Chapter 5. The electron and positron equations (linear approach)

1.0. Introduction. Nonlinear non-Maxwellian electromagnetic field and Dirac equation

The Dirac equation is a relativistic quantum mechanical wave equation, which provides a description of elementary spin-½ particles - leptons. It is the relativistic generalization of the Schrödinger equation.

At present Dirac's equation is written in several identical forms. The usual one equation form is following:

$$\frac{1}{c}\frac{\partial \psi}{\partial t} + \hat{\vec{\alpha}} \cdot \vec{\nabla} \psi = i\hat{\beta}\frac{m_e c}{\hbar}\psi \, , \tag{5.1.1}$$

where ψ is the four-component wavefunction

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix}, \; \psi^+ = \begin{pmatrix} \psi_1 & \psi_2 & \psi_3 & \psi_4 \end{pmatrix}, \tag{5.1.2}$$

m_e is the rest mass of the electron, c is the speed of light, $\hat{\vec{\alpha}}$ and $\hat{\beta}$ are the α -set of $4{\times}4$ Dirac's matrices,

$$\hat{\alpha}_0 = \begin{pmatrix} \hat{\sigma}_0 & 0 \\ 0 & \hat{\sigma}_0 \end{pmatrix}; \; \hat{\vec{\alpha}} = \begin{pmatrix} 0 & \hat{\vec{\sigma}} \\ \hat{\vec{\sigma}} & 0 \end{pmatrix}; \; \hat{\beta} \equiv \hat{\alpha}_4 = \begin{pmatrix} \hat{\sigma}_0 & 0 \\ 0 & -\hat{\sigma}_0 \end{pmatrix}, \tag{5.1.3}$$

where $\hat{\sigma}_0, \hat{\vec{\sigma}}$ are Pauli's matrices. The most detailed form is the following:

$$\begin{cases} \frac{1}{c}\frac{\partial \psi_1}{\partial t} - \frac{\partial \psi_4}{\partial y} = -i\frac{m_e c}{\hbar}\psi_1 \\ \frac{1}{c}\frac{\partial \psi_4}{\partial t} - \frac{\partial \psi_1}{\partial y} = i\frac{m_e c}{\hbar}\psi_4 \\ \frac{1}{c}\frac{\partial \psi_2}{\partial t} + \frac{\partial \psi_3}{\partial y} = -i\frac{m_e c}{\hbar}\psi_2 \\ \frac{1}{c}\frac{\partial \psi_3}{\partial t} + \frac{\partial \psi_2}{\partial y} = i\frac{m_e c}{\hbar}\psi_3 \end{cases}, \tag{5.1.4}$$

Taking into account the fact that $r_C = \dfrac{\hbar}{m_e c}$ is the Compton wavelength, Dirac's equation can be rewritten in the form

$$\frac{1}{c}\frac{\partial\psi}{\partial t}+\vec{\alpha}\cdot\vec{\nabla}\psi=i\hat{\beta}\frac{1}{r_C}\psi\,, \tag{5.1.5}$$

The Dirac equation in the form, which is near to originally proposed by Dirac, is:

$$\left(\hat{\alpha}_o\hat{\varepsilon}+c\hat{\vec{\alpha}}\,\hat{\vec{p}}\right)\psi=\hat{\beta}\,m_e c^2\psi\,, \tag{5.1.6}$$

where $\hat{\varepsilon}=i\hbar\frac{\partial}{\partial t}$ and $\hat{\vec{p}}=-i\hbar\vec{\nabla}$ are the operators of energy and momentum.

Frequently the term from the right side of equation is called the "mass term" or "free term" of Dirac's equation.

At present time the explicitly covariant form of the Dirac equation (employing the Einstein summation convention) is often used:

$$-\hbar\hat{\gamma}^{\mu}\partial_{\mu}\psi=m_e c\psi\,, \tag{5.1.7}$$

where the γ-set of the Dirac matrices is used. But here $\hat{\gamma}_4$ is Hermitian, and the $\hat{\gamma}_k$ are anti-Hermitian, with the definition $\hat{\gamma}_4=\hat{\beta},\ \hat{\gamma}_k=\hat{\gamma}_4\hat{\alpha}_k\ \ (k=1,2,3)$; in this case (Madelung, 1943) $\hat{\gamma}_\mu\frac{\partial}{\partial x_\mu}$ is not Hermitian; instead the operator $\hat{\gamma}_4\hat{\gamma}_\mu\frac{\partial}{\partial x_\mu}$ is. Therefore more rationally to write Dirac's equation in the form (using $x_\mu=\{\vec{r},ict\}$ and Compton wavelength r_C): $\left(\sum_\mu\hat{\gamma}_4\hat{\gamma}_\mu\frac{\partial}{\partial x_\mu}+\hat{\gamma}_4\frac{1}{r_C}\right)\psi=0$, which is identical with (5.1.5) and (5.1.6).

Note (Fermi, 1960) that in order for the Dirac electron equation to obeys to the relativistic momentum-energy relation $\varepsilon^2-\vec{p}^2c^2-m^2c^4=0$, the any set of Dirac's matrices must satisfy the requirements:

$$\begin{aligned}&\hat{\delta}_1^2=\hat{\delta}_2^2=\hat{\delta}_3^2=\hat{\delta}_4^2=1\\&\hat{\delta}_\mu\hat{\delta}_\nu+\hat{\delta}_\nu\hat{\delta}_\mu=0\ \text{ for } \mu\neq\nu\end{aligned}\,, \tag{5.1.8}$$

where $\mu,\nu=1,2,3,4$. "*One finds that the lowest order matrices for which* (5.1.8) *can be fulfilled is the 4th. For order four there are many solutions that are essentially equivalent. One can prove that all the physical consequences of Dirac's equation do not depend on the special choice* (5.1.3) *of* $\hat{\alpha}_1,\hat{\alpha}_2,\hat{\alpha}_3,\hat{\beta}$. *They would be the same if a different set of four* 4x4 *matrices with the specifications* (5.1.8) *had been chosen. In particular it is possible by unitary transformation to interchange the roles of the four matrices, so that their differences are only apparent*".

The connection of contemporary theory of elementary particles with nonlinear electromagnetic (EM) theory is known many years. According to modern ideas (Ryder, 1985; Philipov, 1990), the observed substance of the Universe consists of photons, intermediate bosons, leptons and quarks. Besides electromagnetic interactions, there are strong and weak interactions. All of these interactions are described by the unified theory, which is a substantial generalization of Maxwell's theory. Instead of vectors of the usual electrical and magnetic fields $\vec{\mathrm{E}}$ and $\vec{\mathrm{B}}$, the modern theory contains several similar field vectors $\vec{\mathrm{E}}_i$ and $\vec{\mathrm{B}}_i$, and in a natural way, the waves of these vectors are strictly nonlinear.

The first such generalization of Maxwell's theory was made by C. Yang and R. Mills in 1954 (Nambu, 1982): "*The generalization of the Maxwell theory is the theory of the Yang-Mills fields or non-Abelian gauge fields. Its equations are nonlinear. In contrast to this, the equations of Maxwell are linear, in other words, Abelian*". A sufficiently detailed derivation of the Yang-Mills equations in the form of Maxwell's equations can be found in the book of Ryder (Ryder, 1985). According to one of its creators M.Gell-Mann, «*practically, the result of the field theory development was only the generalisation of the quantum electrodynamics*» (Gell-Mann, 1983)

It is necessary to note that the possibility of a formal representation of the Schrödinger and Dirac electron equations in a form of linear Maxwell equations was also mentioned in several articles and books (Schrödinger, 1927; Archibald, 1955; Akhiezer and Berestetskii, 1965; Koga, 1975; Campolattoro, 1980; Rodrigues, 2002). But up to now all these EM representations of Dirac's equation were examined as the random, curious coincidence of mathematical forms. In these studies no attempts were done the to examine the EM forms of Dirac's equation as quantum-mechanical equations, which describe massive fermions. The traditional view (Gsponer, 2002), "which consists in the fact that the particles of spin 1 and spin ½ belong to different irreducible representations of the Poincare group, so that no connection exists between the Maxwell and Dirac equations, describing the dynamics of particles" probably played a role in this.

