

Kantowski-Sachs Universe with Anisotropic Dark Energy in Brans-Dicke Gravity Theory

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Abstract

A spatially homogeneous Kantowski-Sachs universe filled with anisotropic dark energy is investigated within the framework of Brans-Dicke gravity theory. We obtain exact solution to the field equations by utilizing a special form of the Hubble parameter that yields a time-varying deceleration parameter. We discuss the physical and kinematical features of early decelerating and late-time accelerating model of the universe. We observe that the anisotropic universe approaches isotropy for large time through its evolution, which is consistent with the recent observations on present-day universe. For large time, the universe achieves the de-sitter phase and expands forever with the dominance of dark energy .

Keywords: Kantowski-Sachs model, anisotropic dark energy, Brans-Dicke gravity.

1 Introduction

Recent observations of type I_a supernovae [1, 2] with redshift up to about $Z \leq 1$ have led cosmologists to believe that the universe in its present state is in the phase of accelerated expansion. This intriguing discovery has led to the idea of an exotic form of energy, known as dark energy (DE), which is responsible for the possible cosmic acceleration. The observations of large scale structure and cosmic microwave background (CMB) also provide strong evidence in favour of DE [3,4]. The nature and the composition of DE is still a challenging problem to the cosmologists. The thermodynamical studies of DE reveal that the constituents of DE may be massless particles (boson and fermions) whose collective behavior resembles with a kind of radiation fluid with negative pressure acting as antigravity responsible for gearing up the universe. A DE model can be characterized by the equation of state (EoS) parameter ω defined by $p = \omega\rho$, where p is the pressure and ρ is the energy density of the fluid. The EoS parameter is usually assumed to be a constant $-1, 0, \frac{1}{3}$ and 1 for vacuum, dust, radiation and stiff matter dominated universe respectively. However, it can be a function of time or redshift in general.

Several cosmologists have proposed and studied different acceptable candidates for DE to fit the current observations. The simplest candidate for DE is the cosmological constant with

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$\omega = -1$. However, the cosmological constants is entangled with serious puzzles like the time tuning problem and coincidence problem. Some other alternative candidates for DE are quintessence $\omega > -1$ [5,6], phantom $\omega < -1$ [7,8], k-essence [9], Chaplygin gas [10,11], holographic DE [12-14] ageographic DE [15-16], quintom [17-18] and braneworld models [19,20] etc.

The isotropic models are considered to be the most suitable to study large scale structure of the universe. However, it is believed that the early universe may not have been exactly isotropic. This predication motivates cosmologists to describe the early stages of evolution of the universe with models having anisotropic background. The limitations of the general relativity in providing explanation of this phase of the universe have led cosmologists to adopt various other hypotheses and study of their implication in this context. These hypotheses include those assigning (i) time-dependence of gravitational and cosmological constants, (ii) other geometries or physical fields with the universe and (iii) modified or alternative theories of gravitation. Such studies are expected to bring out a number of aspects of mathematical and physical interests. Among the various modifications of general relativity, the Brans-Dicke (BD) gravity theory [21] is most physically acceptable scalar-tensor of gravitation. In this theory the gravitational interaction involves a scalar field ϕ and the metric tensor. The scalar field ϕ has the dimension of the inverse of gravitational constant. Several aspects of BD theory have been extensively studied so far by many cosmologists. Singh and Rai [22] presented a detailed discussion on cosmological models within the framework of BD theory. In particular, spatially homogeneous Bianchi models in the presence of perfect fluid with or without radiation of are quite importance to discuss the early stages of evolution of the universe. Banerjee and Pavon [23] have shown that the BD theory may explain the present accelerated expansion of the universe without resorting to a cosmological constant or quintessence matter. A cosmic fluid obeying a perfect fluid type equation of state can not support the acceleration [24]. The latest inflationary models [25], possible 'graceful exit' problem [26] and the extended chaotic inflations [27] are based on BD theory. In view of these observations it would be worthwhile to study anisotropic dark energy models within the framework of BD theory.

