Chapter 3. The photon theory

1.0. Basic ideas and the results of the contemporary quantum theory of the photon

The quantum theory of the photon is part of the theory of quantum electrodynamics (QED) (Akhiezer and Berestetskiy, 1969). Let us briefly look at its fundamental notions, which are necessary to understand the place of photon theory in the proposed nonlinear field theory.

1.1. Foundations of the theory of photons in QED

There are several approaches to constructing the quantum theory of photons, but they all originate from the same source, Maxwell's equations, and give us identical results.

We will follow some of the earliest ideas, which are not less consistent than later approaches. At the same time, these earliest ideas allow us to unify different physical representations of elementary particles.

The simplest task of quantum field theory (Akhiezer and Berestetskiy, 1969) is to describe the state of a free particle. In this case, the wave function of the particle is a field in three-dimensional space. Let us examine how the wave function of the photon is introduced, and its wave equation is built.

Within the framework of QED (Akhiezer and Berestetskii, 1969; Gottfried, and Weisskopf, 1984), it is natural to consider the Maxwell equations as field equations which describe the quantum mechanical state of photons or a photon system. It is not difficult to show that this assumption, along with Planck's relationship $\varepsilon = \hbar\omega$, is sufficient to build the theory of the photon and its interaction with other particles (here, ε is the photon energy, ω is an angular frequency, $\hbar$ is the Planck constant). Within this approach, the quantized electric and magnetic fields are the wave function of a photon.

Maxwell equations with the sources that take into account the effect of media, are called Maxwell-Lorentz equations. They are the most general equations of electrodynamics (Jackson, 1999; Tonnelat, 1959). According to the first postulate of NTEP the Maxwell equations without the sources are the initial equation of our theory:

$$\frac{1}{c}\frac{\partial \vec{E}}{\partial t} - rot\vec{H} = 0, \qquad (3.1.1)$$

$$\frac{1}{c}\frac{\partial \vec{H}}{\partial t} + rot\vec{E} = 0, \qquad (3.1.2)$$

$$div\vec{E} = 0, \qquad (3.1.3)$$

$$div\vec{H} = 0, \qquad (3.1.4)$$

where $\vec{E}$ and $\vec{H}$ are the vectors of strength of electrical and magnetic fields.

It is not difficult to show (Longmire, 1963) that with respect to time-dependent problems Maxwell-Lorentz equations are in some sense more than complete. Equations (3.1.1) and (3.1.2) determine electrical and magnetic fields for any moment of time based on their initial values. This proves that it is possible to examine the equations (3.1.3) and (3.1.4) as initial conditions.

Also, note the following. In the general case, all four vector equations of Maxwell are used, but in the case of harmonic waves (which will be basic in our theory) the system of these equations is reduced to first two. In this case, the other two equations, which are the generalization of Gauss' laws for electrical and magnetic fields, follow from the two previous ones.

We know that it is possible to obtain the wave equations from Maxwell equations, in which wave functions are vectors of the electromagnetic (EM) field:

$$\begin{cases} \Delta\vec{\mathrm{E}} - \dfrac{1}{c^2}\dfrac{\partial^2\vec{\mathrm{E}}}{\partial t^2} = 0 \\ \Delta\vec{\mathrm{H}} - \dfrac{1}{c^2}\dfrac{\partial^2\vec{\mathrm{H}}}{\partial t^2} = 0 \end{cases}, \qquad (3.1.5)$$

Thus, the Maxwell theory predicts the existence of electromagnetic waves.

Let us examine how the transition from the classical equations of the EM wave to quantum equations of the photon is accomplished within the framework of QED

1.2. Contemporary point of view to the nature of EM waves

According to contemporary ideas, photons are the elements (quanta) of an EM wave, and an EM wave is a flow of photons. Similar to all other elementary particles, photons simultaneously have both wave and corpuscular properties. These properties do not contradict each other.

We will briefly list the results obtained and verified by experiments in the classical and quantum theories.

1.2.1. *Classical EM description of a wave*

An electromagnetic wave is the propagated in the space disturbance of interconnected electrical and magnetic fields. According to Fourier's theorem, an electromagnetic wave of any form can be decomposed into elementary components - the harmonic EM waves of the type $\vec{F} = \vec{F}_0 \sin\left(\omega t - \vec{k}\cdot\vec{r}\right)$ or, in a complex form: $\vec{F} = \vec{F}_0 e^{i\left(\omega t - \vec{k}\cdot\vec{r}\right)}$, where $\vec{F} = \left\{\vec{\mathrm{E}}, \vec{\mathrm{H}}\right\}$ is any vector of an electrical $\vec{\mathrm{E}}$ or a magnetic $\vec{\mathrm{H}}$ field, $\vec{F}_0$ is the wave amplitude, ω is an angular frequency, t is time, $\vec{k}$ is a wave vector, $\vec{r}$ is the radius-vector of the propagating wave.