Indeed, the Dirac equations cannot be equivalent to the classical linear equations of Maxwell, and a spinor cannot be equivalent to a vector. However, the unified theory of electromagnetic and weak interaction, described by the Yang-Mills theory, makes it possible to assume that a connection exists between the electromagnetic non-Maxwelian, nonlinear equations and the Dirac equations. In other words, we can assume that Dirac's equation is a quantized nonlinear non-Maxwelian electromagnetic equation, described in linear form.

In the present chapter we will derive the linear quantum equation of electron – the Dirac electron equation - and give the numerous proofs of its electromagnetic origin.

2.0. The equations of particles, generated from the breaking of intermediate massive photon

2.1. Derivation of equations

Our analysis of an initial stage of photoproduction of electron-positron pair, made in the previous chapter, shows that an intermediate photon can be divided into two parts, in order to produce an electron and a positron. Let us describe this process mathematically in order to find equations for these particles.

Let us begin with the equation of an intermediate photon (see equation (4.4.2) of the previous chapter):

$$\left(\hat{\varepsilon}^2 - c^2\hat{\vec{p}}^2 - K^2\right)\Phi' = 0, \tag{4.4.2}$$

Here as it follows from the previous sections, the term $K = \hat{\beta}\, m_p c^2$ corresponds to the tangent displacement current (4.3.9):

$$j_\tau = \omega_p \frac{1}{4\pi}\mathrm{E} \equiv \frac{m_p c^2}{\hbar}\frac{1}{4\pi}\mathrm{E} \equiv \frac{\upsilon_p}{r_p}\frac{1}{4\pi}\mathrm{E} \equiv \mathrm{K}\frac{c}{4\pi}\mathrm{E},$$

where $K = 1/r_p$ is the curvature of the fields' motion trajectory, $\upsilon_p = c$ is wave field velocity, $r_p = \dfrac{\hbar}{m_p c}$ is the curvature radius, $\omega_p = \dfrac{m_p c^2}{\hbar} = \dfrac{\upsilon_p}{r_p} \equiv cK$ is an angular velocity. Furthermore, here, $m_p c^2 = \varepsilon_p$ is intermediate photon own energy, where m_p is intermediate boson mass, corresponding to the energy ε_p.

Factorizing (4.4.2) and multiplying it on the left side by Φ'^{+}, we obtain:

$$\Phi'^{+}\left(\hat{\alpha}_o\hat{\varepsilon} - c\hat{\vec{\alpha}}\cdot\hat{\vec{p}} - K\right)\left(\hat{\alpha}_o\hat{\varepsilon} + c\hat{\vec{\alpha}}\cdot\hat{\vec{p}} + K\right)\Phi' = 0, \qquad (5.2.1)$$

or

$$\Phi'^{+}\left(\hat{\alpha}_o\hat{\varepsilon} - c\hat{\vec{\alpha}}\cdot\hat{\vec{p}} - \hat{\beta}m_p c^2\right)\left(\hat{\alpha}_o\hat{\varepsilon} + c\hat{\vec{\alpha}}\cdot\hat{\vec{p}} + \hat{\beta}m_p c^2\right)\Phi' = 0, \qquad (5.2.1')$$

Now, we can separate the intermediate photon equation (5.2.1) into two transformed waves, *advanced* and *retarded*, in order to obtain two new equations for the massive particles:

$$\left[\left(\hat{\alpha}_o\hat{\varepsilon} + c\hat{\vec{\alpha}}\,\hat{\vec{p}}\right) + \hat{\beta}\, m_p c^2\right]\psi = 0, \qquad (5.2.2')$$

$$\psi^{+}\left[\left(\hat{\alpha}_o\hat{\varepsilon} - c\hat{\vec{\alpha}}\,\hat{\vec{p}}\right) - \hat{\beta}\, m_p c^2\right] = 0, \qquad (5.2.2'')$$

where

$$\psi = \begin{pmatrix}\psi_1\\ \psi_2\\ \psi_3\\ \psi_4\end{pmatrix} = \begin{pmatrix}E_x\\ E_z\\ iH_x\\ iH_z\end{pmatrix}, \quad \psi^{+} = \begin{pmatrix}E_x & E_z & iH_x & iH_z\end{pmatrix}, \qquad (5.2.3)$$

is some new transformed EM wave function which appears after the intermediate photon breaking. Further, in this connection, we will conditionally name the equations (5.2.2) as *semi-photon equations*, and the passage from (4.4.2) to (5.2.2) as the *symmetry breaking* of an intermediate photon).

Now, we will analyze the peculiarities of equations (5.2.2). We can see that the latter are similar to the Dirac electron and positron equations. However, instead of electron mass m_e, equations (5.2.2) contain the intermediate photon mass m_p. The question is, what type of particles do equations (5.2.2) describe?

In the case of an electron-positron pair production, it must be $m_p = 2m_e$. So, we have from (5.2.2):

$$\left[\left(\hat{\alpha}_o\hat{\varepsilon} + c\hat{\vec{\alpha}}\,\hat{\vec{p}}\right) + 2\hat{\beta}\, m_e c^2\right]\psi = 0, \qquad (5.2.4')$$

$$\psi^{+}\left[\left(\hat{\alpha}_o\hat{\varepsilon} - c\hat{\vec{\alpha}}\,\hat{\vec{p}}\right) - 2\hat{\beta}\, m_e c^2\right] = 0, \qquad (5.2.4'')$$

and after the breaking of the intermediate photon, the non-charged massive particle must be divided into two charged massive semi-photons, the positively and negatively charged particles acquire electric fields. At the same moment each particle begins to move in the field of the other. In order to become independent (i.e. free) particles, the electron and positron must be drawn sufficiently far away from each other (Fig. 5.1):

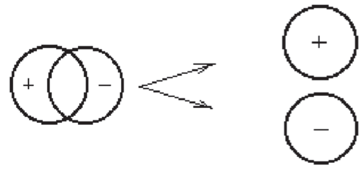


Fig. 5.1.

Therefore, the equations, which originate after the breaking-up of the intermediate photon, cannot be free positive and negative (electron and positron) particle equations, but they have to be the particle equations with an external field. In this case, the energy must be expended to the charged particles begin move apart. This is the energy that creates an electric field.

In fact, if the particles are combined, the system won't have an electric field (Fig. 5.1). At a very small distance, the particles will create the dipole field (see Fig. 5.2)

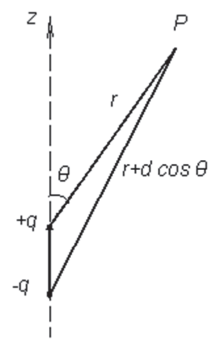


Fig. 5.2

At a distance much greater than the particle size, the positive and negative particles (plus and minus particles of Fig. 5.1) acquire full electric fields. It is known (Jackson, 1999) that the potential V_P of positive and negative charges at point P is defined as follows:

$$V_P = \frac{e}{4\pi}\left(\frac{1}{r} - \frac{1}{r + d\cos\theta}\right), \tag{5.2.5}$$

where $\pm e$ are dipole charges, d is the distance between the charges, and θ is the angle between the axes and radius-vector of the plus particle. When $d = 0$, we have $V_p = 0$. When $d \to \infty$, we obtain, as the limit case, the Coulomb potential for each free particle:

$$\lim_{d\to\infty} V_P = \frac{1}{4\pi}\frac{e}{r}, \tag{5.2.6}$$

Thus, during the breaking process, the particle charges appear. If the particles are moved apart to an infinite distance, the work to be done against the attraction forces is as follows:

$$\varepsilon_{rel} = \oint e V_P d\upsilon = \frac{1}{4\pi}\oint \frac{e^2}{r} d\upsilon, \tag{5.2.7}$$

The external field of particles defines the amount of work, so that the release energy is the field's production energy, and at the same time this is annihilation energy. Therefore, due to the law of energy conservation, this value of energy for each particle must be equal to $\varepsilon_{rel} = m_e c^2$.