In recent years, many cosmologists have studied the universe with variable equation of state. Shamir and Bhatti [28] investigated Bianchi type-III models with anisotropic DE in BD theory. Katore et al [29]. presented a hypersurface homogeneous space-time with anisotropic DE in BD theory of gravitation. Pawar and Solenke[30] have explored the solution of exact Kantowski-Sachs cosmological models in BD theory in the background of anisotropic dark energy. Tripathy *et al.* [31] constructed plane symmetric Bianchi type-I anisotropic dark energy cosmological models in the framework of generalized Brans-Dicke theory with a self-interacting potential. Recently, Rao *et al.* [32] obtained a spatially homogeneous Kantowski-Sachs two-fluid radiating cosmological model filled with barotropic fluid and dark energy in BD theory. Shri Ram *et al.* [33] investigated a Kantowski-Sachs universe with anisotropic dark energy within the framework of Lyra geometry. Chand *et al.* [34] have investigated flat, open and closed FRW models and the effect of dynamic cosmological term on the evolution of the universe in BD theory. Motivated by above discussions we obtain, in this Letter, a new spatially homogeneous Kantowski-Sachs cosmological model filled with an anisotropic dark energy within the framework of BD theory of gravitation.

2 Kantowski-Sachs Model and Field Equations.

The line-element for the spatially homogeneous and anisotropic Kantowski-Sachs model is given by

$$ds^2 = dt^2 - A^2(t)dr^2 - B^2(t)(d\theta^2 + \sin^2 \theta d\phi^2) \quad (2.1)$$

where A and B are cosmic scale factors. The energy-momentum tensor for an anisotropic dark energy is taken as

$$T_i^j = \text{diag} [\rho, -p_r, -p_\theta, -p_\phi] = \text{diag} [1, -\omega_r, -\omega_\theta, -\omega_\phi] \rho \quad (2.2)$$

where ρ is the energy density of the fluid ; p_r , p_θ and p_ϕ are the pressures on r , θ and ϕ axes respectively. Here ω_r , ω_θ and ω_ϕ are the EoS parameter in the directions of r , θ and ϕ axes respectively. Further, we parametrize the energy-momentum tensor as

$$T_i^j = \text{diag} [1, -\omega, -(\omega + \gamma), -(\omega + \delta)] \rho \quad (2.3)$$

where we have chosen $\omega_r = \omega$ and the skewness parameters γ and δ are deviations from ω on θ and ϕ -axes respectively. Since $T_2^2 = T_3^3$, (2.3) can be written as

$$T_i^j = \text{diag} [1, -\omega, -(\omega + \delta), -(\omega + \delta)] \rho. \quad (2.4)$$

The field equations in Brans-Dicke theory are

$$R_{ij} - \frac{1}{2}Rg_{ij} = -\frac{8\pi}{\phi}T_{ij} + \frac{\mu}{\phi^2} \left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}^{\cdot k} \right) + \frac{1}{\phi} \left(\phi_{,i}\phi_{,j} - g_{ij}\phi_{,k}^{\cdot k} \right) \quad (2.5)$$

where

$$\phi_{;k}^{\cdot k} = \frac{8\pi T}{(3 + 2\omega)\phi} \quad (2.6)$$

and μ is a dimensionless coupling constant. A comma and a semicolon denote ordinary and covariant derivative respectively.

In comoving coordinates, Brand-Dicke field equations (2.5) and (2.6) for the metric given by (2.1) with the help of (2.4) yield the following set of field equations:

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{1}{B^2} = -\frac{8\pi\omega\rho}{\phi} - \frac{\mu}{2} \left(\frac{\dot{\phi}}{\phi} \right)^2 - 2\frac{\dot{\phi}\dot{B}}{\phi B} - \frac{\ddot{\phi}}{\phi}, \quad (2.7)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -\frac{8\pi(\omega + \delta)\rho}{\phi} - \frac{\mu}{2} \left(\frac{\dot{\phi}}{\phi} \right)^2 - \frac{\dot{\phi}}{\phi} \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} \right) - \frac{\ddot{\phi}}{\phi}, \quad (2.8)$$

$$\frac{2\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} + \frac{1}{B^2} = \frac{8\pi\rho}{\phi} + \frac{\mu}{2} \left(\frac{\dot{\phi}}{\phi} \right)^2 - \frac{\dot{\phi}}{\phi} \left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B} \right). \quad (2.9)$$

An over dot denotes derivative with respect to t .

