Chapter 7. Classical nonlinear electron theories and their connection with QED

1.0. Introduction

In the previous chapters we showed that a question about the formation of elementary particles and the appearance of its characteristics, in particular, mass and charge, is inseparably connected with the self-action of field of a particle. The self-action requires nonlinear description. The physicists arrived not immediately at these conclusions.

The theory of charge, mass and other characteristics of electron has arisen originally on the basis of classical electrodynamics and was developed by W. Kelvin, J. Larmor, H. Lorentz, M. Abraham, A. Poincare, and many others (see the reviews: (Pauli, 1958; Ivanenko and Sokolov, 1949)).

The first example of a theory (Coll. of articles, 1959) which unified electrodynamics and mechanics was H. A. Lorenz's attempt to explain the inertia of an electron on the basis of classical (linear) electrodynamics. Here, the electron was presented as a "clot" of electromagnetic field. The purpose of the theory was to show that the equation of an electron's motion follows from the equation which describes the field of the electron.. Important results have been achieved within the framework of this theory. However, the description of an electron required the introduction of an additional non-electromagnetic field. Alternatively, self-action fields inside the electron could be introduced, but this requires the creation of a nonlinear theory of the electron.

The idea of nonlinearity appeared as answer to the difficulties, emergent in the linear theory of electron (which was the first elementary particle).

1.1. The general results and difficulties of the classical electron theory

According to the hypothesis, which has been put forward in the end of the 19th century by J.J. Thomson and advanced by H. Lorentz, M. Abraham, A. Poincare, etc. (Lorentz, 1916; Ivanenko and Sokolov, 1949), the electron's own energy (or its mass) is completely caused by the energy of the electromagnetic field of electron. In the same way it is supposed that the electron momentum is obliged to the momentum of the field. Since electron, as any mechanical particle, possesses the momentum and energy, which are together the 4-vector of the generalized momentum, the necessary condition of success of the theory will be the proof that the generalized momentum of an electromagnetic field is a 4-vector.

Thus, for the success of the field mass theory the following conditions should be satisfied at least:

At first, it is necessary to receive final value of the field energy, generated by a particle, which could be precisely equated to final energy of a particle (i.e. product of the mass by the square of the light speed).

At second, the value of a momentum of the field, generated by a particle, must not only be final, but also has the proper correlation with energy, forming with the last a four-dimensional vector.

Thirdly, the theory should manage to deduce the equation of movement of electron.

Fourthly, it is necessary to obtain of electron spin, as a spin of a field (that needs the quantum generalization of the theory of field mass, since a spin is quantum effect).

All the parameters in classical electrodynamics can be expressed through the symmetrical energy-momentum tensor of electromagnetic field τ_μ^ν (Tonnelat, 1959; Ivanenko and Sokolov, 1949) τ_μ^ν is determined by the following expressions:

$$\tau_{ij} = -\left(\mathrm{E}_i \mathrm{E}_j + \mathrm{H}_i \mathrm{H}_j\right) + \frac{1}{2}\delta_{ij}\left(\vec{\mathrm{E}}^2 + \vec{\mathrm{H}}^2\right), \quad (7.1.1)$$

$$\tau_{i4} = 4\pi\, S = \left[\vec{\mathrm{E}} \times \vec{\mathrm{H}}\right]_i, \quad (7.1.2)$$

$$\tau_{44} = 4\pi\, u = \frac{1}{2}\left(\vec{\mathrm{E}}^2 + \vec{\mathrm{H}}^2\right), \quad (7.1.3)$$

were, indices $\mu,\nu = 1,2,3,4$, $i, j = 1,2,3$; $\delta_{ij} = 0$, when $i = j$ and $\delta_{ij} = 1$ for $i \neq j$. Moreover, a 4-vector of the space-time has the form $x_\mu = \{x_i, x_4\} = \{\vec{r}, x_4\} = \{x, y, z, ict\}$.

The analysis shows, that there are two conditions, by which the generalized field momentum G_μ is a 4-vector.

In case of space without charges the size

$$G_\mu = \frac{i}{c}\int \tau_{\mu 4}(dr), \quad (7.1.4.)$$

will represent a 4-vector if divergence of energy tensor of a field turns into zero:

$$\frac{\partial \tau_{\mu\lambda}}{\partial x_\lambda} = 0, \quad (7.1.5.)$$

For example, the electromagnetic field, which is located in a space without charges, satisfies similar conditions. In particular, due to this fact, in the photon theory, EM field is characterized not only by energy, but also by momentum.

2) The condition, by which the energy and momentum of an electromagnetic field form a 4-vector at the presence of charges, is formulated by the Laue theorem. According to the last, at the presence of charges the size G_μ is a 4-vector only in the case when in the coordinate system, relatively to which electron is in rest, for all the energy tensor components the following parity is observed

$$\int \tau^0_{\mu\nu}(d\vec{r}_o) = 0, \quad (7.1.6.)$$

except for the component T^0_{44}, the integral of which is a constant and is equal to full energy of the field, generated by particle (here $(d\vec{r}_o)$ is elementary volume in reference system, in which the electron is in rest). The equality (1.3) expresses a necessary condition, by which the whole particle charge should be in balance.

We can equate this field energy to the particle's own energy, expressing in this way the basic idea of a field hypothesis. According to the last:

$$m_e = \frac{\varepsilon_e}{c^2} = \frac{1}{c^2}\int \tau^0_{44}(d\vec{r}_o), \quad (7.1.7.)$$

Thus, the mass of a particle from the field point of view can be defined in two ways:

1) proceeding from EM momentum of a field G_1 it is possible to define mass as factor of proportionality between a field momentum and three-dimensional speed of a particle.

2) if we consider the electron's own energy as equal or conterminous to the energy of a field, and mass as the ratio of a field energy $\frac{c}{i}G_4$, to a square of light speed (i.e. as the fourth component of a generalized momentum).

The attempts to execute this program, proceeding from classical linear Maxwell theory, have led to difficulties. In particular, it was not possible to prove the Laue theorem (Tonnelat, 1959). In the classical theory the dynamics (mechanics) and electrodynamics are completely independent from each other. Electromagnetic actions are characterized by component T_0^0 of an energy-momentum tensor of an electromagnetic field. It does not include the energy and momentum of the substance, which should be subsequently inserted. The attempts of Lorentz and Poincare to coordinate the theory on the basis of the assumption that energy of substance has an electromagnetic origin, have not led to a positive result. In Lorentz electron theory (linear in essence) existence of charges it is possible to explain only by introduction of forces of non-electromagnetic origin.

Nevertheless (Sokolov and Ivanenko, 1949), there were also a number of successes, which carried a hope to solve this problem by some change of the theory. The most perspective change of Maxwell-Lorentz theory appeared to be its non-linear generalization by Gustav Mie..