So, equations (5.2.2) can be written in the following form:

$$\left[\left(\hat{\alpha}_0 \hat{\varepsilon} + c\hat{\vec{\alpha}}\,\hat{\vec{p}}\right) + \hat{\beta}\, m_e c^2 + \hat{\beta}\, m_e c^2\right]\psi = 0\,, \qquad (5.2.8')$$

$$\psi^+\left[\left(\hat{\alpha}_o \hat{\varepsilon} - c\hat{\vec{\alpha}}\,\hat{\vec{p}}\right) - \hat{\beta}\, m_e c^2 - \hat{\beta}\, m_e c^2\right] = 0\,, \qquad (5.2.8'')$$

Using a linear equation for the description of the law of energy conservation, we can write:

$$\pm\,\hat{\beta}\, m_e c^2 = -\varepsilon_{ex} - c\hat{\vec{\alpha}}\,\vec{p}_{ex} = -e\varphi_{ex} - e\hat{\vec{\alpha}}\,\vec{A}_{ex}\,, \qquad (5.2.9)$$

where "*ex*" means "external". Substituting (5.2.9) into (5.2.8), we obtain the Dirac equations with an external field:

$$\left[\hat{\alpha}_0\left(\hat{\varepsilon} \mp \varepsilon_{ex}\right) + c\hat{\vec{\alpha}}\cdot\left(\hat{\vec{p}} \mp \vec{p}_{ex}\right) + \hat{\beta}\, m_e c^2\right]\psi = 0\,, \qquad (5.2.10)$$

which at $d \to \infty$ gives the Dirac free - plus and minus - particle equations:

$$\left[\left(\hat{\alpha}_o \hat{\varepsilon} + c\hat{\vec{\alpha}}\,\hat{\vec{p}}\right) + \hat{\beta}\, m_e c^2\right]\psi = 0\,, \qquad (5.2.11')$$

$$\psi^+\left[\left(\hat{\alpha}_o \hat{\varepsilon} - c\hat{\vec{\alpha}}\,\hat{\vec{p}}\right) - \hat{\beta}\, m_e c^2\right] = 0\,, \qquad (5.2.11'')$$

Some interesting consequences follow from the above analysis:

1. an intermediate photon is not an absolutely neutral particle before the breaking-up, but a dipole; therefore, it must have a dipole moment (its experimental detection would give confirmation of the nonlinear theory).

2. the relationship (5.2.9) shows that in NTEP the mass is not equivalent to energy, but to a 4-vector of the energy-momentum; it follows from this that in NTEP the energy has a kinetic origin.

3. the following formula is valid within the framework of NTEP for the free term of the particle equation:

$$\pm\,\hat{\beta} m_e c^2 = -\varepsilon_{in} - c\hat{\vec{\alpha}}\vec{p}_{in} = -e\varphi_{in} - e\hat{\vec{\alpha}}\vec{A}_{in}\,, \qquad (5.2.12)$$

where "*in*" means "internal". In other words, values $(\varepsilon_{in}, \vec{p}_{in})$ describe the inner fields and values $(\varepsilon_{ex}, \vec{p}_{ex})$ of the external fields of the electron-positron particles. When we consider an electron particle from a large distance, the fields $(\varepsilon_{in}, \vec{p}_{in})$ act as the mass, then we will have linear Dirac equations of particles. Inside the electron, the term $(\varepsilon_{in}, \vec{p}_{in})$ is required for the detailed description of the inner fields of the particle, which characterize the self-interaction of the particle's parts (it is shown in a further chapter) that this term transforms to a nonlinear equation of the particle).

4. An important additional conclusion following from the above is that the charge, mass and interaction between the particles appear simultaneously in the process of the rotation transformation and division of the "linear" photon.

5. Subsequently we will show that, although the choice of Dirac's matrices does not influence the solutions of Dirac's equation, it has the physical sense.

2.2. Electromagnetic representation of Dirac's equations

Using the electromagnetic representation (5.2.3) of the semi-photon wave function ψ and the displacement electric tangential currents (4.3.7) from previous chapter $\vec{j}^{\,e}_{dis} = \dfrac{1}{4\pi}\omega_p \mathrm{E}\cdot\vec{\tau}$, we obtain an electromagnetic form of equations (5.2.11).

$$\begin{cases} \frac{1}{c}\frac{\partial E_x}{\partial t} - \frac{\partial H_z}{\partial y} = -i\frac{4\pi}{c} j_x^e \\ \frac{1}{c}\frac{\partial H_z}{\partial t} - \frac{\partial E_x}{\partial y} = i\frac{4\pi}{c} j_z^m \\ \frac{1}{c}\frac{\partial E_z}{\partial t} + \frac{\partial H_x}{\partial y} = -i\frac{4\pi}{c} j_z^e \\ \frac{1}{c}\frac{\partial H_x}{\partial t} + \frac{\partial E_z}{\partial y} = i\frac{4\pi}{c} j_x^m \end{cases}, \quad (5.2.13') \qquad \begin{cases} \frac{1}{c}\frac{\partial E_x}{\partial t} + \frac{\partial H_z}{\partial y} = -i\frac{4\pi}{c} j_x^e \\ \frac{1}{c}\frac{\partial H_z}{\partial t} + \frac{\partial E_x}{\partial y} = i\frac{4\pi}{c} j_z^m \\ \frac{1}{c}\frac{\partial E_z}{\partial t} - \frac{\partial H_x}{\partial y} = -i\frac{4\pi}{c} j_z^e \\ \frac{1}{c}\frac{\partial H_x}{\partial t} - \frac{\partial E_z}{\partial y} = i\frac{4\pi}{c} j_x^m \end{cases}, \quad (5.2.13'')$$

Let us note that for the symmetry we included in the equations the displacement magnetic tangential currents (4.3.7) $\vec{j}_{dis}^{\,m} = \frac{1}{4\pi}\omega_p \mathrm{H}\cdot\vec{\tau}$. It is known that the existence of the magnetic current $\vec{j}^{\,m}$ does not contradict to quantum theory (see Dirac's theory of a magnetic monopole (Dirac, 1931)). In case of the plane polarized wave (see previous chapters), the magnetic currents are equal to zero (but not for other polarizations, as we will see further).

According to the results of the previous chapter of the book, the current terms of Dirac's equation are own electrical and magnetic currents of electron. Let us write down them in different identical representations:

$$\begin{aligned} j_j^e &= \frac{1}{2}\frac{1}{4\pi}\omega_p E_l \equiv \frac{1}{4\pi}\frac{m_e c^2}{\hbar} E_l \equiv \frac{1}{2}\frac{c}{4\pi}\frac{1}{r_p} E_l \equiv \frac{1}{2}\frac{c}{4\pi}\mathrm{K}E_l \\ j_j^m &= \frac{1}{2}\frac{1}{4\pi}\omega_p H_l \equiv \frac{1}{4\pi}\frac{m_e c^2}{\hbar} H_l \equiv \frac{1}{2}\frac{c}{4\pi}\frac{1}{r_p} H_l \equiv \frac{1}{2}\frac{c}{4\pi}\mathrm{K}H_l \end{aligned}, \qquad (5.2.14)$$

where the subscript l in general case are $l = (x, y, z)$.