An electromagnetic wave has an energy density equal to $u = \dfrac{1}{8\pi}\left(\vec{\mathrm{E}}^2 + \vec{\mathrm{H}}^2\right) = \dfrac{1}{4\pi}\vec{\mathrm{E}}^2 = \dfrac{1}{4\pi}\vec{\mathrm{H}}^2$. The energy flow density (or intensity) I is the EM energy transferred by the wave in one unit of time through a unit of surface. The energy flow density can be presented in the form of Poynting's vector: $\vec{S} = \dfrac{c}{4\pi}\left[\vec{\mathrm{E}} \times \vec{\mathrm{H}}\right]$. Its absolute value can be expressed as follows: $I = u \cdot c$, where c is the velocity of light.

The vector of the energy flow density allows us to introduce the density of the mechanical momentum of an EM wave as follows: $\vec{g} = \dfrac{\vec{S}}{c^2}$. The presence of the linear momentum makes it possible to conditionally introduce the concept of density of EM mass as the value $\rho_{EM} = \dfrac{g}{c} = \dfrac{u}{c^2}$. The mass-energy relationship of EM field follows from this as $u = \rho_{EM} \cdot c^2$, which agrees with SRT, and leads to a conclusion about the EM origin of matter.

It is interesting that the use of these transformations for the wave train or wave packet (Becker, 1933) leads to a formula analogous to Planck's formula. Namely, the energy of the wave packet becomes equal to the following: $\varepsilon_z = C \cdot \omega$, where C is a constant with the dimensionality of action, which is not difficult to associate with Planck's constant. This analysis also agrees with the conclusion about the EM origin of matter.

1.2.2. *Quantum description of the EM wave*

According to Planck and Einstein (Akhiezer and Berestetskiy, 1969; Frauenfelder and Henley, 1974), a monochromatic electromagnetic (EM) wave consists of N monoenergetic photons, each of which has zero mass, an energy ε, a momentum $\vec{p}$, and a wavelength λ. It is remarkable that all photon characteristics are in one-to-one relationships, that is $\varepsilon = \hbar\omega$, $\vec{p} = \hbar\vec{k}$, $\varepsilon = cp$, $\varepsilon = c\hbar k$, $\vec{k} = 2\pi\,\vec{k}^0/\lambda$ (here, $\vec{k}^0$ is a unit vector of the wave vector).

The number of photons in an EM wave is such that their total energy is equal to $\varepsilon_{tot} = N\varepsilon = N\hbar\omega$. Photons are bosons and coherent photons are capable of condensing in an EM wave (for example, in the form of a laser beam), which has a specific frequency.

The relationship between the classical and quantum descriptions is given by the probabilistic interpretation of an EM wave as a photon system.

1.2.3. *A probabilistic interpretation of the EM wave*

Obviously, the energy density of N photons with a given frequency ω, which are in a volume τ at a given moment, is equal to $u = \dfrac{N\hbar\omega}{\tau}$. On the other hand, $u = \dfrac{1}{4\pi}\vec{\mathrm{E}}^2 = \dfrac{1}{4\pi}\vec{\mathrm{H}}^2$. The comparison of these expressions leads to the conclusion that the number of photons per unit of volume is proportional to the square of the strength of the EM wave field: $\dfrac{N}{\tau} \sim \mathrm{E}^2$.

If the average density of photons during a fixed time interval is large, then two different interpretations of the energy density – the wave and corpuscular ones – lead to the same observed values for the energy density. The difference is caused by the fact that in the first interpretation we consider this energy as the energy of EM wave, stored in the fields, while in the other case we look at it as the total energy of the photons that are located in a volume.

The general relationship between the wave and corpuscular views is explained by the examination of an EM wave of very small intensity. When the value E^2 is so small that the average number of photons (proportional to E^2) per unit of volume becomes less than one, then the value E^2 is interpreted as the probability P of finding a photon in the given volume τ. In this case, we can write $P \sim \mathrm{E}^2\tau$; $P/\tau \sim \mathrm{E}^2$ is called the probability density of the photon distribution.

Based on this approach, Max Born eventually proposed the probabilistic interpretation of the wave function of an electron in quantum mechanics.

