Exploration

On the Physical Interpretation of the Velocity Parameter in the Formula for Gravitational Planck Constant

Matti Pitkänen [1]

Abstract

Nottale's formula for the gravitational Planck constant $\hbar_{gr} = GMm/v_0$ involves parameter v_0 with dimensions of velocity. I have worked with the quantum interpretation of the formula but the physical origin of v_0 - or equivalently the dimensionless parameter $\beta_0 = v_0/c$ (to be used in the sequel) appearing in the formula has remained open hitherto. In the following a possible interpretation based on many-sheeted space-time concept, many-sheeted cosmology, and zero energy ontology (ZEO) is discussed. In ZEO the non-changing parts of zero energy states are assigned to the passive boundary of CD and β_0 should be assigned to it. There are two measures for the size of the system. The M^4 size L_{M^4} is identifiable as the maximum of the radial M^4 distance from the tip of CD associated with the center of mass of the system along the light-like geodesic at the boundary of CD. System has also size L_{ind} defined defined in terms of the induced metric of the space-time surface, which is space-like at the boundary of CD. One has $L_{ind} < L_H$. The identification $\beta_0 = L_{M^4}/L_H$ does not allow the identification of $L_H = L_{M^4}$. L_H would however naturally corresponds to the size of the magnetic body of the system in turn identifiable as the size of CD. One can deduce an estimate for β_0 by approximating the space-time surface as Robertson-Walker cosmology expected to be a good approximation near the passive light-like boundary of CD. The resulting formula is tested for planetary system and Earth. The dark matter assignable to Earth can be identified as the innermost part of inner core with volume, which is .01 per cent of the volume of Earth. Also the consistency of the Bohr quantization for dark and ordinary matter is discussed and leads to a number theoretical condition on the ratio of the ordinary and dark masses. $\beta_0/4\pi$ is analogous to gravitational fine structure constant for $h_{eff} = h_{gr}$. Could one see it as fundamental coupling parameter appearing also in other interactions at quantum criticality in which ordinary perturbation series diverges? Remarkably, the value of G does not appear at all in the perturbative expansion in this region! Could G have several values? This suggests the generalization $G = l_P^2/\hbar \rightarrow G = R^2/\hbar_{eff}$ so that G would indeed have a spectrum and that Planck length l_P would be equal to CP_2 radius R so that only one fundamental length would be associated with twistorialization. Ordinary Newton's constant would be given by $G = R^2/h_{eff}$ with h_{eff}/h_0 having value in the range $10^7 - 10^8$.

Keywords: Velocity parameter, interpretation, formula, gravitation, Planck Constant.

1 Introduction

Nottale's formula [1] for the gravitational Planck constant $\hbar_{gr} = GMm/v_0$ involves parameter v_0 with dimensions of velocity. I have worked with the quantum interpretation of the formula [5, 4, 13, 12] but the physical origin of v_0 - or equivalently the dimensionless parameter $\beta_0 = v_0/c$ (to be used in the sequel) appearing in the formula has remained open hitherto. In the following a possible interpretation based on many-sheeted space-time concept, many-sheeted cosmology, and zero energy ontology (ZEO) is discussed.

A generalization of the Hubble formula $\beta = L/L_H$ for the cosmic recession velocity, where $L_H = c/H$ is Hubble length and L is radial distance to the object, is suggestive. This interpretation would suggest that some kind of expansion is present. The fact however is that stars, planetary systems, and planets do *not* seem to participate cosmic expansion. In TGD framework this is interpreted in terms of quantal jerk-wise expansion taking place as relative rapid expansions analogous to atomic transitions or quantum

[1] Correspondence: Matti Pitkänen `http://tgdtheory.fi/`. Address: Rinnekatu 2-4 8A, 03620, Karkkila, Finland. Email: matpitka6@gmail.com.

phase transitions. The TGD based variant of Expanding Earth model assumes that during Cambrian explosion the radius of Earth expanded by factor 2 [2] [20, 19, 21].