Using the currents' representation with electrical conductivity $\omega_p = 2\omega_e = 2m_e c^2/\hbar$ (which has the CGSE units of inverse second (s^{-1})), we obtain the following electromagnetic form of Dirac equations:

$$\begin{cases} \frac{1}{c}\frac{\partial E_x}{\partial t} - \frac{\partial H_z}{\partial y} = -i\frac{1}{c}\omega_e E_x \\ \frac{1}{c}\frac{\partial H_z}{\partial t} - \frac{\partial E_x}{\partial y} = i\frac{1}{c}\omega_e H_z \\ \frac{1}{c}\frac{\partial E_z}{\partial t} + \frac{\partial H_x}{\partial y} = -i\frac{1}{c}\omega_e E_z \\ \frac{1}{c}\frac{\partial H_x}{\partial t} + \frac{\partial E_z}{\partial y} = i\frac{1}{c}\omega_e H_x \end{cases}, \quad (5.2.15') \qquad \begin{cases} \frac{1}{c}\frac{\partial E_x}{\partial t} + \frac{\partial H_z}{\partial y} = -i\frac{1}{c}\omega_e E_x \\ \frac{1}{c}\frac{\partial H_z}{\partial t} + \frac{\partial E_x}{\partial y} = i\frac{1}{c}\omega_e H_z \\ \frac{1}{c}\frac{\partial E_z}{\partial t} - \frac{\partial H_x}{\partial y} = -i\frac{1}{c}\omega_e E_z \\ \frac{1}{c}\frac{\partial H_x}{\partial t} - \frac{\partial E_z}{\partial y} = i\frac{1}{c}\omega_e H_x \end{cases}, \quad (5.2.15'')$$

As it is easily see, the currents' forms (5.2.13) and (5.2.15) of the electron equation is similar to Maxwell-Lorentz's equations with complex fields, which is frequently used in the classical theory of electromagnetic waves (especially in the theory of ultra-high frequency EM waves). Moreover, beginning from O. Heaviside, magnetic currents are introduced into the complex Maxwell's equations for the symmetry of equations and facilitation of problem solutions. The essential difference between equations (5.2.15) and the Maxwell-Lorentz equations is that the electrical conductivity ω_p contains the Planck constant..

Using the currents' representation with m_e, we obtain the other electromagnetic form of Dirac equations:

$$\begin{cases} \frac{1}{c}\frac{\partial E_x}{\partial t} - \frac{\partial H_z}{\partial y} = -i\frac{m_e c}{\hbar}E_x \\ \frac{1}{c}\frac{\partial H_z}{\partial t} - \frac{\partial E_x}{\partial y} = i\frac{m_e c}{\hbar}H_z \\ \frac{1}{c}\frac{\partial E_z}{\partial t} + \frac{\partial H_x}{\partial y} = -i\frac{m_e c}{\hbar}E_z \\ \frac{1}{c}\frac{\partial H_x}{\partial t} + \frac{\partial E_z}{\partial y} = i\frac{m_e c}{\hbar}H_x \end{cases}, (5.2.16') \quad \begin{cases} \frac{1}{c}\frac{\partial E_x}{\partial t} + \frac{\partial H_z}{\partial y} = -i\frac{m_e c}{\hbar}E_x \\ \frac{1}{c}\frac{\partial H_z}{\partial t} + \frac{\partial E_x}{\partial y} = i\frac{m_e c}{\hbar}H_z \\ \frac{1}{c}\frac{\partial E_z}{\partial t} - \frac{\partial H_x}{\partial y} = -i\frac{m_e c}{\hbar}E_z \\ \frac{1}{c}\frac{\partial H_x}{\partial t} - \frac{\partial E_z}{\partial y} = i\frac{m_e c}{\hbar}H_x \end{cases}, (5.2.16'')$$

which means that the electron has electromagnetic origin and its mass is generated as circular electromagnetic field. Thus, the derivation of Dirac's equation on the basis of quantized electromagnetic equations proves Lorentz's hypothesis about the electromagnetic origin of the electron. (Subsequently we will prove the same relatively to all leptons and hadrons).

Note also that the forms of source term (5.2.14) with curvature radius or curvature

$$j_j^e = \frac{1}{2}\frac{c}{4\pi}\frac{1}{r_p}E_l \equiv \frac{1}{2}\frac{c}{4\pi}\mathrm{K}E_l,\ j_j^m = \frac{1}{2}\frac{c}{4\pi}\frac{1}{r_p}H_l \equiv \frac{1}{2}\frac{c}{4\pi}\mathrm{K}H_l,$$

the electron equation is similar to Gilbert-Einstein gravitational equation, but here the mass-energy term appear due to curvilinearity of field, not of space-time).

At the same time there are essential differences between the classical and quantum form of electromagnetic equations.

- In the nonlinear theory the quantization of the energy-momentum of electromagnetic field is introduced, whereas in the classical theory this limitation is absent.

- In the nonlinear theory the complex forms have literal physical sense: they describe the fields' rotation (in other words, they ensure passage from linear forms to nonlinear forms of theory). In the classical theory complex forms are convenient for calculation, but as final results is considered only one of the projections of rotary motion - the so-called, real part of the complex number. Thus, the complex form of the equations of nonlinear theory is the generalization of the linear theory of Maxwell-Lorentz.

- In the classical theory there are no magnetic currents and charges. At the same time, as we will show subsequently, in the theories of neutrino and quarks, magnetic currents actually exist. But they are not found out of the particles because of the mutual compensation for magnetic currents inside the particles and therefore they do not create magnetic monopoles. On one hand, this explains, why the introduction of magnetic currents does not disrupt the classical results, but on the other hand, it explains, why magnetic currents and charges in it are absent.

2.3. The “field diagram” of the photoproduction of the electron-positron pair

Using the results, obtained above and in the previous chapter, we can give Feynman's diagram of the electron-positron pair photoproduction (see the figs of 4.1 and 4.4 of previous chapter) more real physical sense. Thus we correspond to each element of this diagram the graphical representation of wave fields of the corresponding elementary particles. Let us name this diagram “**field diagram**” of interaction of particles.

Thus, we can conditionally represent the transformation of photon fields during the process of electron-positron pair photoproduction as following *field diagram* (Fig. 5.3):

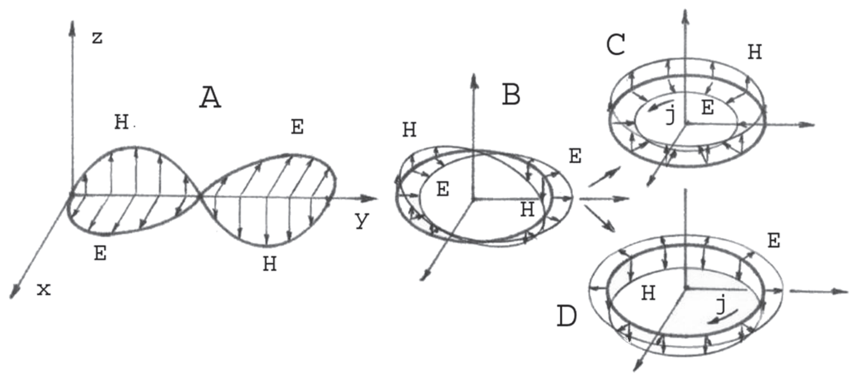


Fig. 5.3.

The part A here represents a “linear” photon (that obeys the linear equation); part B depicts the intermediate massive boson (“nonlinear” photon); and parts C and D represent the electron and positron.

We can make some conclusions from this field diagram without calculations.

a) It is clear that the intermediate photon breakdown process, according to Fig. 5.3, corresponds to the process of particle-antiparticle pair production. Closed currents $j = \rho_e \vec{\upsilon}$ (where in our case $\upsilon = c$), that emerge in this case, create electrical charges of particle $e = \int_\tau \rho_e d\tau$, where τ is volume. This means that the field diagram Fig. 5.3 actually describes the generation of the charged particles.