Indication of the physical sense of the probabilistic interpretation of wave function consists in the comparison of theory of radiation of electromagnetic waves by atoms in the classical and quantum description. As is known (Physical encyclopedia 1962. V. 2, p.129. Radiation theory) "*the sequential quantum theory of emission is built on basis of QED. But if we do not consider the small effects (so-called radiation corrections), then the quantum theory of radiation leads to the same results as the classical theory. In this case it sufficient to replace in the formulas of the classical amplitudes the electrical and magnetic moments of different order with the appropriate matrix elements of electrical* *or magnetic moments, which are taken with respect to the wave functions of the quantum system states, between which occurs the transition*".

In this case from the EM energy, radiated per unit time, it is possible to pass to the probabilities of radiation of the photon of the same frequency by division of EM energy by the photon energy.

As we will show further, namely such relative value determines the probabilistic interpretation of wave function, both in the case of the electrons and other elementary particles.

1.3. The wave function of the photon

Recall that a photon is the quantum of an EM wave, and in classical theory of the EM wave, it is described by a wave equation of the second order, which is a consequence of Maxwell's equations. Here, the EM field vectors $\vec{\mathrm{E}}$ and $\vec{\mathrm{H}}$ are wave functions.

Thus, from a physical point of view, the most simple and consistent way to introduce the wave function of a photon is through quantization of the EM field vectors $\vec{\mathrm{E}}$ and $\vec{\mathrm{H}}$ (Akhiezer and Berestetskiy, 1969).

Since the electric vector can be expressed through the magnetic vector, we can use only the generalized electric vector $\vec{\mathcal{E}}(\vec{r},t)$.

1.3.1. *Wave function of a photon in a momentum space*

According to (Akhiezer and Berestetskiy, 1969; Levich, Myamlin, Vdovin, 1973), the wave function of a photon can be introduced as follows.

Quanta of light or photons are the elementary particles, as the distinctive special feature of which serves the fact that their rest mass is equal to zero Therefore they always move with speed c in vacuum. This fact leads to some important features in the method of describing their behavior. Specifically, the connection between energy and momentum of photon is defined by the general formula

$$\varepsilon = cp = \hbar ck\,, \qquad (3.1.6)$$

If we replace the momentum of photon with an operator, then the operator of energy in the momentum representation takes the form $\hat{H} = c\hat{p} = \hbar c\hat{k}$, (3.1.7)Respectively it is possible to write the Schroedinger equation in the momentum representation

$$i\hbar\frac{\partial\psi}{\partial t} = \hat{H}\psi_p\,, \qquad (3.1.8)$$

where ψ_p is the wave function of photon in the momentum representation.

Operator $\hat{H}$ is connected with the photon energy with the general formula

$$\varepsilon = \int \psi_p^+ \hat{H}\psi_p d\vec{p} = \hbar c\int \psi_p^+ \hat{k}\psi_p d\vec{p}\,, \qquad (3.1.9)$$

From the other side, it is possible to consider that the photon is adequate to the EM field, which exists in the entire space. Its energy is:

$$\varepsilon = \int \frac{\vec{\mathrm{E}}^2 + \vec{\mathrm{H}}^2}{8\pi} d\tau = \frac{1}{4\pi}\int \vec{\mathrm{E}}^2 d\tau\,, \qquad (3.1.10)$$

It is natural to identify the photon energy with the energy of EM field. Both field vectors satisfy Maxwell's equations, which are reduced to the form

$$\Delta\vec{F} - \frac{1}{c^2}\frac{\partial^2 \vec{F}}{\partial t^2} = 0\,, \tag{3.1.11}$$

where the vector $\vec{F}$ corresponds to the electrical and magnetic vectors: $\vec{F} = \{\vec{\mathrm{E}}, \vec{\mathrm{H}}\}$.

Decomposing $\vec{F}$ into the Fourier integral

$$\vec{F}(\vec{r},t) = \int \vec{F}(\vec{k},t) e^{i\vec{k}\vec{r}} d\vec{k}\,, \tag{3.1.12}$$

we obtain for Fourier's components $\vec{F}(\vec{k},t)$ the equation

$$\frac{\partial^2 \vec{F}(\vec{k},t)}{\partial t^2} + k^2 \vec{F}(\vec{k},t) = 0\,, \tag{3.1.13}$$

or

$$\left[\frac{\partial \vec{F}(\vec{k},t)}{\partial t} - ikF(\vec{k},t)\right]\left[\frac{\partial \vec{F}(\vec{k},t)}{\partial t} + ikF(\vec{k},t)\right] = 0\,, \tag{3.1.14}$$

Since EM field is real quantity, the condition must be satisfied:

$$\vec{F}(\vec{k}) = \vec{F}(-\vec{k}), \tag{3.1.15}$$

Let us introduce instead of Fourier's component $\vec{F}(\vec{k},t)$ the new function $\vec{f}(\vec{k},t)$ by the specific relationships

$$\begin{cases} \vec{F}(\vec{k},t) = N(k)\left[\vec{f}(\vec{k},t) + \vec{f}^{\,+}(-\vec{k},t)\right] \\ \dot{\vec{F}}(\vec{k},t) = -ikN(k)\left[\vec{f}(\vec{k},t) + \vec{f}^{\,+}(-\vec{k},t)\right] \end{cases}, \tag{3.1.16}$$

where N is the proportionality factor. The dot is used in order to mark the differentiation with respect to time.