There are two measures for the size of the system. The M^4 size L_{M^4} is identifiable as the maximum of the radial M^4 distance from the tip of CD associated with the center of mass of the system along the light-like geodesic at the boundary of CD. System has also size L_{ind} defined defined in terms of the induced metric of the space-time surface, which is space-like at the boundary of CD. One has $L_{ind} < L_{M^4}$. The identification $\beta_0 = L_{M^4}/L_H < 1$ does not allow the identification $L_H = L_{M^4}$. L_H would however naturally corresponds to the size of the magnetic body of the system in turn identifiable as the size of CD.

One can deduce an estimate for β_0 by approximating the space-time surface near the light-cone boundary as Robertson-Walker cosmology, and expressing the mass density ρ defined as $\rho = M/V_{M^4}$, where $V_{M^4} = (4\pi/3)L^3_{M^4}$ is the M^4 volume of the system. ρ can be expressed as a fraction ϵ^2 of the critical mass density $\rho_{cr} = 3H^2/8\pi G$. This leads to the formula $\beta_0 = \sqrt{r_S/L_{M^4}} \times (1/\epsilon)$, where r_S is Schwartschild radius.

This formula is tested for planetary system and Earth. The dark matter assignable to Earth can be identified as the innermost part of inner core with volume, which is .01 per cent of the volume of Earth. Also the consistency of the Bohr quantization for dark and ordinary matter is discussed and leads to a number theoretical condition on the ratio of the ordinary and dark masses.

$\beta_0/4\pi$ is analogous to gravitational fine structure constant for $h_{eff} = h_{gr}$. Could one see it as fundamental coupling parameter appearing also in other interactions at quantum criticality in which ordinary perturbation series diverges? Remarkably, the value of G does not appear at all in the perturbative expansion in quantum critical phase! Could G can have several values?

There is also a problem: the twistorialization of TGD [10] leads to the conclusion that the radius of twistor sphere for M^4 is given by Planck length l_P so that - contrary to the view held for decades - one would have two fundamental lengths - l_P and CP_2 radius R and there is no idea about how they are related. Quantum criticality cannot relate them since they are not coupling parameters.

The formula for $G = l_p^2/\hbar$ however suggests a generalization $G = R^2/h_{eff}$ with h_{eff}/h_0 having value in the range $10^7 - 10^8$: one would have $l_P = R$! Also classical gravitation could tolerate the spectrum of G since Newton's equations in gravitational field is invariant under scaling $h_{eff} \to xh_{eff}$ inducing $G \to G/x$ and $t \to t/x, r \to r/x$ with scales up the size scale of space-time sheets as the proportionality of Compton length to h_{eff} requires.

2 About TGD based interpretation for the parameter v_0 appearing in Nottale's formula

2.1 Formula for the gravitational Planck constant and some background

The formula

$$\hbar_{gr} = \frac{GMm}{v_0} \tag{2.1}$$

for the gravitational Planck constant was originally introduced by Nottale [1]. Here v_0 is a parameter with dimensions of velocity.

The formula is expected to hold true at the magnetic flux tubes mediating gravitational interaction and obeying also the general formula

$$h_{gr} = h_{eff} \quad , \quad h_{eff} = nh_0 \quad , \quad h = 6h_0 \quad . \tag{2.2}$$

The support for the formula $h = 6h_0$ is discussed in [14, 18]. The value of h_{gr} can be very large unlike the value of h_{eff} associated with say valence bonds.

There are two kinds of flux tubes - homologically non-trivial and trivial ones corresponding to two kinds of geodesic spheres of CP_2, and they seem to correspond to small and large values of h_{eff}.

1. Since the Kähler magnetic energy of homologically non-trivial flux tubes carrying monopole magnetic flux is large, the natural expectation is that gravitation and presumably also other long range interactions mediated by massless particles - with color interactions perhaps forming an exception - correspond to homologically trivial flux tubes for which only the volume energy due to cosmological constant is non-vanishing. Massive particles would correspond to flux tubes carrying monopole magnetic flux associated with homologically non-trivial flux tubes. Homology could therefore define a key difference between massive and massless bosons at space-time level.