Note that in the electromagnetic interpretation for understandable reasons the free term of Dirac’s equation can be called “a source” of electric field of electron. The electric charge appears here as the gauge coupling constant, like as in Standard Model theory.

b) It follows from Fig. 5.3 that parts *C* and *D* (two ‘semi-photons’) contain currents of opposite directions. Thus, we can assume, that *the cause of an intermediate photon breaking is a mutual repulsion of oppositely directed currents*.

It is not difficult to see that in both parts *C* and *D* (electron and positron) magnetic and electric forces appear, which have reciprocal directions that is necessary to ensure the equilibrium of particle.

c) Note also that the “daughter” semi-photons *C* and *D*, i.e. electron and positron, are completely anti-symmetric, and one cannot be transformed into the other by any co-ordinate transformation (if this transformations are not accompanied by the change of fields’ directions).

d) From the above also follows that the angular momentum of the intermediate photon is equal to:

$$\sigma_p = p_p \cdot r_p = 2m_e c \cdot \frac{\hbar}{2m_e c} = 1\hbar ,$$

In accordance with the law of conservation of angular momentum, we have $\sigma_s^+ + \sigma_s^- = \sigma_p$, where σ_s^+, σ_s^- are the spins of the plus and minus semi-photons (i.e. of the electron and positron). Then, we obtain the known value for the angular momentum of the electron:

$$\sigma_s = \frac{1}{2}\sigma_p = \frac{1}{2}\hbar,$$

e) It is interesting that *both the semi-photons' and intermediate photons' radii must be the same.* Since $\sigma_s = p_s \cdot r_s$, where r_s is the semi-photon (electron) radius, and $p_s = m_e c$ is the inner semi-photon (electron) linear momentum, we have:

$$r_s = \frac{\sigma_s}{p_s} = \frac{1}{2}\frac{\hbar}{m_e c} = \frac{\hbar}{2m_e c} = r_p,$$

Thus, *the torus size of the intermediate photon doesn't change after its breaking*.

f) Using this result, we can show that during the breaking *the angular velocity (angular frequency) also does not change*:

$$\omega_s = \frac{c}{r_s} = \frac{2m_e c^2}{\hbar} = \omega_p.$$

g) Linear velocity of rotation of fields' of intermediate boson and electron are equal to speed of light:

$$\upsilon = \omega_p \cdot r_p = \omega_s \cdot r_s = c$$

h) Magnetic moment of ring electron accordingly with definition is $\mu_s = I \cdot S_I$, where I is electron ring current and S_I is the current ring square. In our case we have $I = q_s \frac{\omega_s}{2\pi} = q_s \frac{1}{2\pi}\frac{2m_e c^2}{\hbar}$, $S_I = \pi\, r_s^{\,2} = \pi \left(\frac{\hbar}{2m_e c}\right)^2$. Using these formulae, we find:

$$\mu_s = \frac{1}{2}\frac{q\hbar}{2m_e c},$$

If we put $q_s = e$, the value is equal to half of the experimental value of the magnetic momentum of the electron. Taking into account the Thomas's precession (Thomas, 1926) we obtain the experimental value of the electron magnetic momentum.

The breakdown of the intermediate photon makes it possible to explain some fundamental experimental results:

1) *the origin of the law of charge conservation*: a total *electric charge* of an isolated system remains constant regardless of the changes within the system itself. Since there are the same numbers of plus and minus half-periods of photons in nature, the sum of all created or destroyed charges must be equal to zero.

2) *the difference between the positive and negative charges*: this difference follows from the asymmetry of fields and the difference in directions of tangent currents of semi-photons after the pair production.

3) *the explanation of "Zittertbewegung"*: E. Schroedinger showed in his well-known articles about the relativistic electron (Schrödinger, 1929;1930;1931a;1931b;1932; Bethe, 1964) that the rest electron has a special inner motion "Zitterbewegung", which has a frequency $\omega_Z = \frac{2m_e c^2}{\hbar}$, an amplitude $r_Z = \frac{\hbar}{2m_e c}$, and velocity of light $\upsilon = c$. The attempts to explain this motion within the framework of QED did not produce results. However, if the electron is a semi-photon, then we receive a simple explanation of Schroedinger's analysis.

4) *the difference between the bosons and fermions*: the bosons contain an even number, while the fermions contain an odd number of semi-photons.

5) in nonlinear theory the problem of the *infinite electron energy* does not exist, because the space distribution of particle field is continuous.

6) The characteristic feature of quantum theory is the non-commutativity of canonical variables. This is easily explained by the fact that non-commutativity appears as the consequence of motion of vectors along the curvilinear trajectory.

7) In this case the optics-mechanical analogy of Hamilton, from which all quantum theory began, finds its substantiation (actually NTEP is the optics of nonlinear waves, which simultaneously can describe the motion of the material objects).

8) The occurrence of Pauli's matrixes, which describe the rotation in classical mechanics in 2D space in the Dirac electron and positron equations, receives an explanation as well as the occurrence of Gell-Mann matrixes in the Yang-Mills equations, which describe the rotation in 3D space.

9) The necessity of a nucleus electromagnetic field receives an explanation: it serves as the medium with the big refraction number, leaning on which the light string bents (obviously this requirement is identical to the requirement of conservation of system momentum).

10) The formed EM particles are simultaneously both waves and particles (i.e. the wave - particle dualism is inherent to them).

11) Since the twirled photon has integer spin (i.e., it is a boson), but the twirled semi-photons have spin half (i.e,. they are fermions), we automatically receive an explanation of division of all elementary particles into bosons and fermions.

12) It is easy to see, that the fig. 4 reflects the process of spontaneous symmetry breakdown of an initial photon and occurrence of mass of elementary particles, which have place in presence of a nucleus field, as some catalyst of the reaction (playing here the role of Higgs boson).

13) If in the theory of static spherical electron of Lorentz classical theory there are no the electromagnetic forces, capable to constrain the repulsion of electron parts from each other and it is necessary to enter Poincare's forces of non electromagnetic origin, then it is easy to see, that here, owing to presence of a current, there is the magnetic part of full Lorentz force directed against electrostatic forces of repulsion and counterbalancing them. Thus, such electron does not demand the introduction of extraneous forces of an unknown origin and is stable.

3.0. Analysis of the free electron's equation solution from EM point of view

According to the above results, an electromagnetic form of a solution of the free electron Dirac equation must be a transformed electromagnetic wave.

As we saw above, for the appearance of electric current and, as a result, charge, it is necessary that the electric vector moved in the trajectory plane of the motion of electromagnetic wave.

If this supposition is correct, then two solutions must exist for the y-direction of a photon:

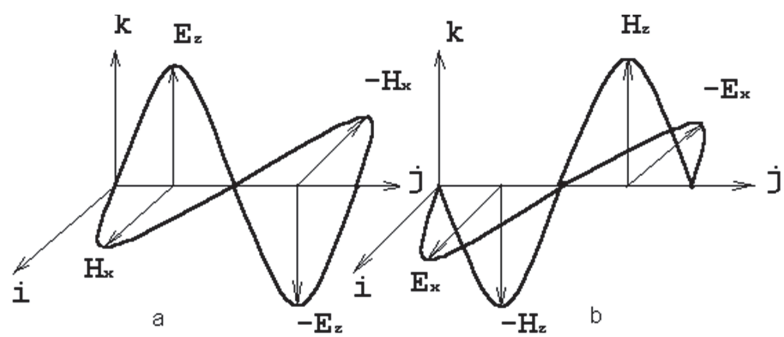
1) for the wave rotated around the OZ-axis development of electromagnetic theory

$$^{oz}\psi = \begin{pmatrix} E_x \\ 0 \\ 0 \\ iH_z \end{pmatrix} = \begin{pmatrix} \psi_1 \\ 0 \\ 0 \\ \psi_4 \end{pmatrix}, \tag{5.3.1}$$

2) for the wave rotated around the OX-axis

$$^{ox}\psi = \begin{pmatrix} 0 \\ E_z \\ iH_x \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ \psi_2 \\ \psi_3 \\ 0 \end{pmatrix}, \tag{5.3.2}$$

For clarity, we will depict both orientations of the original photon in one figure 5.4, as it corresponds to classical electrodynamics:



a Fig.5.4 b

The ψ- functions (5.3.1) and (5.3.2), as solutions of equations (5.2.11), must have the same expressions as solutions of the Dirac's equation for an electron (Schiff, 1955). Let us analyze solutions of the Dirac's wave equation for an electron from the NTEP point of view.