It is not difficult to see that condition (3.1.15) is satisfied automatically with this representation of $\vec{F}(\vec{k},t)$.

Substituting values $\vec{F}(\vec{k},t)$ and $\dot{\vec{F}}(\vec{k},t)$ in (3.1.14), we come to two equations:

$$\begin{cases} i\dfrac{\partial \vec{f}}{\partial t} = k\vec{f} \\ -i\dfrac{\partial \vec{f}^{\,+}}{\partial t} = k\vec{f}^{\,+} \end{cases}, \tag{3.1.17}$$

Let us emphasize that the equations (3.1.17) are nothing else except for another writing form of Maxwell's equations.

Multiplying (3.1.17) on $\hbar$, we obtain

$$\begin{cases} i\hbar\dfrac{\partial \vec{f}}{\partial t} = p\vec{f} \\ -i\hbar\dfrac{\partial \vec{f}^{\,+}}{\partial t} = p\vec{f}^{\,+} \end{cases}, \tag{3.1.18}$$

As we see, function $\vec{f}(\vec{k},t)$ satisfies the equation (3.1.18), which by its form is identical with the Schroedinger equation. If we replace p with operator $\hat{H}$, then function $\vec{f}(\vec{k},t)$ should be identified with the wave function of photon in k - representation.

Proportionality factor N, which remained, until now, arbitrary, can be determined from the comparison of (3.1.9) and (3.1.10).

Substituting in (3.1.10) the decomposition (3.1.16), we obtain after the appropriate conversions:

$$\varepsilon = 4\pi^2 \int N^2(k) f(\vec{k}) f^+(\vec{k}) d\vec{k}\,, \qquad (3.1.19)$$

With $N = \sqrt{\frac{ck\hbar}{4\pi^2}}$ the energy of EM field and photon energy are proved to be identical. Thus, in the k - representation photon is described by the wave function

$$\psi(\vec{k},t) = \vec{f}(\vec{k},t)$$

moreover the following condition is satisfied

$$\int \vec{f}^+ \vec{f} d\vec{k} = 1\,, \qquad (3.1.20)$$

In this case Maxwell's equations for the EM field of simple harmonic wave is proved to be identical with the Schroedinger equations for the separate photon. Introducing explicit dependence on the time, it is possible to write

$$\psi(\vec{k},t) = f_0(\vec{k})\exp(-i\omega t) = f_0(\vec{k})\exp\left(-\frac{i}{\hbar}\varepsilon t\right), \qquad (3.1.21)$$

Let us emphasize that since Maxwell's equations are relativistic-invariant, the Schroedinger equation for the photon is also relativistic-invariant

The vectors of the EM field $\vec{\mathrm{E}}$ and $\vec{\mathrm{H}}$ are considered as classical wave functions. After representing $\vec{\mathrm{E}}$ and $\vec{\mathrm{H}}$ in the form of a Fourier integral or sum, we can pass to complex wave vectors $\vec{f}_{\vec{k}}$. These vectors correspond to the wave vector $\vec{k}$, and represent a certain generalization of vectors of the EM field.

Using the EM wave equation, it is easy to show that $\vec{f}_{\vec{k}}$ also satisfies a similar wave equation. Representing this wave equation as a product of two equations for the advanced and retarded waves, we obtain a system of two linear equations with respect to the wave function $\vec{f}_k$.

For the quantization of the function $\vec{f}_{\vec{k}}$, we postulate that in a monochromatic solution of the function $\vec{f}_{\vec{k}}$, namely $\vec{f}_{\vec{k}} = \vec{f}_0(\vec{k})e^{-i\omega t}$, the frequency ω obeys Planck's formula $\varepsilon = \hbar\omega$; hence it can be represented through the wave number $\omega = ck$.

In the case of one photon, the function $\vec{f}_{\vec{k}}$ satisfies the condition $\int \vec{f}^*_{\vec{k}} \vec{f}_{\vec{k}} d^3k = 1$, which can be considered to be a normalization condition. We can then interpret the square of wave function $\left|\vec{f}_{\vec{k}}\right|^2$ as the probability density of the photon with a linear momentum, equal to $\hbar\vec{k}$. In this case, the expressions for the energy and momentum of a photon, represented through the wave function $\vec{f}_{\vec{k}}$, acquire the meaning of the usual quantum mechanical expressions for the average values of energy and momentum. Thus, according to the above, the function $\vec{f}_k$ can be interpreted as a quantum wave function of the photon in the *momentum space*.