We now define some physical parameters for the Kantowski-Sachs space-time (2.1) to discuss

the physical features of the cosmological model. The average scale factor and scalar volume are given by

$$V = a^3 = AB^2. \tag{2.10}$$

The scalar expansion θ and shear scalar σ have expressions

$$\theta = \left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B} \right), \tag{2.11}$$

$$\sigma^2 = \frac{1}{3} \left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B} \right)^2. \tag{2.12}$$

The anisotropy parameter A_m is given by

$$A_m = \frac{1}{3} \sum_{i=1}^3 \left(\frac{H_i - H}{H} \right)^2 \tag{2.13}$$

where

$$H = \frac{1}{3} (H_1 + H_2 + H_3) \tag{2.14}$$

is the mean Hubble parameter and H_1, H_2, H_3 are directional Hubble parameters in the direction of r, θ, ϕ axes given by

$$H_1 = \frac{\dot{A}}{A}, H_2 = H_3 = \frac{\dot{B}}{B}. \tag{2.15}$$

The deceleration parameter q in cosmology is the measure of the cosmic acceleration of the universe expansion and is defined as

$$q = -\frac{a\ddot{a}}{a} = -\left(\frac{\dot{H} + H^2}{H^2} \right). \tag{2.16}$$

3 Solution of Field Equations

The field equations (2.7)-(2.9) are a system of three independent equations with six unknown parameters A, B, ϕ, ρ, ω and δ . Therefore to find the exact solutions of the field equations, we use three constraints successively. Subtracting (2.7) from (2.8), we get

$$\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{\dot{B}^2}{B^2} - \frac{1}{B^2} = \frac{8\pi\delta\rho}{\phi} - \frac{\dot{\phi}}{\phi} \left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B} \right). \tag{3.1}$$

The above equation further reduces to

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = \frac{k_1}{V\phi} + \frac{1}{V\phi} \int \left[\frac{\phi}{B^2} - 8\pi\delta\rho \right] V dt \tag{3.2}$$

where k_1 is the constant of integration. The integral term in (3.2) vanishes for

$$\delta = \frac{\phi}{8\pi B^2 \rho}. \tag{3.3}$$

Using (3.3) in (3.2), we get

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = \frac{k_1}{V\phi}. \tag{3.4}$$

Integration of (3.4) yields

$$\frac{A}{B} = k_2 \exp\left(\int \frac{k_1}{a^3\phi} dt\right) \tag{3.5}$$

where k_2 is an integration constant .

We now use the power-law assumption to solve the integral part in (3.5). The power-law relation between scale factor a and scalar function ϕ , as used by Johri and Desikan [35] in the context of Robertson-Walker Brans-Dicke model, is given as

$$\phi = ba^m \tag{3.6}$$

where b and m are constants. Shamir and Bhatti[28] have also used (3.6) to obtain an anisotropic DE model of Bianchi type-III in BD theory of gravitation.

From (3.5) it is clear that we can find the scale factors A and B if the average scale factor known function of time. To determine a , we consider a specific form of the Hubble parameter H . Following the work of Ellis and Madsen [36], Singh [37] has chosen a functional form of H as:

$$H = \frac{\dot{a}}{a} = k_3(1 + a^{-n}) \tag{3.7}$$

where $k_3 > 0$ and $n(> 1)$ are constants. With this choice of H , he has presented an universe with early deceleration and late-time acceleration; and thus describing unified description of evolution of the universe.

Combining (2.16) and (3.7) and integrating, we get the time-varying form of the deceleration parameter as

$$q = -1 + \frac{n}{1 + a^n}. \tag{3.8}$$

A similar form of q has also been proposed by Banerjee and Das [38] in the case of Robertson-Walker space-time. From (3.8), we see that when $a = 0, q = n - 1; q = 0$ for $a^n = n - 1$ and $q < 0$ for $a^n > n - 1$. From (3.9), we observe that $a = 0$ when $t = 0$. Solving (3.8) for the average scale factor, one obtains

$$a = (e^{knt} - 1)^{k/n}. \tag{3.9}$$

Thus, the universe starts evolving with a decelerating expansion to an accelerating expansion at late time. Adhav *et al.* [39]utilized this form of the average scale factor to study LRS Bianchi type- I, III, V, VI₀ and Kantowski-Sachs cosmological models with variable EoS parameter in general relativity.