It is known (Schiff, 1955) that the solution of the Dirac free electron's equation (5.2.1) has a form of a plane wave:

$$\psi_j = B_j \exp\left(-\frac{i}{\hbar}(\varepsilon t - \vec{p}\vec{r})\right), \tag{5.3.3}$$

where $j = 1, 2, 3, 4$; $B_j = b_j e^{i\phi}$; amplitudes b_j are numbers, and ϕ is the initial wave phase. Functions (5.3.3) are eigenfunctions of energy-momentum operators, where ε and $\vec{p}$ are the energy-momentum eigenvalues. Here, for each $\vec{p}$, the energy ε has either positive or negative values according to equation, representing the law of energy-momentum's conservation $\varepsilon_{\pm} = \pm\sqrt{c^2\vec{p}^2 + m^2c^4}$.

We have two linear-independent sets of four orthogonal normalizing amplitudes for ε_+:

$$1)\, B_1 = -\frac{cp_z}{\varepsilon_+ + mc^2},\ B_2 = -\frac{c\left(p_x + ip_y\right)}{\varepsilon_+ + mc^2},\ B_3 = 1,\ B_4 = 0, \tag{5.3.4}$$

$$2)\ B_1 = -\frac{c\left(p_x - ip_y\right)}{\varepsilon_+ + mc^2},\ B_2 = \frac{cp_z}{\varepsilon_+ + mc^2},\ B_3 = 0,\ B_4 = 1, \tag{5.3.5}$$

Accordingly, for ε_- :

$$3)\ B_1 = 1,\ B_2 = 0,\ B_3 = \frac{cp_z}{-\varepsilon_- + mc^2},\ B_4 = \frac{c\left(p_x + ip_y\right)}{-\varepsilon_- + mc^2}, \tag{5.3.6}$$

$$4)\, B_1 = 0,\ B_2 = 1,\ B_3 = \frac{c\left(p_x - ip_y\right)}{-\varepsilon_- + mc^2},\ B_4 = -\frac{cp_z}{-\varepsilon_- + mc^2}, \tag{5.3.7}$$

Each of these four solutions (Schiff, 1955) can be normalized by multiplying it by normalization factor:

$$\kappa = \left[1 + \frac{c^2 \vec{p}^2}{\left(\varepsilon_+ + m_e c^2\right)^2}\right]^{-\frac{1}{2}},$$

which gives $\psi^+\psi = 1$.

Let us discuss these results.

1) The existence of two linear independent solutions corresponds to two independent orientations of electromagnetic wave vectors, and gives a unique logical explanation for this fact.

2) Since $\psi = \psi(y)$, we have $p_x = p_z = 0$, $p_y = mc$, and we obtain for field vectors the following: for the "positive" energy from (5.2.4) and (5.2.5) :

$$B_+^{(1)} = \begin{pmatrix} 0 \\ b_2 \\ b_3 \\ 0 \end{pmatrix} \cdot e^{i\phi},\ B_+^{(2)} = \begin{pmatrix} b_1 \\ 0 \\ 0 \\ b_4 \end{pmatrix} \cdot e^{i\phi}, \tag{5.3.8}$$

For the "negative" energy, we obtain from (5.2.6) and (5.2.7):

$$B_-^{(1)} = \begin{pmatrix} b_1 \\ 0 \\ 0 \\ b_4 \end{pmatrix} \cdot e^{i\phi},\ B_-^{(2)} = \begin{pmatrix} 0 \\ b_2 \\ b_3 \\ 0 \end{pmatrix} \cdot e^{i\phi}, \tag{5.3.9}$$

which corresponds exactly to (5.3.1) and (5.3.2).

3) Calculating correlations between the components of the field vectors. Substituting $\phi = \frac{\pi}{2}$ for $\varepsilon_+ = mc^2$ and $\varepsilon_- = -mc^2$, we obtain accordingly:

$$B_+^{(1)} = \begin{pmatrix} 0 \\ \frac{1}{2} \\ i \cdot 1 \\ 0 \end{pmatrix}, \; B_+^{(2)} = \begin{pmatrix} -\frac{1}{2} \\ 0 \\ 0 \\ i \cdot 1 \end{pmatrix}, \tag{5.3.10}$$

$$B_-^{(1)} = \begin{pmatrix} i \cdot 1 \\ 0 \\ 0 \\ -\frac{1}{2} \end{pmatrix}, \; B_-^{(2)} = \begin{pmatrix} 0 \\ i \cdot 1 \\ \frac{1}{2} \\ 0 \end{pmatrix}, \tag{5.3.11}$$

Obviously, the imaginary unit in these solutions indicates that the field vectors $\vec{E}$ and $\vec{H}$ are mutually orthogonal.

Also, we see that the amplitude of an electric field is two times less than the magnetic field amplitude. This fact demonstrates that the electromagnetic field's values, which correspond to solution of Dirac equation, are different in comparison to the fields of a linear wave of Maxwell's theory, where $\vec{E} = \vec{H}$. (We can show that this result provides an electron's stability).

4) It is easy to show that in electromagnetic form, the solution of Dirac's equation is a standing wave. Actually, whenever the wave rotates on a circle, we have $\vec{p} \perp \vec{r}$ and, therefore, $\vec{p} \cdot \vec{r} = 0$. Then, instead of (5.2.2), we obtain the standing wave:

$$\psi_j = b_j \exp\left(-\frac{i}{\hbar}\varepsilon t\right), \tag{5.3.12}$$

5) According to Euler's formula $e^{i\varphi} = \cos\varphi + i\sin\varphi$, the solution of Dirac's equation (5.3.12) describes a circle. This corresponds to our theory.

6) Let us calculate the normalization factor κ , substituting $p = mc, \varepsilon = mc^2$:

$$\kappa = \left(\frac{5}{4}\right)^{-\frac{1}{2}}, \tag{5.3.13}$$

Now, we will compare it with the normalization factor, which is obtained from the electromagnetic representation of the theory. In view of the fact that the electric field is twice as small as the magnetic field, the energy density of a semi-photon will be equal to:

$$W_{s-ph} = \frac{1}{8\pi}\left(E_{s-ph}^2 + H_{s-ph}^2\right) = \frac{1}{8\pi}\left[\left(\frac{1}{2}H_{s-ph}\right)^2 + H_{s-ph}^2\right] = \frac{1}{8\pi}\frac{5}{4}H_{s-ph}^2, \tag{5.3.14}$$

Using the non-normalized expression for the wave function:

$$\psi_j = B_0 B_j e^{i(\vec{k}\vec{r} - \omega t)} = B_0 \begin{pmatrix} 0 \\ i\frac{1}{2} \\ 1 \\ 0 \end{pmatrix} e^{i(\vec{k}\vec{r} - \omega t)}, \tag{5.3.15}$$

(where B_0 is some constant, generally dimensional), and the Hermitian-conjugate function:

$$\psi_j^+ = B_0 B_j^+ e^{-i\left(\vec{k}\vec{r}-\omega t\right)} = B_0 \begin{pmatrix} 0 & -i\frac{1}{2} & 1 & 0 \end{pmatrix} e^{-i\left(\vec{k}\vec{r}-\omega t\right)}, \tag{5.3.16}$$

for the field energy, we obtain the following expression:

$$W = \frac{1}{8\pi}\psi_j^+\psi_j = \frac{1}{8\pi}\cdot\frac{5}{4}\cdot B_0^2\,, \tag{5.3.17}$$

which precisely corresponds to the result of quantum theory.