2.0. The Gustav Mie non-linear electron theory

Within the framework of classical physics, the first completely successful nonlinear theory was created by Gustav Mie (Mie, 1912a, 1912b, 1913, 2007; Pauli, 1958; Tonnela, 1959; Sommerfeld, 1964). The most widely known variant of this theory was obtained by M. Born and L. Infeld (Born and Infeld, 1934b). Similar variant was also obtained by E. Schrödinger and others.

Gustav Mie took the first step in the direction of the generalization of Maxwell's equations and to the theory of the elementary particles in 1912 in his famous papers' "Foundations of a Theory of Matter". Their goal is no less than the generalisation of the Maxwell equations so that they include the *existence of the electron.* This generalisation is subjected from the start to the principle of relativity and derived from a "world function" (Lagrange function), which may depend only on Lorentz-invariant quantities. Here a distinction is made possibly for the first time in a consistent fashion - between entities of intensity and entities of quantity, i.e. the difference between $F=\left(\vec{B},\ -i\vec{E}\right)$ and $A_\mu=\left(\vec{A},\ i\varphi\right)$, on the one hand, and $f=\left(\vec{H},\ -i\vec{D}\right)$ and $j_\mu=\left(\rho\vec{\upsilon}, i\rho\right)$ on the other. Mie tested the invariants, which may be set up with the entities of intensity and the entities of quantity respectively.

Mie set himself the task to generalize the field equations and the energy-momentum tensor in the Maxwell-Lorentz theory in such a way that the Coulomb repulsive forces in the interior of the electrical elementary particles are held in equilibrium by other, equally electrical, forces, whereas the deviations from ordinary electrodynamics remain undetectable in regions outside the particles. In other words Mie introduced a uniform view of the field and substance, and got rid of Poincare–Lorentz forces, which have a non-electromagnetic origin.

2.1. The G. Mie theory Lagrangian

Mie was the first who suggested that the theory should be constructed on the basis of a Lagrangian that depends on fundamental invariants.

It is possible to make some general statements about the form of the Lagrangian L, which is often called the world function. Independent invariants, which can be formed from the bivector $F_{\mu\nu}$ (where $F_{\mu\nu}$ are the tensor components of EM field strengths) and the vector-potential $A_\mu=\left(i\varphi,\vec{A}\right)=\left(i\varphi,A_i\right)=\left(A_4,A_i\right)$ of an EM field, are the following:

1. A square of the bivector $F_{\mu\nu}$: $I_1=\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$;

2. A square of the pseudo-vector $I_2 = \frac{1}{4} F_{\mu\nu} F^{*\mu\nu}$ (where $F^{*\mu\nu}$ is dual electromagnetic tensor).

3. A square of a 4-vector of EM potential A_μ : $I_3 = A_\mu A^\mu$;

4. A square of the vector $F_{\mu\nu} A^\nu$: $I_4 = F_{\mu\rho} A_\sigma F^{\mu\sigma} A^\rho$;

5. A square of the vector $F^*_{\mu\nu} A^\nu$: $I_5 = F^*_{\mu\rho} A_\sigma F^{*\mu\sigma} A^\rho$.

Therefore, L can depend only on these five invariants. If L is equal to the first of these, the field equations are the ordinary equations of EM theory for a space without charges. Thus, L can differ substantially from I_1 only inside material particles. Invariant 2 can be included into L only when squared, in order not to break the invariance related to spatial reflections. Invariants 3-5 break the gauge invariance.

We cannot make further statements about the world function L. Thus, there is an infinite number of ways in which we can select L.

Mie supposed that only the invariants (1) and (3) need to be considered for the description of quasistationary processes and construct a world function (Lagrangian), such that at large distance from the electron the ordinary Maxwell equations apply, whereas the equations are modified at the electron and in its immediate neighbourhood. The entities of intensity are obtained from Mie's world function by differentiation with respect to the entities of quantity. The Lagrangian is to be integrated over an arbitrary region of the four-dimensional world and to be varied in suitable manner.

G. Mie accepted the following Lagrangian as the initial one:

$$L_{Mi} = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - f\left(\pm\sqrt{A_\mu A^\mu}\right), \tag{7.2.1'}$$

or

$$L_{Mi} = \frac{1}{8\pi}\left(E^2 - H^2\right) - f\left(\pm\sqrt{A_\mu A^\mu}\right), \tag{7.2.1''}$$

where f is some function, and $\vec{E}$, $\vec{H}$ are the strength vectors of the electric and magnetic fields, respectively.

Using this Lagrangian, Gustav Mie obtained the final energy (or mass) of a charged particle as a value completely determined by the energy of the particle's field. In this theory, Laue's theorem of particle stability is satisfied, and the proper correlation between the energy and momentum of the particle is achieved.

An attempt by Weyl should be mentioned here, in which he tries to make the asymmetry between the two kinds of electricity understandable from the point of view of Mie's theory. If world function L is not rational function of $\sqrt{A_\mu A^\mu}$, we can put

$$L^+_{Mi} = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - f\left(+\sqrt{A_\mu A^\mu}\right), \tag{7.2.2'}$$

$$L^-_{Mi} = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - f\left(-\sqrt{A_\mu A^\mu}\right), \tag{7.2.2''}$$

where f denotes any function which is not even. For the statical case the field equations will not remain invariant for an interchange of electrostatic potential φ with $-\varphi$ (positive and negative electricity). Thus, if L is a multiple-valued function of the invariants, mentioned above, then it is possible to choose various single branches of this function as world function for positive electricity, and another for negative electricity.

2.2. The Mie theory difficulties

Mie assumes (Pauli, 1958) the field of the stationary electron to be static and spherically symmetry. But the latter assumption is admittedly not justified by our experimental knowledge alone, but recommends itself for its simplicity. We will then have to look for those solutions of the field equations which are regular everywhere — for $r = 0$ as well as for $r = \infty$.

A much more serious difficulty is caused by a fact already noticed by Mie. Once we have found a solution for the electrostatic potential φ of a material particle of the required kind, φ+const. will not be another solution, because the field equations of Mie's theory contain the absolute value of the potential. A material particle will therefore not be able to exist in a constant external potential field. This seems to constitute a very weighty argument against Mie's theory. In the theories which we are going to discuss in the following sections, this kind of difficulty does not arise.

3.0. Born-Infeld nonlinear theory

The Born-Infeld field (Born and Infeld, 1934a, 1934b; Pauli, 1958; Tonnela, 1959; Ivanenko and Sokolov, 1949) theory can be considered as a revival of the old idea of the electromagnetic origin of mass. This non-linear field theory (Born, 1953) "is a modification of Maxwell's electrodynamics in which the self-energy of the electron is finite. Mie had shown already in 1912 that the equations of the electromagnetic field can be formally generalized by replacing the linear relations between the two pairs of field vectors *E, B* and *D*, *H* by non-linear ones. Yet he did not specify these relations, and thus his formalism remained empty".