It is quite clear, based on results in (Akhiezer and Berestetskiy, 1969), that the equation of this function is equivalent to the system of Maxwell's equations. For this reason, it is possible to consider Maxwell's equation as the equation of one photon (Gersten, 2001).

1.3.2. *Insurmountable difficulty in the introduction of photon wave function in a coordinate representation. Space nonlocality of photon*

However, the attempt to introduce the function of a photon in a *coordinate representation* revealed an insurmountable difficulty (Akhiezer and Berestetskiy, 1969; Bialynicki-Birula, 1994). According to the analysis of Landau and Peierls (Landau and Peierls, 1930), and later Cook (Cook, 1982a; 1982b) and Inagaki (Inagaki, 1994), the *wave function of a photon is nonlocal by its nature*.

After performing an inverse Fourier transform of the above function $\vec{f}_k$, we obtain (Landau and Peierls, 1930): $\frac{1}{(2\pi)^3}\int \vec{f}_k e^{i\vec{k}\vec{r}} d^3k = \vec{f}(\vec{r},t)$. It seems that it is possible to define $\vec{f}(\vec{r},t)$ as the wave function of photon in a coordinate representation. In fact, because of the normalization condition for $\vec{f}_k$, the function $\vec{f}(\vec{r},t)$ can also be normalized by the usual method: $\int\left|\vec{f}(\vec{r},t)\right|^2 d^3x = 1$. However, the value $\left|f(\vec{r},t)\right|^2$ can no longer be interpreted as the probability density of finding the photon at a given point of space.

It is known that the presence of a photon can be established only by its interaction with charges. This interaction is determined by values of the EM field vectors $\vec{\mathrm{E}}$ and $\vec{\mathrm{H}}$ at a given point. However, these fields are not determined by the value of the wave function $\vec{f}(\vec{r},t)$ at the same point, because they are defined by its values in the entire space.

In fact, components of the Fourier field vectors expressed by f_k contain the factor $\sqrt{k}$. This can formally be written in the following form:

$$\vec{\mathcal{E}}(\vec{r},t) = \sqrt[4]{-\Delta}\,\vec{f}(\vec{r},t), \tag{3.1.22}$$

where Δ is the Laplace operator. However, $\sqrt[4]{-\Delta}$ is an integral operator, and therefore the relationship between the $\vec{\mathcal{E}}(\vec{r},t)$ and $\vec{f}(\vec{r},t)$ is not local but integral. In other words, $\vec{f}(\vec{r},t)$ is not determined by the field value $\vec{\mathrm{E}}(\vec{r},t)$ at the same point, but depends on the field distribution in a certain region, *which has the size on the order of wavelength*.

This means that the *localization of a photon in a smaller region is impossible* and, therefore, the *concept of a probability density distribution that could be used to find the photon at a fixed point of space does not make sense*.

With respect to this fact, it is remarkable that the accuracy of all experiments with light is limited by the wavelength of electromagnetic waves. The same conclusion follows from the theory.

Let us also note that the probability density distribution, according to Lorenz's transformation, must behave as a temporary component of the four-dimensional vector. However, its divergence is equal to zero. At the same time, it is not possible to compose a bilinear combination from the vectors of an electromagnetic field; this combination forms a four-dimensional vector whose divergence can be equal to zero.

The reason for this is grounded in the fact that the values of energy density $u = \frac{1}{8\pi}\left(\vec{E}^2 + \vec{H}^2\right)$, and the momentum density of the field $\vec{S} = c^2\vec{g} = \frac{c}{4\pi}\left[\vec{E}\times\vec{H}\right]$, which satisfy the continuity equation, do not form a four-dimensional vector (the four-dimensional vector is formed only by the energy-momentum of an EM field).

Thus (Blokhintsev, 1982), in the case of a free electromagnetic field, it is not possible to construct a value that could play the role of the probability density of finding the photon in one or another point of space at a given moment of time. However, this value can be constructed, relying on the integral values - energy and momentum, which compose 4- vector and can be related to any chosen part of space.

2.0. Linear EM wave equation in matrix form

Within the framework of the proposed nonlinear theory of elementary particles (NTEP), the photon is considered as an object of QED described by a known linear wave equation. The only differences of our theory from the QED relate to the form of equations and the interpretation of their characteristics.