4.0. An equation of the electron field motion

Within the framework of NEPT, the 4-vector $\left\{e\varphi,\ \frac{e}{c}\vec{A}\right\}$ is a 4-vector of the energy-momentum of curvilinear wave field $\{\varepsilon, \vec{p}\}$. Therefore, a well-known analysis of the Dirac's electron equation in external field can be used to analyze equations of the inner semi-photon field, if we use the following:

$$\frac{e}{c}\vec{A} = \vec{p}, \quad e\varphi = \varepsilon\,, \tag{5.4.1}$$

As it is known (Akhiezer and Berestetskii, 1965; Schiff, 1955), an equation of the electron's motion in the external field can be found from the next operator equation that has Poisson brackets:

$$\frac{d\hat{O}}{dt} = \frac{\partial\hat{O}}{\partial t} + \frac{1}{i\hbar}\left(\hat{O}\hat{\mathrm{H}} - \hat{\mathrm{H}}\hat{O}\right), \tag{5.4.2}$$

where $\hat{O}$ is a physical value operator whose variation we want to find, and $\hat{\mathrm{H}}$ is the Hamilton operator of Dirac's equation, which in general case is equal to (Akhiezer and Berestetskii, 1965; Bethe, 1964; Schiff, 1955):

$$\hat{\mathrm{H}} = -c\hat{\vec{\alpha}}\,\hat{\vec{P}} - \hat{\beta}\,mc^2 + \varepsilon\,, \tag{5.4.3}$$

where $\hat{\vec{P}} = \hat{\vec{p}} - \vec{p}_p$ is a full momentum of a semi-photon.

Let us note that since the mass is an integral characteristic of an electron, it cannot participate in the internal motion of matter of electron, and must be assumed equal to zero.

If we assume $\hat{O} = \vec{r}$, then we obtain on the basis (5.4.2) and (5.4.3):

$$\dot{\vec{r}} = \frac{d\vec{r}}{dt} = c\hat{\vec{\alpha}}$$

which means that the eigenvalue of electron velocity is equal to $\pm c$. In quantum theory (Fock, 1932), "a question about does have this paradox result physical sense, it remains open". V. Fock and others "proposed to see here the defect of Dirac's theory". In NEPT, this result will be completely precise if the velocity of fixed electron is not the speed of a whole particle, but the speed of the electron's wave field along a curvilinear trajectory.

For $\hat{O} = \hat{\vec{P}}$, we have the motion equation:

$$\frac{d\hat{\vec{P}}}{dt} = \left[-grad\left(e\varphi\right) - \frac{e}{c}\frac{\partial \vec{A}}{\partial t}\right] + \frac{e}{c}\left[\vec{\upsilon} \times rot\,\vec{A}\right], \tag{5.4.4}$$

Substituting $\vec{\upsilon} = c\hat{\vec{\alpha}}$, where $\vec{\upsilon}$ - velocity of the electron matter, we obtain the Newton's law for the motion of electrical charge:

$$\frac{d\vec{P}}{dt} = e\vec{E} + \frac{e}{c}\left[\vec{\upsilon} \times \vec{H}\right] = f\,, \tag{5.4.5}$$

where f is the Lorentz's force. Since for a motionless electron $\frac{d\hat{\vec{P}}}{dt} = 0$, then it follows from (5.4.5):

$$\left(\frac{\partial\,\vec{p}}{\partial\,t} + grad\;\varepsilon\right) - \left[\vec{\upsilon} \times rot\;\vec{p}\right] = 0\,, \tag{5.4.6}$$

Assuming that (5.4.6) is correct for any small volume of the particle $\Delta\tau$, we can pass to the densities of EM fields of electron:

$$\vec{g} = \frac{d\vec{p}}{d\tau},\; u = \frac{d\varepsilon}{d\tau}\,, \tag{5.4.7}$$

Then we obtain the equation of matter motion of a semi-photon:

$$\left(\frac{\partial\,\vec{g}}{\partial\,t} + grad\;u\right) - \left[\vec{\upsilon} \times rot\;\vec{g}\right] = 0\,, \tag{5.4.8}$$

Let us analyze a physical meaning of (5.4.8). Let's remember the motion equation of an ideal liquid in the form of Lamb's-Gromek's equation (Lamb, 1931). In the case when the external forces are absent, this equation is as follows:

$$\left(\frac{\partial\,\vec{g}_l}{\partial\,t} + grad\;u_l\right) - \left[\vec{\upsilon} \times rot\;\vec{g}_l\right] = 0\,, \tag{5.4.9}$$

where u_l, $\vec{g}_l$ are the energy and momentum density of an ideal liquid.

Comparing (5.4.8) and (5.4.9), it is not difficult to see their mathematical identity. An interesting conclusion follows from this result: *the EM wave's field motion may be interpreted as a motion of ideal liquid.*

Additionally, according to (5.4.5) and (5.4.6), we have from (5.4.9)

$$\frac{\partial\,\vec{g}}{\partial\,t} + grad\;u = \vec{f}\,, \tag{5.4.10}$$

As it is known, the term $\left[\vec{\upsilon} \times rot\;\vec{g}\right]$ in (5.4.9) is responsible for the centripetal acceleration. Probably, we have the same in (5.4.8). If the "photon liquid" moves along the ring with radius r, then the angular velocity ω of the ring motion of field is tied to $rot\;\vec{\upsilon}$ by expression:

$$rot\vec{\upsilon} = 2\vec{\omega} = 2\omega\,\vec{e}_z\,, \tag{5.4.11}$$

and the centripetal acceleration is

$$\vec{a}_n = \frac{1}{2}\vec{\upsilon} \times rot\,\vec{\upsilon} = \frac{\upsilon^2}{r}\vec{e}_r = c\omega\,\vec{e}_r\,, \tag{5.4.12}$$

where $\vec{e}_r$ is a unit radius-vector, $\vec{e}_z$ is a unit vector of *OZ*-axis. As a result, the equation (5.4.25) has a form of Newton's law:

$$\rho\,\vec{a}_n = \vec{f}\,, \tag{5.4.13}$$

The results (5.4.5) and (5.4.13) can be considered (Shiff, 1955) as a representation of the Ehrenfest theorem for the motion of electron's inner fields. These results show also that within the framework of NEPT the electron is a stable object.

Relatively to centripetal acceleration a_n it is possible to come to an additional interesting conclusion.

As it follows from the previous paragraph, the total acceleration of the EM field of convoluted semi-photon is centripetal it, i.e., has only a radial component.

Using expression (8.2) for describing the change in quantum values (Fock, 1932), it is not difficult to obtain the expression for the centripetal acceleration in NTEP. According to determination, the total acceleration is written as follows: $\vec{a} = \dfrac{d^2\vec{r}}{dt^2}$. Since $\dfrac{dr}{dt} = c\hat{\vec{\alpha}}$, we have

$$\vec{a} = \frac{d}{dt}\left(c\hat{\vec{\alpha}}\right) = \frac{c}{i\hbar}\left[\hat{\vec{\alpha}}, \hat{H}\right], \tag{5.4.14}$$

Substituting the expression for the Hamiltonian and taking into account that the spin matrices $\hat{\vec{\sigma}}$ are connected with the $\hat{\vec{\alpha}}$-matrices by the relationships:

$$\begin{cases} \hat{\sigma}_1 = -i\hat{\alpha}_2\hat{\alpha}_3 \\ \hat{\sigma}_2 = -i\hat{\alpha}_3\hat{\alpha}_1\,, \\ \hat{\sigma}_3 = -i\hat{\alpha}_1\hat{\alpha}_2 \end{cases} \tag{5.4.15}$$

we will obtain after calculations the formula, which expresses the connection of acceleration with pulse and spin of twisted semi-photon:

$$\hat{\vec{a}} = \frac{2c^2}{\hbar}\left[\hat{\vec{P}}_{ph} \times \hat{\vec{\sigma}}\right], \tag{5.4.16}$$