The idea which Born applied to Mie plan is, as he note, "*a special case of what Whittaker has called the principle of impotence. If research leads to an obstacle which in spite of all efforts cannot be removed, theory declares it as insurmountable in principle… Examples are relativity, where the impossibility of material and signal velocities larger than the velocity of light is declared, and the uncertainty relations of quantum mechanics, which forbid the simultaneous determination of position and velocity and of similar pairs.*

In the case of the electromagnetic field the self-energy can be made finite by prohibiting the increase of E the electric vector beyond a certain limit, the absolute field. This can be done by imitating relativity where the classical Lagrangian of a free particle $L = \frac{1}{2} m\upsilon^2$ *is replaced by*

$$L = mc^2\left[1 - \left(1 - \upsilon^2/c^2\right)^{1/2}\right], \tag{7.3.1}$$

from which $\upsilon < c$ *follows. In a similar way the Lagrangian density of Maxwell's electrodynamics can be replaced by a square root expression. Thus a finite self-energy of a point charge is obtained which represents not only the inertial mass but also, as Schrödinger has shown, the gravitational mass*".

To obtain the laws of nature Born and Infeld use a variation principle of least action of the form

$$\delta \int L\, d\tau' = 0\,, \ \left(d\tau'\right) = dx^1 dx^2 dx^3 dx^4\,, \tag{7.3.2}$$

and postulate that the action integral has to be an invariant. The problem to find the form of L satisfying this condition arises here.

Born and Infeld consider a covariant tensor field a_{kl}. The question is to define L to be such a function of a_{kl} that (7.3.2) is invariant. The well-known answer is that L must have the form

$$L = \sqrt{|a_{kl}|}\,; \quad (|a_{kl}| = \text{determinant of } a_{kl}), \tag{7.3.3}$$

If the field is determined by several tensors of the second order, L can be any homogeneous function of the determinants of the covariant tensors of the order ½ .

Each arbitrary tensor a_{kl} can be split up into a symmetrical and anti-symmetrical part:

$$a_{kl} = g_{kl} + f_{kl}\,; \quad g_{kl} = g_{lk}\,; \quad f_{kl} = -f_{lk}\,, \tag{7.3.4}$$

The simplest simultaneous description of the metrical and electromagnetic field is the introduction of *one* arbitrary (unsymmetrical) tensor a_{kl}; it can be identified its symmetrical part g_{kl} with the metrical field, its antisymmetrical part with the electromagnetic field.

The quotient of the field strength expressed in the conventional units divided by the field strength in the natural units may be denoted by b. This constant of a dimension of a field strength may be called the *absolute field*.

The Born-Infeld field equations are formally identical with Maxwell's equations for a substance which has a dielectric constant and a susceptibility, being certain functions of the field strength, but without a spatial distribution of charge and current.

As Born and Infeld in the summary write (Born-Infeld, 1934b), "*The new field theory can be considered as a revival of the old idea of the electromagnetic origin of mass. The field equations derive from the postulate that there exists an "absolute field" b which is the natural unit for all field components and the upper limit of a purely electric field. The field equations have the form of Maxwell's equations for a polarizable medium for which the dielectric constant and the magnetic susceptibility are special functions of the field components. The conservation laws of energy and momentum can be derived. The static solution with spherical symmetry corresponds to an electron with finite energy (or mass); the true charge can be considered as concentrated in a point, but it is also possible to introduce a free charge with a spatial distribution law. The motion of the electron in an external field obeys a law of the Lorentz type where the force is the integral of the product of the field and the free charge density*".

3.1. Born-Infeld field and Lagrangian

As in Maxwell's electromagnetism the Born-Infeld field $F_{\mu\nu}$ is derived from a potential $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. This condition cancels the rotor of the electric field $\vec{E}$ and the divergence of die magnetic field $\vec{B}$ **:**

$$\partial_\nu F_{\lambda\mu} + \partial_\mu F_{\nu\lambda} + \partial_\lambda F_{\mu\nu} = 0\,, \tag{7.3.5}$$

The Born-Infeld field differs from the Maxwell field in the dynamic equations, which are written in terms of the tensor

$$\tilde{F}_{\mu\nu} = \frac{F_{\mu\nu} - \frac{I_2}{b^2}{}^*F_{\mu\nu}}{\sqrt{1 + \frac{2I_1}{b^2} - \frac{I_2^2}{b^4}}}\,, \tag{7.3.6}$$

where *S* and *P* are the scalar and pseudoscalar field invariants:

$$I_1 = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} = \frac{1}{2}\left(\left|\vec{B}\right|^2 - \left|\vec{E}\right|^2 \right), \tag{7.3.7}$$

$$I_2 = \frac{1}{4} {}^*F_{\mu\nu} F^{\mu\nu} = \vec{E} \cdot \vec{B}, \tag{7.3.8}$$

(${}^*F_{\mu\nu}$ is the dual field tensor, i.e. the tensor resulting from exchanging the roles of $\vec{E}$ and $-\vec{B}$). Born-Infeld dynamical equations are

$$\partial_\nu F^{\mu\nu} = 0, \tag{7.3.9}$$

which is obtained from the Born-lnfeld Lagrangian

$$L = -\frac{b^2}{4\pi}\left(1 - \sqrt{1 + \frac{2I_1}{b^2} - \frac{I_2^2}{b^4}} \right), \tag{7.3.10}$$

or:

$$L_{BI} = \frac{b^2}{4\pi}\left(1 - \sqrt{1 - \frac{\vec{E}^2 - \vec{B}^2}{b^2} - \frac{\left(\vec{E} \cdot \vec{B}\right)^2}{b^4}} \right), \tag{7.3.11}$$

The constant b in equations (7.3.6), (7.3.10) and (7.3.11) is a new universal constant with units of held that controls the scale for passing from Maxwell's theory to the nonlinear Born-Infeld theory, in the same way as the light speed c is the velocity scale that indicates the range of validity of Newtonian mechanics. The Maxwell Lagrangian and its related dynamical equations are recovered in the limit $b \to \infty$, or in regions where the held is small compared with b. Besides. Born-Infeld solutions having $S = 0 = P$ ('free waves') also solve Maxwell's equations. As it follows from Born-Infeld solution the constant b is a maximum electric field of an electron $b \equiv E_0$. Thus, if we consider E and B as wave functions, then the ratios E/b and B/b can be considered as the normalized wave functions.