2.1. Wave equation of a photon in matrix form

Let us consider the general case of a circularly polarized electromagnetic (EM) wave that is moving, for instance, along the y - axis. This wave is the superposition of two plane-polarized waves with mutually perpendicular vectors of the EM fields: $\vec{E}_x, \vec{H}_z$ and $\vec{E}_z, \vec{H}_x$. The electric and magnetic wave fields can be written in a complex form as follows:

$$\begin{cases} \vec{E} = \vec{E}_o e^{-i(\omega t - ky)} + \vec{E}_o^* e^{i(\omega t - ky)} \\ \vec{H} = \vec{H}_o e^{-i(\omega t - ky)} + \vec{H}_o^* e^{i(\omega t - ky)} \end{cases}, \tag{3.2.1}$$

An electromagnetic wave propagating in any direction can have two plane polarizations; it contains only four field vectors. For example, in the case of y -direction, we have:

$$\vec{\Phi}(y) = \{E_x, E_z, H_x, H_z\}, \tag{3.2.2}$$

and $E_y = H_y = 0$ for all transformations. Here, note that the Dirac bispinor also has only four components.

The EM wave equation has the following known form (Jackson, 1999):

$$\left(\frac{\partial^2}{\partial t^2} - c^2\vec{\nabla}^2\right)\vec{\Phi}(y) = 0, \tag{3.2.3}$$

where $\vec{\Phi}(y)$ is any of the above electromagnetic wave field vectors (3.2.2). In other words, this equation represents four equations: one for each vector of the electromagnetic field.

We can also write this equation in the following operator form:

$$\left(\hat{\varepsilon}^{\,2} - c^2\hat{\vec{p}}^{\,2}\right)\Phi(y) = 0, \tag{3.2.4}$$

where $\hat{\varepsilon} = i\hbar \frac{\partial}{\partial t}, \hat{\vec{p}} = -i\hbar\vec{\nabla}$ are correspondingly the operators of energy and momentum; Φ is a matrix which consists of the four components $\vec{\Phi}(y)$.

Taking into account that $\left(\hat{\alpha}_o \hat{\varepsilon}\right)^2 = \hat{\varepsilon}^2$, $\left(\hat{\vec{\alpha}}\ \hat{\vec{p}}\right)^2 = \hat{\vec{p}}^2$, where $\hat{\alpha}_0 = \begin{pmatrix} \hat{\sigma}_0 & 0 \\ 0 & \hat{\sigma}_0 \end{pmatrix}$; $\hat{\vec{\alpha}} = \begin{pmatrix} 0 & \hat{\vec{\sigma}} \\ \hat{\vec{\sigma}} & 0 \end{pmatrix}$; $\hat{\beta} \equiv \hat{\alpha}_4 = \begin{pmatrix} \hat{\sigma}_0 & 0 \\ 0 & -\hat{\sigma}_0 \end{pmatrix}$ are Dirac's matrices and $\hat{\sigma}_0, \hat{\vec{\sigma}}$ are Pauli's matrices, equation (3.2.4) can also be represented in a matrix form:

$$\left[\left(\hat{\alpha}_o \hat{\varepsilon}\right)^2 - c^2\left(\hat{\vec{\alpha}}\ \hat{\vec{p}}\right)^2\right] \Phi = 0\,, \qquad (3.2.5)$$

Recall that in case of a photon $\omega = \varepsilon/\hbar$ and $k = p/\hbar$. From equation (3.2.5), using (3.2.1), we obtain $\varepsilon = cp$, which is the same as for a photon. Therefore, we can consider the wave function Φ of the equation (3.2.5) both as that of an EM wave and (taking into account its quantization) of a photon.

Factoring (3.2.5) and multiplying it on the left by the Hermitian-conjugate function Φ^+, we get:

$$\Phi^+\left(\hat{\alpha}_o \hat{\varepsilon} - c\hat{\vec{\alpha}}\ \hat{\vec{p}}\right)\left(\hat{\alpha}_o \hat{\varepsilon} + c\hat{\vec{\alpha}}\ \hat{\vec{p}}\right) \Phi = 0\,, \qquad (3.2.6)$$

Equation (3.2.6) may be broken down into two Dirac-like equations without mass:

$$\begin{cases} \Phi^+\left(\hat{\alpha}_o \hat{\varepsilon} - c\hat{\vec{\alpha}}\ \hat{\vec{p}}\right) = 0 \\ \left(\hat{\alpha}_o \hat{\varepsilon} + c\hat{\vec{\alpha}}\ \hat{\vec{p}}\right) \Phi = 0 \end{cases}, \qquad (3.2.7)$$

Note that the system of equations (3.2.7) is identical to the equation (3.2.5), and can be represented (Akhiezer and Berestetskiy, 1969; Levich, Myamlin and Vdovin, 1973) as a system of quantum equations for a photon in Hamilton's form. At the same time in the electromagnetic interpretation they are the equations of EM waves.