2. One can argue the flux tubes accompanying flux tubes with non-trivial homological charge are relatively short: since the length of the flux tube is expected to be proportional to h_{eff} or its positive power, this would suggest small values of h_{eff} for them. For instance, valence bonds for which non-standard value of h_{eff} is suggestive could correspond to relatively flux tubes carrying monopole flux [15].

3. Suppose that the value of exponent of Kähler function for the "world of classical worlds" (WCW) is exponent of Kähler function expressible as the 6-D variant of Kähler action for the twistor lift of 4-D Kähler action reducing to the sum of 4-D Kähler action and volume term in the dimensional reduction of the 6-surface to S^2 bundle over space-time surface required by the induction of twistor structure [8, 10, 9]. If so, the shortness of homologically non-trivial flux tubes could be forced by the large values of Kähler magnetic action and energy making the exponent small.

2.2 A formula for β_0 from ZEO

I have made some attempts relate the value of $\beta_0 = v_0/c$ appearing in the formula for h_{gr} to some typical rotation velocity in the system [5, 4] but although orders of magnitude are reasonable, these attempts have not led to a prediction of v_0. It might be that the explanation is hidden at deeper level and involves many-sheeted space-time and the view about quantum theory based on zero energy ontology (ZEO) in an essential manner.

A generalization of the Hubble formula $\beta = L/L_H$ for the cosmic recession velocity, where $L_H = c/H$ is Hubble length and L is radial distance to the object, is suggestive. Some kind of expansion suggests itself. The fact is however that stars, planetary systems, and planets do *not* seem to participate cosmic expansion. In TGD framework this is interpreted in terms of quantal jerk-wise expansion taking place as relative rapid expansions analogous to atomic transitions or quantum phase transitions. The TGD based variant of Expanding Earth model assumes that during Cambrian explosion the radius of Earth expanded by factor 2 [2] [20, 19, 21].

The interpretation of the velocity parameter β_0 to be discussed involves in an essential manner ZEO based quantum measurement theory giving rise to a quantum theory of consciousness [16]. The causal diamond CD assignable to given conscious entity expands state function reduction by state function and this expansion is very much analogous to cosmic expansion.

In TGD inspired theory of consciousness, which is essentially quantum measurement theory in ZEO [16], self as a conscious entity corresponds to a sequence of analogs of weak measurements changing the members of state pairs at active boundary of CD and increasing the size of CD by shifting the active boundary farther away from the passive boundary. Passive boundary and the members of state pairs at it remain invariant. This produces a generalized Zeno effect leaving both passive boundary and states at it invariant. This gives the unchanging contribution to the consciousness that one might call "soul". Experienced time corresponds to the increasing distance between the tips of CD and experienced time to the sequence of weak measurements. Active boundary gives rise to changing part in the contents of consciousness. Self dies and reincarnates in opposite time direction when the big state function reduction changing the roles of the boundaries of CD occurs and CD begins to increase in opposite time direction.

To make progress one must consider more precisely what space-time as 4-surface property means in ZEO. The unchanging part of the consciousness corresponds to the passive light-like boundary of CD and various constant parameters should be assigned with the quantum state at it.

There are two measures for the size of the system at the passive boundary and also a measure for the size of its magnetic body mediating gravitational interactions.

1. One can identify M^4 size L_{M^4} as the maximum of the radial M^4 distance from the tip of CD associated with center of mass of the system to the boundary of the system along the light-like geodesic at the passive boundary of CD.

2. System has also size L_{ind} defined as the maximum distance in the induced metric of the space-time surface, which is space-like at the boundary of CD. L_{ind} cannot correspond to Hubble length L_H since this would give $\beta > 0$.