3.2. The point and non-point solutions of Born-Infeld's nonlinear theory

The results presented above led them to the following Lagrangian considered as a function of invariants I_1 and I_2:

$$L_{BI} = \frac{E_0^2}{4\pi}\left(1 - \sqrt{1 - \frac{E^2 - H^2}{E_0^2} - \frac{\left(\vec{E} \cdot \vec{H}\right)^2}{E_0^4}} \right), \tag{7.3.12}$$

where E_0 is a maximum electric field of an electron.

Using $\vec{H} = 0$, $\vec{E} = -grad\varphi$, $\rho(\vec{x} - \xi) = \delta(\vec{r})\delta(t - s)$, where δ is the Dirac fuction, we can find the following Lagrangian form:

$$L_n = \frac{E_0^2}{4\pi}\left(1 - \sqrt{1 - \frac{E_r^2}{E_0^2}} \right) - e\varphi\delta(\vec{r})$$

Then, with the help of the variation principle, we can obtain:

$$-\frac{1}{4\pi}\frac{\partial D_r}{\partial E_r}-\frac{\partial L}{\partial \varphi}=0$$

where $\vec{D}$ is an electrical induction vector (D-field):

$$D_r = 4\pi\frac{\partial L}{\partial E_r}=\frac{E_r}{\sqrt{1-\frac{E_r^2}{E_0^2}}}$$

which corresponds to equation:

$$div\vec{D}_r = 4\pi e\delta(\vec{r})$$

Solution of this equation, which corresponds to linear Maxwell theory, is as follows:

$$D_r = \frac{e\vec{r}}{r^3}, \qquad (7.3.13)$$

As we can see, *from the point of view of D-field, the electron should be considered as a point particle*.

For the electric field (E-field), we obtain:

$$\vec{E}_r = \frac{\vec{D}_r}{\sqrt{1+\frac{D_r^{\,2}}{E_0^2}}}=\frac{e\vec{r}}{r\sqrt{r^4+r_0^4}}, \qquad (7.3.14)$$

where $r_0=\sqrt{\frac{e}{E_0}}$. Thus, *from the point of view of electric field (E-field), the electron is not the point particle*.

In comparison to a linear theory, these results present very important specifics of a nonlinear theory. This can explain why experiments on dispersion of electron can be interpreted so that the non-point electron can look as a point particle.

We can obtain the potential's value at the center of a particle as follows:

$$\varphi_0 = \int_0^\infty E_r dr = 1.8541\ldots\frac{e}{r_0}$$

The charge's density distribution of the non-point electron can be found in the following way:

$$\rho = \frac{divE}{4\pi}=\frac{er_0^4}{2\pi\ r(r^4+r_0^4)^{3/2}}, \qquad (7.3.15)$$

For $r >> r_0$, $\rho \propto r^{-7}$, therefore diminishing very rapidly as r increases. For $r < r_0$, $\rho \propto 1/r$, therefore $\rho\to\infty$, but $r^2\rho \to 0$. It easy to verify that the space integral of ρ is equal to e. For a full charge we have:

$$\int \rho\, d\tau = \frac{1}{4\pi}\int div\vec{E}d\tau = \frac{1}{4\pi}\oint Eds = \lim_{r\to\infty} r^2E(r) = e$$

The charge can be considered as distributed in a sphere of radius r_0, since, because of the condition $r >> r_0$, the density will quickly go to zero. Therefore, the size r_0 can be considered as an effective radius of an electron.

The value of an electromagnetic mass of an electron can be found based on the equality condition:

$$m_{EM} = \frac{\int \tau_{44} d\tau}{c^2} = \frac{2}{3}\frac{e\varphi_0}{c^2} = 1.2361...\frac{e^2}{c^2 r_0}$$

Using experimental values for the mass and the charge of an electron, it is possible to obtain for an effective electron radius the value of $r_0 = 2{,}28 \cdot 10^{-13}$ cm, which is practically equal to the classical radius of an electron. In this case, we have for the electron energy $\varepsilon = \int \tau_{44} d\tau = m_{EM} c^2$.

Also, it is easy to find the maximal field of an electron, which is in the center of an electron at $r = 0$:

$$E_0 = \frac{e}{r_0^2} = 9{,}18 \cdot 10^{15}\, CGS = 2{,}75 \cdot 10^{20} \frac{V}{m} .$$

As is known two types of fields and two definitions for the charge density, corresponding to them, present in theory of dielectrics. The ratio of $\vec{D}$ to $\vec{E}$ can be considered as a dielectric constant δ_d:

$$\delta_d = \frac{D}{E} = \sqrt{\frac{r^4 + r_0^4}{r^4}} , \qquad (7.3.16)$$

which in this case is the function of a position. On large distances from the charge, when $\frac{r_0}{r} \to 0$, δ_d is equal to one, the same as in conventional electrodynamics. We can say that instead of the energy expression $\frac{e^2}{r^2}$ the value of $\frac{e^2}{\delta_d\, r^2}$ is used, and that the reduction of r is compensated by the increase of δ_d, so that the full energy remains final.

Thus, we proved that it is possible to obtain the final self-energy (or the mass) of the charged particle as a value completely conditioned by the energy of the field of this particle, when one uses a certain formally irreproachable hypothetical nonlinear generalization of electrodynamics as the basis. Furthermore, the Laue theory of stability is valid within this theory, so that a correct relationship between the energy and the particle's momentum is established.

Thus, basing on some formal hypothetical nonlinear generalization of electrodynamics, it appeared possible (Ivanenko and Sokolov, 1949):

1. to prove the theorem of stability, i.e. to prove, that in the nonlinear theory the electron is stable without introduction of forces of non-electromagnetic origin;
2. to receive the final energy (mass) of a particle;
3. to receive the final size of an electric charge;
4. to receive the final size of an electromagnetic field.

4.0. Schrödinger variant of Born-Infeld theory without root

As E. Schrödinger has noted (Schrödinger, 1935), Born's theory starts from describing the field by two vectors (or a "six-vector"), $\vec{B}, \vec{E}$, the magnetic induction and electric field-strength respectively. A second pair of vectors (or a second six-vector) $\vec{H}, \vec{D}$, is introduced, merely an abbreviation, if you please, for the partial derivatives of the Lagrange function with respect to the components of $\vec{B}$ and $\vec{E}$ respectively (though with the negative sign for $\vec{E}$). $\vec{H}$ is called magnetic field and $\vec{D}$ dielectric displacement. It was pointed out by Born that it is possible to choose the independent vectors in different ways. *Four* different and, to a certain extent, equivalent and symmetrical representations of the theory can be given by combining each of the two "magnetic" vectors with each of the two "electric" vectors to form the set of six independent variables. Every one of these four representations can he derived from a variation principle, using, of course, entirely different Lagrange functions physically different, that is, though their analytic expressions by the respective variables are either identical or very similar to each other.