Actually, it is not difficult to show that only in the case when the Φ-matrix has the form:

$$\Phi = \begin{pmatrix} E_x \\ E_z \\ iH_x \\ iH_z \end{pmatrix}, \quad \Phi^+ = \begin{pmatrix} E_x & E_z & -iH_x & -iH_z \end{pmatrix}, \qquad (3.2.8)$$

the equations (3.2.7) are the right Maxwell-like equations of the retarded and advanced electromagnetic waves. Using (3.2.8), and substituting it into (3.2.7), we obtain:

$$\begin{cases} \frac{1}{c}\frac{\partial \mathrm{E}_x}{\partial t} - \frac{\partial \mathrm{H}_z}{\partial y} = 0 \\ \frac{1}{c}\frac{\partial \mathrm{H}_z}{\partial t} - \frac{\partial \mathrm{E}_x}{\partial y} = 0 \\ \frac{1}{c}\frac{\partial \mathrm{E}_z}{\partial t} + \frac{\partial \mathrm{H}_x}{\partial y} = 0 \\ \frac{1}{c}\frac{\partial \mathrm{H}_x}{\partial t} + \frac{\partial \mathrm{E}_z}{\partial y} = 0 \end{cases}, \quad (3.2.9') \qquad \begin{cases} \frac{1}{c}\frac{\partial \mathrm{E}_x}{\partial t} + \frac{\partial \mathrm{H}_z}{\partial y} = 0 \\ \frac{1}{c}\frac{\partial \mathrm{H}_z}{\partial t} + \frac{\partial \mathrm{E}_x}{\partial y} = 0 \\ \frac{1}{c}\frac{\partial \mathrm{E}_z}{\partial t} - \frac{\partial \mathrm{H}_x}{\partial y} = 0 \\ \frac{1}{c}\frac{\partial \mathrm{H}_x}{\partial t} - \frac{\partial \mathrm{E}_z}{\partial y} = 0 \end{cases}, \quad (3.2.9'')$$

For waves of any other direction the same results can be obtained by cyclic transposition of indices, or by a canonical transformation of matrices and wave functions.

We will further conditionally name each of (3.7) equations the linear semi-photon equations, remembering that it was obtained by division of one wave equation of a photon into two equations of the electromagnetic waves: retarded and advanced.

Let us make two important remarks.

1) To describe the circularly polarized EM wave, as is known, two pairs of mutual-perpendicular vectors are required: electric vectors (in our case, $\mathrm{E}_x, \mathrm{E}_z$) and magnetic vector ($\mathrm{H}_x, \mathrm{H}_z$). To this corresponds the fact that the system of four equations (3.2.9) describes one photon with circular polarization.

In the case of plane polarization there are two separate photons, that move along the y-axis (in our case with the vectors $\mathrm{E}_x, \mathrm{H}_z$ and $\mathrm{E}_z, \mathrm{H}_x$), which are described by two independent systems of equations:

$$\begin{cases} \frac{1}{c}\frac{\partial \mathrm{E}_x}{\partial t} \mp \frac{\partial \mathrm{H}_z}{\partial y} = 0 \\ \frac{1}{c}\frac{\partial \mathrm{H}_z}{\partial t} \mp \frac{\partial \mathrm{E}_x}{\partial y} = 0 \end{cases}, \quad (3.2.10)$$

and

$$\begin{cases} \frac{1}{c}\frac{\partial \mathrm{E}_z}{\partial t} \pm \frac{\partial \mathrm{H}_x}{\partial y} = 0 \\ \frac{1}{c}\frac{\partial \mathrm{H}_x}{\partial t} \pm \frac{\partial \mathrm{E}_z}{\partial y} = 0 \end{cases}, \quad (3.2.11)$$

It is possible to say that system (3.2.9) includes this pair of photons; i.e., it is the general case of describing photons of different polarization. As we will see in the following chapters, this has an important physical meaning.

2) At present as the wave vector of photon is more frequently used the 4- vector potential $A_\mu = \{i\varphi, A_i\}$ of EM of field, and the system of four equations is used as the initial wave equations:

$$\begin{aligned} \vec{\nabla}^2 \vec{A} - \frac{1}{c^2}\frac{\partial^2 \vec{A}}{\partial t^2} = 0 \\ \vec{\nabla}^2 \varphi - \frac{1}{c^2}\frac{\partial^2 \varphi}{\partial t^2} = 0 \end{aligned}, \qquad (3.2.12)$$

which undergoes quantization in this or in another way.

There is an essential difference between systems (3.2.9) and (3.2.12). Wave functions in the first case are the characteristics of the field of EM of wave, and a quantity of equations is determined by the polarization of EM of wave. In the second case the potentials are auxiliary mathematical characteristics, and a number of equations is determined by the formal association of the physical values.