Substitution of (3.6) and (3.9) in (3.5) leads to

$$\frac{A}{B} = k_2 \exp\left(\int \frac{k_1 dt}{b(e^{knt} - 1)^{\frac{3k(m+3)}{n}}}\right). \tag{3.10}$$

It is difficult to evaluate the integral in (3.10) for general values of k , m and n . In order to evaluate this integral and consequently to determine the scale factors A and B , we assume $k = 1$, $n = 3$ and $m = -5/2$. Then (3.10) takes the form

$$\frac{A}{B} = k_2 \exp \left(\int \frac{k_1 dt}{b(e^{3t} - 1)^{1/2}} \right). \tag{3.11}$$

This equation, on integration, yields

$$\frac{A}{B} = k_2 \exp \left[\frac{2k_1}{9b} \left(\tan^{-1} \sqrt{e^{3t} - 1} \right) \right] \tag{3.12}$$

We can take $k_2 = 1$ without loss of generality. From (2.10) and (3.12), we obtain the solutions of the scale factors A and B of the forms

$$A = (e^{3t} - 1)^{1/3} \exp \left[\frac{4k_1}{9b} \left(\tan^{-1} \sqrt{e^{3t} - 1} \right) \right], \tag{3.13}$$

$$B = (e^{3t} - 1)^{1/3} \exp \left[\frac{-2k_1}{9b} \left(\tan^{-1} \sqrt{e^{3t} - 1} \right) \right]. \tag{3.14}$$

From (3.6) the solution of scalar function ϕ is given by

$$\phi = \frac{b}{(e^{3t} - 1)^{5/6}}. \tag{3.15}$$

Hence the metric of the present cosmological model can be written in the form

$$\begin{aligned} ds^2 = & dt^2 - (e^{3t} - 1)^{2/3} \exp \left[\frac{8k_1}{9b} \left(\tan^{-1} \sqrt{e^{3t} - 1} \right) \right] dr^2 \\ & - (e^{3t} - 1)^{2/3} \exp \left[-\frac{4k_1}{9b} \left(\tan^{-1} \sqrt{e^{3t} - 1} \right) \right] (d\theta^2 + \sin^2 \theta d\phi^2). \end{aligned} \tag{3.16}$$

4 Physical Features of the Model

We now discuss the physical and dynamical behaviours of the model (3.16). The directional Hubble parameters and the mean Hubble parameter are obtained as

$$H_1 = \frac{e^{3t}}{(e^{3t} - 1)} + \frac{2k_1}{3b\sqrt{(e^{3t} - 1)}}, \tag{4.1}$$

$$H_2 = H_3 = \frac{e^{3t}}{(e^{3t} - 1)} - \frac{k_1}{3b\sqrt{(e^{3t} - 1)}}, \tag{4.2}$$

$$H = \frac{e^{3t}}{(e^{3t} - 1)}. \tag{4.3}$$

The values of the directional Hubble parameters are infinite at $t = 0$ and tend to zero as $t \rightarrow \infty$. The mean Hubble parameter is also infinite at $t = 0$ and $H \rightarrow 0$ as $t \rightarrow \infty$ i.e. the rate of

expansion of universe is decreasing.

The expansion scalar, shear scalar and the anisotropic parameter have the values given by

$$\theta = \frac{3e^{3t}}{(e^{3t} - 1)}, \tag{4.4}$$

$$\sigma = \sqrt{\frac{2}{3}} \frac{k_1}{b\sqrt{(e^{3t} - 1)}}, \tag{4.5}$$

$$A_m = \frac{5k_1^2(e^{3t} - 1)}{27b^2e^{6t}}. \tag{4.6}$$

From (4.6) it is clear that the mean anisotropic parameter is very large at $t = 0$ and its value tends to zero as $t \rightarrow \infty$. This means that initially the universe is highly anisotropic and it tends to isotropy as late-time. The expansion scalar and shear scalar are infinite at $t = 0$ and are decreasing functions of time. The expansion scalar tends to a constant value as $t \rightarrow \infty$ where as the shear scalar tends to zero. The ratio $\frac{\sigma}{\theta}$ tends to zero as $t \rightarrow \infty$ which also indicates that the universe approaches isotropy for large time.