In studying Born's theory E. Schrödinger cams across a further representation, which is entirely different from all the aforementioned, and presents curious analytical aspects. The idea is to use two complex combinations of $\vec{B}, \vec{E}, \vec{H}, \vec{D}$ as independent variables, but in such a way that their "conjugates," *i.e.,* the partial derivatives of $\widetilde{L}$, equal their *complex* conjugates. Choosing the following pair of independent variables

$$\widetilde{F} = \vec{B} - i\vec{D}\ ,\ \widetilde{G} = \vec{E} + i\vec{H}\ , \qquad (7.4.1)$$

(which form a true six-vector) the appropriate Lagrangian works out

$$L = \frac{\widetilde{F}^2 - \widetilde{G}^2}{\left(\widetilde{F}\widetilde{G}\right)}, \qquad (7.4.2)$$

and one has

$$\widetilde{F}^* = \frac{\partial L}{\partial \widetilde{G}}, \quad \widetilde{G}^* = \frac{\partial L}{\partial \widetilde{F}}, \qquad (7.4.3)$$

The * indicates the complex conjugate. The derivative with respect to a vector is short for: a vector, of which the components are the three derivatives with respect to the components of that vector. The units are "natural" units, Born's constant b being equalled to 1 (in other units $\widetilde{L}$ would take the factor b^2).

What is so very surprising with E. E. Schrödinger's idea (Schrödinger, 1935) is that the square root, which is so characteristic for Born's theory, has disappeared. The Lagrangian is not only rational, but homogeneous of the zeroth degree.

As Schrödinger show, the treatment of the field by the Lagrangian (7.4.2) is entirely equivalent to Born's theory. Therefore it cannot yield any new insight which could not, virtually, be derived from Born's original treatment as well. Moreover, for actual calculation the use of imaginary vectors will hardly prove useful. Yet for certain theoretical considerations of a general kind I am inclined to consider the present treatment as the standard form on account of its extreme simplicity, the Lagrangian being simply the *ratio* of the two invariants, whereas in Maxwell's theory it was equal to one of them.

It is not difficult to observe that the 10 components of $\tau_{\mu\nu}$ (see above) are identical in form with the components of Maxwell's vacuum tensor, $\widetilde{F}$, $\widetilde{G}$ being substituted for $\vec{H}, \vec{E}$.

By investigating the transformations of the real tensor $\tau_{\mu\nu}$, it is easy to find a frame of reference, in which the physical meaning of our "condition of conjugateness" is readily disclosed. What distinguishes a Maxwell tensor from the general symmetrical tensor is only that its roots or eigenvalues have the form $\pm\rho$, each double. The first part of $\tau_{\mu\nu}$, viz., $\frac{i\tau_{\mu\nu}}{\left(\widetilde{F}\widetilde{G}\right)}$ is precisely of this type. At that the value ρ works out to

$$\rho = \pm\sqrt{\left(\frac{i}{2}\widetilde{L}\right)^2 - 1}, \tag{7.4.4}$$

by considering that in Maxwell's case ρ is known to be $\pm\frac{1}{2}\sqrt{\left(\vec{H}^2 - \vec{E}^2\right)^2 + 4\left(\vec{H}\vec{E}\right)^2}$ (see the proof in (Lightman, A.P., et al., 1975)).

4.1. *Other Lagrangians of nonlinear theories*

It was noted (Ivanenko and Sokolov, 1949), that various and arbitrary variants of formal nonlinear electrodynamics lead to close values of coefficients, if we take into account that the electron radius is equal to a classical radius of an electron. For example, in this way E. Schrödinger obtained similar results using the following Lagrangian:

$$L_{Sch} = \frac{E_0^2}{8\pi}\ln\left(1 + \frac{E^2 - H^2}{E_0^2}\right), \tag{7.4.5}$$

The only serious deficiency in these nonlinear theories is that they are not quantum.

5.0. The vacuum polarization as cause of formation of electron

As is known (Wikipedia, Vacuum polarization; Peskin and Schroeder, 1995) in quantum field theory, and specifically quantum electrodynamics, vacuum polarization describes a process in which a background electromagnetic field produces virtual electron-positron pairs that change the distribution of charges and currents that generated the original electromagnetic field. It is also sometimes referred to as the self-energy of the gauge boson (photon). Vacuum polarization was observed experimentally in 1997 using the TRISTAN particle accelerator in Japan.

According to quantum field theory, the ground state of a theory with interacting particles is not simply empty space. Rather, it contains short-lived "virtual" particle-antiparticle pairs which are created out of the vacuum and then annihilate each other.

Some of these particle-antiparticle pairs are charged; e.g., virtual electron-positron pairs. Such charged pairs act as an electric dipole. In the presence of an electric field, e.g., the electromagnetic field around an electron, these particle-antiparticle pairs reposition themselves, thus partially counteracting the field (a partial screening effect, a dielectric effect). The field therefore will be weaker than would be expected if the vacuum were completely empty. This reorientation of the short-lived particle-antiparticle pairs is referred to as *vacuum polarization*.

The vacuum polarisation firstly was discussed 1912 by G. Mie in his classical nonlinear electromagnetic theory of electron. One of the first a student of Werner Heisenberg and Arnold.

Sommerfeld - Dr. Erich Bagge - connected the task of the forming the elementary particle (in particular, electron) with the polarization of physical vacuum

In the simplest case the formulation of problem of formation of an (spherical) electron looks as follows (Bagge, 1951).

The fact of the formation of the electron pairs by high energy light quanta can be considered as deep polarization of vacuum that leads to the breaking of the electric dipole under the action of EM field of light wave. This makes it possible to examine the presence of this polarization also in the case, when field energetically cannot cause a pair formation. This leads to interaction of light by light, as this showed Heisenberg, Euler and Kockel (Euler.and Kockel, 1935; Euler, 1936; Heisenberg and Euler, 1936) in quantum field theory.

Maybe Peter Debye (Debye, 1934) was first, who note, that this approach can be developed also in the framework of Dirac's theory: "If Dirac's picture of pair production in 'hole theory' is correct, then at lower energies a kind of vacuum polarisation should be prepared similar to the polarisation effect in a dielectric medium shortly before an electric discharge occurs."

E. Bagge has shown (Bagge, 1951; 1988; 1990, 1993) that with the reasonable choice of the function $\delta_V(r,t)$ it is possible to describe the rest spherical electron within the framework of Dirac's relativistic theory of particles as electro-magnetic stable, spheroidal particles. Their properties, especially their spins and their magnetic momentum, are exactly those, which have been measured at first and later on derived by Dirac.

This concept theoretically can be realised by the help of the formula for the media dispersion constants of optical resonance effects, allowing the calculation a polarisation function $\varepsilon_V(\vec{r},t)$ of the vacuum. In this case the ratio of $\vec{D}$ to $\vec{E}$ can be considered as a "dielectric constant (permeability) of vacuum": $\delta_V(r,t)=\dfrac{\vec{D}}{\vec{E}}$.