In connection with this let us recall (Bethe, 1964) that the bispinor equation of Dirac also has 4 wave functions and 4 equations for them; moreover this is in no way connected with the 4-dimensional space-time. As we will be convinced, this has a straight succession with the theory of photon and a deep physical sense.

3.0. Normalized and non-normalized representation of the wave function of the photon

Above within the framework of NTEP we used the wave function of photon, which is the strength of electromagnetic field.

The particular interpretation of wave function is one of the special features of quantum mechanics in comparison with the classical electrodynamics. The physical sense of quantum wave function consists in the fact that its square (but more precisely, the product of wave function to its conjugate function) is the probability of finding the particle in some point of space in this instant. In this case the normalization of wave function is presented by one of the basic requirements of wave mechanics.

It is understandable that this meaning of wave function is more mathematical, than physical. But this probabilistic interpretation is confirmed by all experimental data. Thus, we must show that in the framework of NTEP the representation of the wave function in the form of the strengths of electromagnetic field does not contradict the probabilistic representation.

The wave function of photon must satisfy the requirements of the energy conservation law:

$$\int_0^\infty \frac{1}{8\pi}\left(\vec{\mathrm{E}}^2 + \vec{\mathrm{H}}^2\right) d\tau = \varepsilon\,, \qquad (3.3.1.)$$

where in this case ε is photon energy. Taking into account that $\Phi^+\Phi = \left(\vec{\mathrm{E}}^2 + \vec{\mathrm{H}}^2\right)$, we obtain:

$$\int_0^\infty \Phi^+ \Phi d\tau = 8\pi\varepsilon\,, \qquad (3.3.2.)$$

For the passage to the probabilistic representation of wave function we will use the indication, given above in paragraph 1.2.3. “The probability interpretation of EM of wave”: “*From the EM energy, radiated per unit time, it is possible to pass to the probabilities of radiation of the photon by division of EM energy by the photon energy*”. It is not difficult to see that if we write down the wave function of photon in the form:

$$\Psi(\vec{r},t) = \frac{\Phi(\vec{r},t)}{\sqrt{8\pi\varepsilon}}\,, \qquad (3.3.3.)$$

we will obtain wave function in the probabilistic representation. Actually, the value

$$P(\vec{r},t) = \Psi^+\Psi = \frac{\Phi^+\Phi}{8\pi\varepsilon}, \qquad (3.3.4.)$$

is the dimensionless quantity, having the sense of probability density. In this case

$$\int_0^\infty \Psi^+\Psi d\tau = 1, \qquad (3.3.5)$$

As far as one photon is considered, the value P will determine the portion of energy from the photon energy at the particular point of space-time. When we have many photons in a given volume of space, ε should be understand as the total electromagnetic energy of these photons. In this case the formula (3.3.5.) will determine the probability of finding the photon at the particular point of the volume of space at the given point of time. Obviously, in both cases the following normalization conditions are satisfied. Depending on the convenience, the one or the other normalization can be selected, taking into account that the energy of photon ε is a normalized constant, similar to unit.

We will dedicate the following parts of theory to the description of the generation of massive elementary particles. We will ascertain that the physical sense of the wave function of massive particles remains the same (with the only difference that, besides energy, in this case it is possible to use the mass of particles according to Einstein's formula $\varepsilon = mc^2$).

Let us note that since energy, momentum, wave vector, frequency and wavelength of photon are one-to-one connected among themselves, it is possible to speak about normalization in relation to any of these characteristics. Of course in this case it is necessary to use a wave function in the appropriate representation.

One additional question, connected with the normalization of wave function, is, what volume of the space of integration the formula (3.3.5) contains. Usually the infinite space is understood here. On this base it is said that the photon is spread on the infinite space.

As we have seen above, accordingly to QED a photon is a nonlocal object with size, characterized by its wavelength. Since photon is neutral and does not possess external infinite field, physically, integration on infinity is thoughtless. Then a question arises: why the integration for formula (3.3.5) does not cause difficulty in the physics calculations.

Let us assume that photon occupies the limited volume τ_{ph}. It is not difficult to see that:

$$\int_0^\infty \Phi^+\Phi d\tau = \int_0^{\infty-\tau_{ph}} \Phi^+\Phi d\tau + \int_0^{\tau_{ph}} \Phi^+\Phi d\tau = \int_0^{\tau_{ph}} \Phi^+\Phi d\tau = 8\pi\varepsilon, \qquad (3.3.6.)$$

since the integral on the space $\tau = \infty - \tau_{ph}$, where the photon is absent, is equal to zero.