3. A reasonable option is that L_H corresponds to the size scale of the part of the magnetic body of the system responsible for mediation of gravitational interactions. L_H would thus correspond to effective range of gravitational interactions. The simplest guess is that L_H corresponds the maximal radial size of CD given as $L_H = T/2$, where T is the temporal distance between the tips of the CD.

One can deduce an estimate for β_0 by approximating the space-time surface near the passive boundary of CD as Robertson-Walker cosmology. This approximation is indeed natural since space-time surface is small deformation of future/past light-cone near the boundary. The assumption about RW cosmology is *not* needed elsewhere inside CD. This conforms with the holography.

This estimate is only an approximation involving the ratio $\epsilon^2 = \rho/\rho_{cr} < 1$ of the average mass density ρ to the critical mass density

$$\rho_{crit} = \frac{3H^2}{8\pi G}$$

besides H. One can consider at least two options.

1. Option I: ρ corresponds to the average density $\rho = M/V_{M^4}$ within M^4 volume $V_{M^4} = (4\pi/3)L^3_{M^4}$ at the passive boundary. The condition $\rho = \epsilon^2\rho_{cr}$ allows to solve $\beta = L/L_H$ as

$$\beta_0 = \frac{L_{M^4}}{L_H} = \frac{1}{\epsilon}\sqrt{\frac{r_S}{L_{M^4}}} \quad , \quad r_S = 2GM \quad . \tag{2.3}$$

   Here r_S is Schwartschild radius. As noticed, a reasonable identification for L_H would be as the size scale of the gravitational magnetic body given by the size $L_H = T/2$. It turns that this formula is rather reasonable and consistent with earlier results in the case of planetary system and Earth.

2. Option II gives up completely the attempt to interpret the situation in terms of Hubble constant and identifies $\beta_0 = L_{ind}/L_{M^4} < 1$. In this case the expression in terms of mass density in terms of critical mass density does not help to obtain a more detailed formula. If one requires consistency with the previous formula, one obtains L_{ind} as pr $L_{ind} = \sqrt{r_S L_{M^4}}/\epsilon$. For $\epsilon = 1$ one has geometric mean.

2.3 Testing the model in the case of Sun and Earth

One can test these equations for Sun and Earth to see whether they could make sense. The restriction to the option I with volume V identified as the volume in the induced metric at the passsive boundary of CD. Option II is obtained at the limit $\epsilon_! = 1$.

Consider first Sun.

1. In the case of Sun the model for the Bohr quantization of planetary orbits was originally proposed by Nottale [1] and was developed further in TGD framework in [5, 4] assuming that genuine quantum coherence in astrophysical scales possible for dark matter is in question. The value of β_0 is in a reasonable approximation $\beta_0(inner) = 2^{-11}$ for the inner planets and $\beta_0(out) = \beta_0(inner)/5$ for the outer planets.

2. For the 3 inner planets the distance of Earth given by astronomical unit $AU = .149 \times 10^9$ km is the natural estimate for L_H so that one has $L_H = AU$. For outer planets the natural choice is of the order of the orbit of the outer planet with largest orbital radius, which is Neptune with distance of 30 AU for Neptune. The prediction of the model for the orbital radius of Neptune is 25 AU so that the estimate looks reasonable. Note that the radii in Bohr model are proportional to $h_{gr}^2 n^2$, n the principal quantum number, so that the scaling $v_0 \to v_0/5$ scales the radius by factor 5^2. This also means that scaling $n \to kn$ and scaling $v_0 \to v_0/k$ produces the same scaled orbital radius.

3. For the inner planets one obtains

$$\beta_0 = \frac{r_S}{L_H} \times \frac{1}{\epsilon} = 1.1 \times 10^{-4} \times \frac{1}{\epsilon} \ .$$

   The value co-incides with $\beta_0 = 2^{-11}$ providing a reasonable approximation in Nottale model for $r = 4.55$. This leaves open the fraction $\epsilon^2 = \rho/\rho_{crit}$. One would have $\epsilon^2 = .048$. The size scale of CD would be about $1/\beta = 2^{11}$ using AU as a unit.