It is natural therefore to represent in this case the equations of field in that form, which they have for the polarization medium. This means that in particular the dielectric constant δ_V must be considered as the function of space.

This fact naturally constrains the field in the environment of electron, which automatically leads to end value for the self-energy of electron.

Considering the dielectric constant of vacuum with particles $\delta_V(r)$ not as constant, but as the function of three-dimensional variables and field (in simplest case, of scalar potential φ), we must replace the known Laplace equation for the empty space

$$\Delta\varphi_1 = 0\,, \tag{7.5.1}$$

by the equation, which considers the inconstancy of dielectric constant:

$$\frac{\partial}{\partial x}\left(\delta_V\frac{\partial\varphi}{\partial x}\right)+\frac{\partial}{\partial y}\left(\delta_V\frac{\partial\varphi}{\partial y}\right)+\frac{\partial}{\partial z}\left(\delta_V\frac{\partial\varphi}{\partial z}\right)=0\,, \tag{7.5.2}$$

This equation already can not have the infinite solution for the field of spherical charge. Actually, if $\delta_V=\delta_V(r),\quad \varphi=\varphi(r)$,

then the differential equation (7.5.2) takes the form:

$$\frac{d^2\varphi}{dr^2}+\frac{2}{r}\frac{d\varphi}{dr}+\frac{d(\ln\delta_V)}{dr}\frac{d\varphi}{dr}=0, \qquad (7.5.4)$$

If a dielectric constant is constant, the equation (7.5.4) has a solution in the form of the Coulomb potential: $\varphi_C = \frac{e}{r}$.

It is obvious that we must obtain this expression at a great distance from the electron, whereas in the limits of the basic volume, which contains the basic energy (mass) of electron, the solution must not give the divergent result.

By analogy with the dispersion formulas of optical theory Bagge proposed the following expression for the dielectric constant (without taking into account a damping):

$$\delta_V = \frac{C}{(\varepsilon_r - \varepsilon)^2}, \qquad (7.5.5)$$

where C is a constant, which must be determined; ε_r $(\sim \hbar\omega_r)$ is the resonance energy ($\varepsilon = 2m_e c^2$), which is determined by photon energy; ε $(\sim \hbar\omega)$ is certain energy of the fields (which can be identified with the energy of electron-positron, which compose the dipole of intermediate photon (see Kiryakos, 2010a). It is natural to assume $\varepsilon = e\varphi$; then for the dielectric constant we obtain the expression:

$$\delta_V = \frac{C}{(\varepsilon_r - e\varphi)^2}, \qquad (7.5.6)$$

Using (7.5.6), we will obtain from (7.5.4) the following expression:

$$\frac{d^2\varphi}{dr^2}+\frac{2}{r}\frac{d\varphi}{dr}+\frac{2e}{\varepsilon_r - e\varphi}\left(\frac{d\varphi}{dr}\right)^2=0, \qquad (7.5.7)$$

Considering that the strength of the field of electron has an finite maximum quantity, we must to assume that the resonance energy, which corresponds to this strength, is the constant, which can be expressed in the form: $\varepsilon_r = \frac{e^2}{r_0}$, so that if $\varepsilon_\gamma = 2mc^2$, then $r_0 = \frac{e^2}{2mc^2}$ is a radius of the charged sphere, whose charge is distributed uniformly by the volume. Then from (7.5.7) we obtain the nonlinear differential equation:

$$\frac{d^2\varphi}{dr^2}+\frac{2}{r}\frac{d\varphi}{dr}+\frac{2e}{e/r_0 - e\varphi}\left(\frac{d\varphi}{dr}\right)^2=0, \qquad (7.5.8)$$

The general solution of this nonlinear differential equation of the second order, as it is not difficult to verify (Bagge, 1951), takes the form:

$$\varphi = \frac{e}{r_0}\frac{Ar_0+(B-1)r}{Ar_0+Br}, \qquad (7.5.9)$$

where A and B are constants of the integrations, which can be found from the limiting conditions:

a) $\varphi = 0$ with $r \to \infty$, which gives s $B=1$, b) $\varphi \to \frac{e}{r}$ with $r >> r_0$, which gives $A=1$, Thus, the potential of spherically symmetrical electron upon consideration of self-action can be recorded in the form:

$$\varphi = \frac{e}{r + r_0}, \tag{7.5.10}$$

As we see, potential φ at point $r = 0$ does not go to infinity, but is taken the specific value: $\varphi = \frac{e}{r_0}$, which corresponds to the maximum value of field.

Solution (7.5.10) makes it possible to calculate also strength E and electrical displacement D of field, and also energy ε_e of spherical charge.

From (7.5.10), according to the connection of scalar potential and strength of electric field, we obtain for the value:

$$\vec{E} = -grad\varphi = \frac{e}{(r + r_0)^2}\frac{\vec{r}}{r}, \tag{7.5.11}$$

From this the maximum value of field will be equal to:

$$\vec{E} = -grad\varphi = \frac{e}{(r_0)^2}\frac{\vec{r}}{r}, \tag{7.5.11'}$$

Using (7.5.10), from (7.5.6) we obtain for the dielectric constant:

$$\delta_V = \frac{Cr_0^2}{e^4}\frac{(r + r_0)^2}{r^2}, \tag{7.5.12}$$

and for the displacement:

$$\vec{D} = \delta_V \vec{E} = \frac{Cr_0^2}{e^3}\frac{\vec{r}}{r^3}, \tag{7.5.13}$$

Equation (7.5.13) gives the possibility to establish the connection between C and r_0. From one side:

$$\oint \vec{D}d\vec{S} = 4\pi e = 4\pi \frac{Cr_o^2}{e^3}, \tag{7.5.14}$$

From the other side

$$\frac{1}{8\pi}\int \vec{E}\vec{D}d\tau = m_0c^2 = \frac{Cr_0}{2e^2}, \tag{7.5.15}$$

It follows from (7.5.14) and (7.5.15):

$$\varepsilon_r = \frac{e^2}{r_0} = 2m_0c^2, C = \frac{e^4}{r_0^2} = 4m_0^2c^4 ,$$

Substituting, we obtain complete expression for the dielectric constant the expression:

$$\delta_V = \frac{4m_0^2c^4}{(2m_0c^2 - e\varphi)^2}, \tag{7.5.16}$$

which completely satisfies to the physical conditions of the electron-positron pair production: taking into account $\varphi \to 0$ we obtain $\delta_V \to 1$, while the energy $e\varphi = 2mc^2$, which is necessary for the appearance of pair, gives for the dielectric constant the infinite value, which corresponds to the breaking of photon to two parts. Both conclusions correspond precisely to physical statement of problems.