The scalar function ϕ for this model is infinite at $t = 0$ and tends to zero as $t \rightarrow \infty$. This means that the scalar field in the model dies out for large time.

The energy density of anisotropic fluid is obtained as

$$\rho = \frac{b}{8\pi} \left[\frac{k^{2/3}}{(e^{3t} - 1)^{3/2}} \exp \left\{ \frac{4k_1}{9b} \tan^{-1}(\sqrt{e^{3t} - 1}) \right\} - \frac{(25\mu + 36)e^{6t}}{8(e^{3t} - 1)^{17/6}} - \frac{k_1^2}{3b^2(e^{3t} - 1)^{11/6}} \right]. \tag{4.7}$$

From the above equation we observe that the energy density of dark energy is infinite at time $t = 0$ and it decreases as time increases and ultimately assumes a constant value for large-time.

The deviation free part of EoS parameter ω and skewness parameter δ have the values given as

$$\omega = -\frac{b}{8\pi\rho} \left[\frac{k^{2/3}}{(e^{3t} - 1)^{3/2}} \exp \left\{ \frac{4k_1}{9b} \tan^{-1}(\sqrt{e^{3t} - 1}) \right\} + \frac{(25\mu - 31)e^{6t} - 24e^{3t}}{8(e^{3t} - 1)^{17/6}} + \frac{k_1^2}{9b^2(e^{3t} - 1)^{11/6}} + \frac{5k_1e^{3t}}{6b(e^{3t} - 1)^{7/3}} \right], \tag{4.8}$$

$$\delta = \frac{b}{8\pi\rho} \left[\frac{k^{2/3}}{(e^{3t} - 1)^{3/2}} \exp \left\{ \frac{4k_1}{9b} \tan^{-1}(\sqrt{e^{3t} - 1}) \right\} \right]. \tag{4.9}$$

At $t = 0$, ω and δ both are infinite. As t increases, δ decreases and becomes zero at late time. We also find that $\omega = -1$ as $t \rightarrow \infty$.

For the model (3.16), the deceleration parameter has the value given by

$$q = -1 + 3e^{-3t}. \tag{4.10}$$

From (4.10) we find that q is positive for $t < \frac{1}{3} \log 3$ and negative for $t > \frac{1}{3} \log 3$, and it has a signature flip at $t = \frac{1}{3} \log 3$. The signature flip in q is essential for the conclusion that the

present universe is accelerating. Thus, the universe starts evolving from $t = 0$ with decelerating expansion and the expansion changes from deceleration to acceleration at a finite time and then undergoes accelerated expansion forever. We also observe that $q = -1$ as $t \rightarrow \infty$ which indicates the inflationary behaviour of the universe.

From above discussion, we arrive at the conclusion that the universe achieves the de-Sitter phase and expands forever with the dominance of dark energy.

In this paper, we have constructed a Kantowski-Sachs cosmological model of the universe with anisotropic dark energy with the framework of BD gravity theory. The exact solution of the field equations is obtained using the special form of Hubble parameter which represents an early decelerating and late-time accelerating universe in BD theory. The role of BD scalar is to reduce the energy density. The expansion in the universe is found to be infinite at the initial singularity $t = 0$ which decreases with the increase in time and ultimately assumes a constant value at late-time. The anisotropy parameter is found to be infinite at the initial time $t = 0$, which tends to zero as $t \rightarrow \infty$, and thus the universe isotropizes at late-time. The EoS parameter is a decreasing functions of time which tends to -1 as $t \rightarrow \infty$. The skewness parameter is zero for late time. The energy density tend to a constant a late-time. Also we observe that $q = -1$ as $t \rightarrow \infty$ which means that the universe asymptotically the de-Sitter phase and hence expands forever with the dominance of DE. The results obtained in this paper may play important role in the study of structure formation of the universe which has astrophysical significance.

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