Consider next Earth. One can consider two choices for L.

1. Case I: Earth radius $R_E = 6.371 \times 10^3$ km is the first candidate: this choice might be relevant for the applications at Earth's surface such as fountain effect in super-fluidity.

2. Case II: The distance $d_M = 60.3 R_E$ of Moon, is second choice for the scale L. The Schwartschild radius of Earth is $r_S = 9$ mm.

The value of β_0 in these two cases is given by.

$$\beta_0(I) = \sqrt{\frac{r_S}{R_E}}\frac{1}{\epsilon} = .38 \times 10^{-4}\frac{1}{\epsilon} \ ,$$
$$\beta_0(II) = \sqrt{\frac{r_S}{d_M}}\frac{1}{\epsilon} = .04 \times 10^{-4}\frac{1}{\epsilon} \ .$$

The condition $\beta_0(I) = 2^{-11}$ is marginally consistent with the biology related considerations of [17] and requires $r = 13.16$. The size of the CD would be about $2^{11} R_E$ for option I.

For the same value of r for both I and II one has $\beta(I) = 7.76\beta(II) \simeq 8\beta(II)$ so that option II could be obtained from option I by the scaling $\beta(I) \to \beta/8$ inducing the scaling $R_E \to 64 R_E > 60.3 R_E$. By the proportionality of Bohr orbit radius to $1/\beta^2$, the ratio $r(II)/r(I) = \sqrt{64/60.33} = 1.030$ would compensate this error. The mass mass of the moon is $M_M = .012 M_E$ so that the replacement of M_E with the $M_E + M_M$ would produce correction factor 1.012 which is by 2 per cent smaller than the required correction factor.

2.4 Under what conditions the models for dark and ordinary Bohr orbits are consistent with each other?

Under what conditions the Bohr orbitologies for dark and ordinary matter are consistent with each other?

1. The condition $v^2 = GM/r$ determines the relationship between velocity and radius in Newtonian theory. The values of v and r cannot therefore change for ordinary matter, which must coupled to all matter - both ordinary and dark matter of the central system.

2. A natural assumption is that dark matter couples only to the dark matter within the volume closed by its orbit. If dark object corresponds to an object modellable as point-like object (the alternative option is that dark matter is along a closed flux tubes along Bohr orbit) then the above condition reads $v_D^2 = GM_D/r$ so that one has

$$\frac{v_D}{v} = \sqrt{\frac{M_D}{M}} \quad . \tag{2.4}$$

   There seems to be no reason why the velocities of dark matter and ordinary matter could not be different. In the case of dark matter there is also Bohr orbit condition giving for gravitational Bohr radius as a generalization of $a_0 = \hbar/\alpha m_e \rightarrow a_{gr} = \hbar_{gr}/\alpha_{gr} m$ with $\alpha = e^2/4\pi\hbar \rightarrow \alpha_{gr} = GMm/4\pi\hbar_{gr} = v_0/4\pi$. This gives

$$a = a_{gr,D} n_D^2 \quad , \quad a_{gr} = \frac{4\pi G M_D}{v_0^2} \quad . \tag{2.5}$$

   This formula should be consistent with the formula originally derived for matter and motivated by the idea that ordinary matter forms bound states with dark matter. I have considered also the option that dark matter is delocalized along the flux tube associated with the orbit of the planet.

3. The two formulas make sense simultaneously only if one can interpret the Bohr orbit for M_D as Bohr orbit for M having same radius. This condition gives $M_D n_D^2 = Mn^2$ giving

$$n_D^2 = \frac{M}{M_D} n^2 \quad . \tag{2.6}$$

   Therefore M/M_D should be square of integer, which is rather strong constraint.

One can test this formula in the case of planetary system and for Earth.

1. The first guess is that the inner core of Sun with radius in the range $.2R_S$ and $.25R_S$ corresponds mostly to dark matter. Solar core contains about 34 cent of solar mass (see `http://tinyurl.com/nrcojr2`). This gives in excellent approximation $M/M_D = 3$, which is however not square. $M/M_D = 4$ would satisfy the condition and would have $n_D = 2n$.