It is not difficult to see that (5.4.16) gives the right direction for the acceleration (see Fig. 5.5)

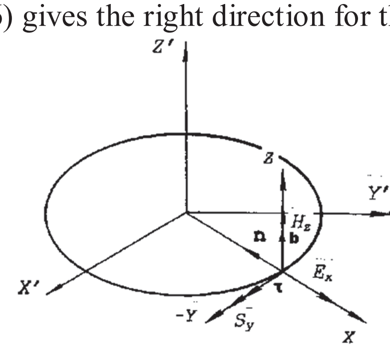


Fig. 5.5

Actually, since the direction of $\hat{\vec{P}}_{ph}$ coincides with the direction of Poynting's vector (in the figure: $-\vec{S}_y$), and $\hat{\vec{\sigma}}$ is directed along the rotational axis, the acceleration is directed along a radius towards center. It, naturally, follows from this that the acceleration is perpendicular to the speed of the motion of field $\vec{a}_n \perp \vec{\upsilon}$ ($\upsilon = c$), and therefore the scalar product of acceleration to the speed is equal to zero: $\vec{a}_n \cdot \vec{\upsilon} = 0$.

Thus, according to our calculations, the velocity of fixed electron (in reality, of its fields) is equal to the speed of light and is directed tangentially toward the circular path, and the product of its acceleration to the speed is equal to zero. These special features of the motion of fixed electron can explain some results of the 4-dimensional kinematics of the theory of relativity. There we have two mysterious results (Landau and Lifshitz, 1977): 1) the square of 4- speed (in units of the speed of light) is equal to one; and 2) the product of 4-speed for the 4-acceleration is equal to zero. From the comparison with the obtained above results we can assume that the kinematics of the theory of relativity correspond to the kinematics of the theory of elementary particles. Subsequently we will examine this fact in more detail.

5.0. The physical and mathematical differences between vector and spinor wave functions

From the theory it follows that the wave functions of photon and intermediate boson are vectors, whereas the wave functions of electron is spinor. What are the physical and mathematical differences between these two objects? We can see that these differences appear during the breaking of an intermediate photon and the production of an electron-positron pair because of the change of the transformation properties of electromagnetic fields.

Let us attempt to describe the differences between the electromagnetic fields $\{E'_x, E'_z, H'_x, H'_z\}$ of the vector Φ'-wavefunction of an intermediate photon and electromagnetic fields $\{E_x, E_z, H_x, H_z\}$ of the spinor ψ -wavefunction of semi-photon (i.e. electron) from different points of view.

5.1. The topological differences

The spinor's invariant transformation has the form (Ryder, 1987; Gottfried & Weisskopf, 1984):

$$\psi' = U\psi , \qquad (5.5.1)$$

where

$$U(\vec{n}\,\theta) = \cos\frac{1}{2}\theta - i\vec{n}\cdot\vec{\sigma}'\sin\frac{1}{2}\theta , \qquad (5.5.2)$$

is the transformation operator, $\vec{n}$ is a unit vector on some axis, θ is the angle of rotation around this axis, and $\vec{\sigma}' = (\sigma_x', \sigma_y', \sigma_z')$ is a spin vector. The rotation matrix (5.5.2) possesses a remarkable property. If the angle of rotation is $\theta = 2\pi$ (that is returning to the initial reference system), then $U = -1$, instead of $U = 1$, as we would otherwise expect. In other words, the state function of a system with a half spin in the usual three-dimensional space returns to its initial state only after turning by 4π .

This result can be explained only if we assume that the *electron and positron are plus and minus half-cycles(halfs of wave period) of an intermediate photon, and, therefore, their fields need to be rotated twice to return to the initial state.* This confirms the fact that the semi-photon is a transformed half-cycle of the photon. Thus in the framework of nonlinear electromagnetic representation of theory we can name the spinor wave function as "electromagnetic spinor".

5.2. The differences from the tensor representation point of view

The Dirac spinor cannot be equivalent to vector. But already in the early articles it was noted that between them there is a specific correspondence. Therefore, previously (A. Sokolov, D. Ivanenko, 1952; Goenner, 2004) the spinors were also called "half-vectors" (or tensors of half rank), and the equation of Dirac - "half-vector equation". The spinors obtained this name because of the comparison of their transformation law with the transformation law of the vectors. Let us describe briefly this comparison (in detail see bibliography).

In Dirac's theory the 4-vectors are some bispinor constructions. For example, let us find transformation law for the 4-vector of energy-momentum dencity $\{u, \vec{g}\}$ of electron

$$u = \psi^+ \hat{\alpha}_0 \psi = \psi^+ \psi , \quad \vec{g} = \psi^+ \hat{\vec{\alpha}} \psi , \tag{5.5.3}$$

where ψ is a spinor (or bispinor), $\hat{\alpha}_0, \hat{\vec{\alpha}}$ are Dirac's matrices. Using the correlation (5.2.3) we can obtain the electromagnetic representation of above relations:

$$\psi^+ \hat{\alpha}_o \psi = \vec{E}^2 + \vec{H}^2 = 8\pi u , \tag{5.5.4}$$

$$\psi^+ \hat{\vec{\alpha}} \psi = 2\vec{E} \times \vec{H} = 8\pi \, c\vec{g} , \tag{5.5.5}$$

which actually correspond to relations of electromagnetic theory.

The rotation transformation law in the xt-plane (the Lorentz transformation) for Dirac wave function can be write in the form:

$$\psi' = \left(ch\frac{\gamma}{2} - \alpha_1 sh\frac{\gamma}{2} \right)\psi , \quad \psi = \left(ch\frac{\gamma}{2} + \alpha_1 sh\frac{\gamma}{2} \right)\psi' , \tag{5.5.6}$$

where γ is the imaginary angle of rotation. Let us find now transformation law for the 4- vector of energy-momentum:

$$u = \psi^+ \psi , \quad g_x = \psi^+ \hat{\alpha}_1 \psi , \tag{5.5.7}$$

In this case, using (5.5.6), we can write:

$$\psi = \Omega_{xt} \psi' ; \quad \psi^+ = \psi'^+ \Omega_{xt}^+ , \tag{5.5.8}$$

where

$$\Omega_{xt} = \Omega_{xt}^+ = ch\frac{\gamma}{2} + \hat{\alpha}_1 sh\frac{\gamma}{2} , \tag{5.5.9}$$

are operators of the transformation of the Dirac wave function. Then, substituting (5.5.8) into (5.5.7) we obtain, using (5.5.9):

$$u = u' ch\gamma + g'_x \, sh\gamma ,$$

$$g_x = g'_x \, ch\gamma + u' sh\gamma ,$$

as it must be in this case.

Since

$$\psi^+ \psi = \psi'^+ \Omega_{xt}^2 \psi = \psi'^+ Q_{xt} \psi' , \tag{5.5.10}$$

the operator

$$Q_{xt} = ch\gamma + \hat{\alpha}_1 sh\gamma , \tag{5.5.11}$$

can be considered as the transformation operator of the vectors. Thus, we have:

$$\Omega_{xt} = \sqrt{Q_{xt}} \,, \tag{5.5.12}$$

Consequently, the Dirac wavefunction is not tensor in the usual sense; however, the observed physical quantities, which are quadratic combinations from the wavefunctions, are real tensors.

According to above we can name the transformation law for the wavefunctions as the half-vector transformation law, and the equation, to which they obey - the "half-vector equation"

Thus, there are no foundations to doubts that the special (non-Maxwellian) electromagnetic field can organize objects with the spin ½.

5.3. The differences from a point of view of the group theory

It is known that the vector fields of a photon are transformed as elements of the group O(3). At the same time, the spinor fields of the Dirac equation are transformed as elements of the group SU(2), and (Ryder, 1987) two spinor transformations correspond to one vector transformation.