Following E. Bagge (Bagge, 1951), let us note also that in the general case the problem must be placed and be solved, taking into account all fields and their precise configuration. In this case the dielectric constant will be not the simple function of potential, but the tensor, determined by the electrical and magnetic fields of the intermediate photon and nucleus.

It was noted (Ivanenko and Sokolov, 1949), that various and, as was outlined, from the physical point of view, arbitrary variants of formal nonlinear electrodynamics lead to close values of coefficients, if to take into account, that the electron radius is equal to classical radius of electron.

It was also noted, that the basic defect of these theories, as well as of Mie theory, was the arbitrary choice of Lagrangian, which had no connection with the quantum theory, in particular, with Dirac theory, and did not take into account properties of electron, revealed by the last.

We will show that these theories can be considered as approach of the NTEP and that they are mathematically connected to the Dirac electron theory.

6.0. Nonlinear classical theories as approximations of NEPT

6.1. A connection of G. Mie's theory with Born-Infeld theory and NEPT

Let us show that the Mie Lagrangian, after some additions, can be represented as a Lagrangian that is similar to the Lagrangian of NEPT (and, consequently, QED).

Recall the Mie Lagrangian (7.2.1):

$$L_{Mi} = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - f\left(\pm \sqrt{A_\mu A^\mu}\right) \quad \text{or} \quad L_{Mi} = \frac{1}{8\pi}\left(E^2 - H^2\right) - f\left(\pm \sqrt{A_\mu A^\mu}\right)$$

As we know (Pauli, 1958; Sommerfeld, 1964), the charge density is not an invariant with respect to Lorentz transformations. However, the charge is an absolute invariant with respect to Lorentz transformations. It is also known that the square of the 4-potential, i.e. $I_3 = A_\mu A^\mu$, is an invariant with respect to Lorentz transformations. However, it is not an invariant with respect to gauge transformations.

It appears that the product of the square of charge and I_3 will be invariant with respect to both Lorentz and gauge transformations.

6.1.1. *Larmor – Schwarzschild's invariant*

According to (Pauli, 1958) and (Sommerfeld, 1964), R. Schwarzschild (Schwarzschild, 1903), introduced the value

$$S_w = \varphi - \frac{\vec{\upsilon}}{c} \cdot \vec{A}, \tag{7.6.1}$$

which he called "electrokinetic potential". He showed that this value, when multiplied by the density of charge, forms a relativistic invariant:

$$L' = \rho(\varphi - \frac{\vec{\upsilon}}{c} \cdot \vec{A}) = -\frac{1}{c} j_{\mu} \cdot A^\mu = \rho S_w, \tag{7.6.2}$$

where $j_\mu = \{ic\rho, \rho\vec{\upsilon}\}$ is a 4-current density, $A^\mu = \{i\varphi, \vec{A}\}$ is a 4-potential. Further, Schwarzschild forms the Lagrange function by integration with respect to space

$$\overline{L} = \frac{1}{2}\int \left(H^2 - E^2\right)dV + \int \rho(\varphi - \frac{\vec{\upsilon}}{c}\cdot\vec{A})dV \tag{7.6.3}$$

He then obtains the action function by integration with respect to time.

Thus, in four dimensions, the Lagrange function density (or Lagrangian) can be written as follows:

$$L = \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{c}j_\mu A^\mu, \tag{7.6.4}$$

while the Lagrange function will be:

$$\overline{L} = \frac{1}{4}\int F_{\mu\nu}F^{\mu\nu}d\tau - \frac{1}{c}\int j_\mu A^\mu d\tau, \tag{7.6.5}$$

(In note 10 to his book (Pauli, 1958) Pauli noted that before Schwarzschild, the same Lagrangian has been suggested by J.J. Larmor (Larmor, 1900)).

6.1.2. *The Mie variant of gauge-invariant theory*

We will now consider the radicand function in Mie's Lagrangian:

$$A_\mu^2 \equiv A_\mu A_\mu = -\varphi^2 + A_i^2, \tag{7.6.6}$$

Multiplying it by the square of an electron's charge, we obtain:

$$e^2 A_\mu^2 = -(e\varphi)^2 + (e\vec{A})^2, \tag{7.6.7}$$

Since the value:

$$\varepsilon_e = e\varphi, \tag{7.6.8}$$

is the energy of electron interaction, and the value:

$$p_{ei} = \frac{1}{c}eA_i, \tag{7.6.9}$$

is the momentum of electron interaction, we obtain from (7.6.9):

$$e^2 A_\mu^2 = -\varepsilon_e^2 + \left(c\vec{p}_e\right)^2, \tag{7.6.10}$$

Taking into account that $\left(\hat{\alpha}_o\varepsilon\right)^2 = \varepsilon^2$, $\left(\hat{\vec{\alpha}}\ \vec{p}\right)^2 = \vec{p}^2$, these expressions can also be written as:

$$e^2 A_\mu^2 = -\left(\varepsilon_e^2 - c^2 p_{ei}^2\right) = -\left(\left(\hat{\alpha}_0\varepsilon_e\right)^2 - c^2\left(\hat{\vec{\alpha}}\vec{p}_{e_i}\right)^2\right), \tag{7.6.11}$$

Using the above results, we will accept the following expression for the nonlinear part of Mie's Lagrangian $L_{Mie}^N = f\left(\pm\sqrt{A_\mu A^\mu}\right)$:

$$L_{Mie}^N = e\left(\pm\sqrt{\varphi^2 - c^2\vec{A}^2}\right), \tag{7.6.12}$$

Using the properties of Dirac matrices, it is easy to obtain the following decomposition:

$$\sqrt{e^2 A_\mu^2} = \mp\left(\hat{\alpha}_0\varepsilon_e \pm c\hat{\vec{\alpha}}\vec{p}_e\right), \tag{7.6.13}$$

which results in the following expression for the nonlinear part of the Lagrangian:

$$L_{Mie}^{Ne} = \mp\left(\hat{\alpha}_0 \varepsilon_e \pm c\hat{\vec{\alpha}}\vec{p}_e\right), \tag{7.6.14}$$

Taking into account that

$$\hat{\beta}\ mc^2 = \mp\left(\hat{\alpha}_0 \varepsilon_e \pm c\hat{\vec{\alpha}}\vec{p}_e\right), \tag{7.6.17}$$

we see that we can put the mass term of Dirac's equation into Mie's Lagrangian.

The usage of these expressions leads to Dirac's equations of electron and positron, and gives the basis to Weyl's attempt to interpret an asymmetry of both types of electricity not with regard to a mass, but with respect to the difference between the particle and antiparticle.

