the Schrödinger stationary equation.

From this follows, that the existing calculation methods of the energy, momentum, angular momentum and other characteristics of particles state in the quantum field theory are calculations of resonance states of elementary particles in the various types of resonators (boxes), which in the quantum theory are usually named the potential wells. From the mathematical point of view these problems refer to as eigenvalues problems.

Consider the connection of these problems with NTEP.

6.2. Particle in a box

In quantum mechanics, the particle in a box model describes a particle free to move in a small space surrounded by impenetrable barriers. The particle in a box model provides one of the very few problems in quantum mechanics which can be solved analytically, without approximations. We will use this model as an example to illustrate the similarity between our and quantum approaches.

In quantum mechanics, the wavefunction, as electromagnetic field vectors in classical electrodynamics, gives the most fundamental description of the behavior of a wave-particle. The wavefunction $\psi(\vec{r}, t)$ can be found by solving the Schrödinger equation (11.6.3).
Inside the box, no forces act upon the particle, which means that the part of the wavefunction inside the box oscillates through space and time with the same form as a free particle. For a three dimensional box, the solutions are

$$\psi_{n_x,n_y,n_z} = \sqrt{\frac{8}{L_x L_y L_z}} \sin\left(k_{n_x} x\right)\cdot \sin\left(k_{n_y} y\right)\cdot \sin\left(k_{n_z} z\right), \qquad (13.6.4)$$

If a particle is trapped in a three-dimensional box, it may freely move in the *x, y, z* - directions, between barriers separated by lengths L_x , L_y , L_z respectively

The energies which correspond with each of the permitted wave numbers may be written as

$$\varepsilon_{n_x,n_y,n_z} = \frac{\hbar^2 k^2_{n_x,n_y,n_z}}{2m}, \qquad (13.6.5),$$

where the three-dimensional wavevector is given by

$$k_{n_x,n_y,n_z} = k_{n_x}\vec{e}_x + k_{,n_{y_z}}\vec{e}_y + k_{n_z}\vec{e}_z = \frac{n_x\pi}{L_x}\vec{e}_x + \frac{n_y\pi}{L_y}\vec{e}_y + \frac{n_z\pi}{L_z}\vec{e}_z\,, \qquad (13.6.6),$$

As we can see, the obtained solution coincides with the above solution for electromagnetic waves in the waveguide.

7.0. Formation of elementary particles' spectra

The first calculations of quantum systems related to the electron motion in the orbits of the hydrogen atom (Shpolskii, 1951). The formulas of quantization of electron characteristics in the hydrogen atom have been first found empirically (formulas of Balmer, Paschen, etc.). Then, it has been shown that they turn out as consequence of conditions of Bohr quantization.

Wilson and Sommerfeld have made the generalization of Bohr quantization rules independently. They have shown that in case of systems with any number of degree of freedom it is possible to find such generalized coordinates $q_1, q_2, ..., q_i$, in which the motion of system is separated on i harmonic oscillations; in this case a known rule of oscillator quantization can be applied for any of degrees of freedom.

The representation of any quantum system as the set of oscillators completely corresponds to the representation of the elementary particles in the form of nonlinear electromagnetic waves within the framework of NTEP.

Due to Wilson-Sommerfeld's theorem can receive f quantum conditions:

$$\oint p_1 dq_1 = \left(n_1 + \frac{1}{2}\right)h,\ \oint p_2 dq_2 = \left(n_2 + \frac{1}{2}\right)h,\ \ldots,\ \oint p_i dq_i = \left(n_i + \frac{1}{2}\right)h, \qquad (13.7.1)$$

where the integers $n_1, n_2, ..., n_i$ refer to as quantum numbers.

As an example of the application of these rules we will present the calculation results of the hydrogen-like atom. Electron position in space at its motion around a nucleus is characterized by three polar coordinates r, ϑ, ψ, which describe the radial, equatorial and azimuthal motions, respectively. Therefore quantum conditions in this case take the form

$$\oint p_r dr = \left(n_r + \frac{1}{2}\right)h, \oint p_\vartheta d\vartheta = \left(n_\vartheta + \frac{1}{2}\right)h, \oint p_\psi d\psi = \left(n_\psi + \frac{1}{2}\right)h, \qquad (13.7.2)$$

The generalized momentums p_r, p_ϑ, p_ψ are calculated by the following rules: at first it is necessary to write the expression of kinetic energy in polar coordinates r, ϑ, ψ :

$$\varepsilon_k = \frac{m}{2}\upsilon^2 = \frac{m}{2}\left(\dot{r}^2 + r^2\dot{\vartheta}^2 + r^2 \sin^2\vartheta \cdot \dot{\psi}^2\right), \qquad (13.7.3)$$

and next to find the derivatives regarding the generalized velocities (which correspond to linear momentums):

$$p_r = \frac{\partial \varepsilon_k}{\partial \dot{r}} = m\dot{r},\ p_\vartheta = \frac{\partial \varepsilon_k}{\partial \dot{\vartheta}} = mr^2\dot{\vartheta},\ p_\psi = \frac{\partial \varepsilon_k}{\partial \dot{\psi}} = mr^2 \sin^2\vartheta\dot{\psi}, \qquad (13.7.4)$$

Then, using (13.7.2) it is possible to obtain the formulas of the momentums' quantization, defined by radial, equatorial and azimuthal quantum numbers: n_r, n_ϑ, n_ψ

As de Broglie showed later, the Bohr and Wilson-Sommerfeld's quantisation rules define the conditions of *integrality* of the electron wavelengths in different closed trajectories.

Obviously within the framework of NTEP these rules determine the resonance conditions of the nonlinear electromagnetic waves, if we take into account a quantization of their energy according to Planck. Since any field can be represented as the oscillators' sum, we can consider this rule as true for any quantum systems.

The results of Wilson and Sommerfeld were obtained later as solutions of the wave equation for standing de Broglie waves, i.e. of the Schroedinger equation for different potential wells. Note, that also the boundary states are expressed here by the same way, as in the classical EM theory:

$$\psi(a) = 0,\ \psi(b) = 0,\ \psi(d) = 0, \qquad (13.7.5)$$

Thus, we can say that Schrödinger's equation is the equation for calculation of resonance states of nonlinear EM wave in potential wells (resonators) of different type, in which the boundary of wave motion are defined by potential energy of the system.

This problem is identical to the problem of standing EM wave in resonator. The difference consists only in that the *wave vector is not constant here, but by some complex way depends on spatial coordinates; in other words, the dispersion relation is here defined by the potential of system, which varies from a point to point according* (13.6.2). Moreover, the mathematical descriptions would coincide here completely if we imagine that medium in the EM resonator can have dispersion, depending on spatial coordinates under the same law as potential energy in a potential well of quantum-mechanical problem.

Conclusion. On calculation of the spectra of the elementary particles

Thus we showed that the wave spectra in NTEP appear in the same manner as into CED and QED, but at the same time these spectra are the spectra of elementary particles.

The calculation of own particle spectra in NTEP has important peculiarity in comparison with the calculation of stable states of the particle in the externel field, i.e., in field of other particles: the *particle itself acts here as potential well*. In order to calculate the energy-mass spectrum, we must calculate the resonance states of the particle itself as resonator.

Additionally, in this case the initial equation must be a nonlinear equation. But as the studies showed, solution of nonlinear tasks has great mathematical difficulties. In spite of a number of the successes, final solutions are not obtained until now.

Therefore we will attempt firstly to obtain the solution on the basis of the resonance conditions, described above. This way corresponds to calculation of energy levels of hydrogen atom in the early time of development of quantum theory.