Also, this way we can relate the Mie's Lagrangian to Born-Infeld's Lagrangian. We can write using (7.6.11) the following:

$$\left(eA_\mu\right)^2 = -\left[\left(\int_0^{+\infty} u d\tau\right)^2 - c\left(\int_0^{+\infty} \vec{g} d\tau\right)^2\right] = -\left[\int_o^{+\infty}(u - c\vec{g})d\tau \int_0^{+\infty}(u + c\vec{g})d\tau\right], \tag{7.6.18}$$

Then, using the approximation in the previous chapter, we have:

$$\left(eA_\mu\right)^2 = -\left(u^2 - c^2\vec{g}^2\right)\left(\Delta\tau\right)^2, \tag{7.6.19}$$

Thus, in approximate form, the Mie's Lagrangian can be written in the following form:

$$L_{Mi} = \frac{1}{8\pi}\left(\mathrm{E}^2 - \mathrm{H}^2\right) + \left(\Delta\tau\right)\sqrt{\left(u^2 - c^2\vec{g}^2\right)}, \tag{7.6.20}$$

Recalling electromagnetic representation of the Fierz identity:

$$\begin{aligned}(8\pi)^2\left(u^2 - c^2\vec{g}^2\right) &= (8\pi)^2\left(u^2 - \vec{S}^2\right) = \\ &= \left(\vec{E}^2 + \vec{H}^2\right) - 4\left[\vec{E}\times\vec{H}\right]^2 = \left(\vec{E}^2 - \vec{H}^2\right) - 4\left(\vec{E}\cdot\vec{H}\right)^2\end{aligned}, \tag{7.6.21}$$

we can write an approximation of the Mie Lagrangian as

$$L_{Mi} = \frac{1}{8\pi}\left(\mathrm{E}^2 - \mathrm{H}^2\right) + 8\pi\ \Delta\tau\sqrt{\left(\vec{E}^2 - \vec{H}^2\right) - 4\left(\vec{E}\cdot\vec{H}\right)^2}, \tag{7.6.22}$$

As it is not difficult to see this approximation is similar to Born-Infeld Lagrangian. Moreover, in the following chapter we will show that Mie's Lagrangian can be transformed into the form of the Lagrangian of nonlinear field theory, which corresponds to the electron theory of NEPT (see the following chapter "NTEP. 8. Nonlinear quantum electron theory").

7.0. Nonlinear classical theories as approximations of NTEP. General statement of problem

As we has shown already, the condition of quantization in NTEP is caused by the introduction of relationships of the energy (according to Planck) and momentum (according to de Broglie) quantization. Therefore, the classical nonlinear theories, in which the requirement of noncommutativity is valid because of the specific nonlinearity (caused by cyclic transport of field vectors), can be considered as quantum theories if they are supplemented with this condition.

Obviously, the NTEP can include all possible invariants of electromagnetic field. Therefore, its Lagrangian can be written as some function of the field invariants $I_1 = \left(\vec{E}^2 - \vec{H}^2\right)$ and $I_2 = \left(\vec{E}\cdot\vec{H}\right)$:

$$L = f_L(I_1, I_2), \tag{7.7.1}$$

Apparently, the function f_L can have a specific form for each particular problem. However, an expansion of the function $f_L(I_1, I_2)$ in Taylor–Mac-Laurent power series must exist in every case. In general case, these expansions must contain the same set of terms that will differ only by constant coefficients, some of which can be equal to zero (see examples of such expansions in (Akhiezer and Berestetskii, 1965; Weisskopf, 1936; Schwinger, 1951).

Therefore, in general, the expansion will look as follows:

$$L_M = \frac{1}{8\pi}\left(\vec{E}^2 - \vec{B}^2\right) + L', \tag{7.7.2}$$

where

$$\begin{aligned} L' = \alpha \left(\vec{E}^2 - \vec{B}^2\right)^2 + \beta \left(\vec{E}\cdot\vec{B}\right)^2 + \gamma \left(\vec{E}^2 - \vec{B}^2\right)\left(\vec{E}\cdot\vec{B}\right) + \\ + \xi \left(\vec{E}^2 - \vec{B}^2\right)^3 + \zeta \left(\vec{E}^2 - \vec{B}^2\right)\left(\vec{E}\cdot\vec{B}\right)^2 + \ldots \end{aligned}, \tag{7.7.3}$$

is the part responsible for nonlinear interaction (here, $\alpha, \beta, \gamma, \xi, \zeta, \ldots$ are constant coefficients).

On the other hand, the Born-Infeld Lagrangian can be expanded into a series with small parameters $a^2E^2 << 1$ and $a^2B^2 << 1$, where $a^2 = \frac{1}{E_0^2}$:

$$L_{BI} = -\frac{1}{8\pi}\left(\vec{E}^2 - \vec{B}^2\right) + \frac{a^2}{32\pi}\left[\left(\vec{E}^2 - \vec{B}^2\right)^2 + 4\left(\vec{E}\cdot\vec{B}\right)^2\right] + \sum O(\vec{E}^2, \vec{H}^2), \tag{7.7.4}$$

where $\sum O(\vec{E}^2, \vec{H}^2)$ is the series remainder with the terms, containing vectors of electromagnetic field in powers greater than four. Obviously, at a large distance from the center of a particle (where the maximal field is), under the conditions $a^2E^2 << 1$ and $a^2B^2 << 1$, the terms of these series quickly converge. However, the terms with higher powers must be taken into account at a small distance from the particle center.

In the following chapter we will show that the first approximation of Lagrangian of NTEP can be represented in EM form as:

$$L_N = -\frac{1}{8\pi}\left(\vec{E}^2 - \vec{B}^2\right) + A\left[\left(\vec{E}^2 - \vec{B}^2\right) + 4\left(\left(\vec{E}\cdot\vec{B}\right)^2\right)\right], \tag{7.7.5}$$

where A is some constant. Thus, taking into account (7.7.4), we can write:

$$L_N \approx L_{BI}, \tag{7.7.6}$$

Then, within the framework of NTEP, an approximate solution of the electron equation will be similar to the solution of the Born-Infeld theory. Obviously, this assertion will be also correct for other nonlinear theories.

Having this, it is not difficult to answer the question, why the various variants of formal nonlinear electrodynamics lead to close values of coefficients: the expansions of nonlinear Lagrangian (7.7.3) are approximately similar for various variants and consequently they produce close results.

At the same time, since the Lagrangian and equations of NTEP completely coincide with the Lagrangian and equations of quantum electrodynamics, the Mie theory and its variant - the Born-Infeld theory, are closely related to Dirac's electron theory.

In the following chapter we will show that the Dirac electron equation can be expressed in form of nonlinear equation, which in the electromagnetic form is similar to classical nonlinear equations, considered above, and in the quantum form is similar to known nonlinear quantum field equations.