   Since dark matter corresponds to extensions of rationals, one can ask whether one could allow for dark matter algebraic integers as values of n_D so that $n_D = \sqrt{3}n$ would be allowed for an extension containing $\sqrt{3}$. This would be a number theoretic generalization of quantization in terms of in terms of integers somewhat analogous to that associated with quantum groups.

2. For Earth the estimate [17] gives $M/M_D \simeq .5 \times 10^4$ giving $\beta_0 = 4.4 \times 10^{-4}$ rather near to $\beta_0 = 2^{-11} \simeq 5 \times 10^{-4}$. It is enough to find integer sufficiently near to 5000 having the property that it is square. One has $70^2 = 4900$ and $71^2 = 5041$.

   One would have $n_D \simeq 5000 \times n$ and consistency with the formula. Earth has outer core occupying 15 cent of its volume, inner core occupying 1 cent of the volume and innermost inner core with radius 300 km occupying fraction 10^{-4} of the volume (see `http://tinyurl.com/y8vf7vc3`) suggests that the innermost inner core consists of dark mass with density twice the average density.

   Remark: I have considered for M_D a probably too science fictive identification in terms of possibly existing gravitational analog of Dirac monopole. The gravitational flux would emanate radially from the center of the Earth along flux tubes carrying magnetic monopole flux and turn back at certain distance and return back along second space-time sheet and back to the original space-time sheet at wormhole like structure. This field would not be visible at large enough distances.

   If one has $M_D = 2 \times 10^{-4} M_E$, the density of the innermost inner core would be 2ρ, where ρ is the average density of Earth. From Wikipedia (see `http://tinyurl.com/ma6xqnh`) one learns that the average density ρ_E of Earth is $5.52 \times \rho_W$, $\rho_W =$ kg/dm^3 and the density in the inner core varies in the range $\rho/\rho_w \in [12.6 - 13.0]$. The lower limit is approximately $2 \times \rho$. This suggests that the density of the innermost inner core is somewhat larger than 2ρ.

2.5 How could Planck length be actually equal to much larger CP_2 radius?!

The following argument stating that Planck length l_P equals to CP_2 radius R: $l_P = R$ and Newton's constant can be identified $G = R^2/\hbar_{eff}$. This idea looking non-sensical at first glance was inspired by an FB discussion with Stephen Paul King.

First some background.

1. I believed for long time that Planck length l_P would be CP$_2$ length scale R squared multiplied by a numerical constant of order $10^{-3.5}$. Quantum criticality would have fixed the value of l_P and therefore $G = l_P^2/\hbar$.

2. Twistor lift of TGD [8, 9, 10, 11] led to the conclusion that that Planck length l_P is essentially the radius of twistor sphere of M^4 so that in TGD the situation seemed to be settled since l_P would be purely geometric parameter rather than genuine coupling constant. But it is not! One should be able to understand why the ratio l_P/R but here quantum criticality, which should determine only the values of genuine coupling parameters, does not seem to help.

   Remark: M^4 has twistor space as the usual conformal sense with metric determined only apart from a conformal factor and in geometric sense as $M^4 \times S^2$: these two twistor spaces are part of double fibering.

Could CP_2 radius R be the radius of M^4 twistor sphere, and could one say that Planck length l_P is actually equal to R: $l_P = R$? One might get $G = l_P^2/\hbar$ from $G = R^2/\hbar_{eff}$!

1. It is indeed important to notice that one has $G = l_P^2/\hbar$. $\hbar$ is in TGD replaced with a spectrum of $\hbar_{eff} = n\hbar_0$, where $\hbar = 6\hbar_0$ is a good guess [14, 18]. At flux tubes mediating gravitational interactions one has

$$\hbar_{eff} = \hbar_{gr} = \frac{GMm}{v_0} \ ,$$

   where v_0 is a parameter with dimensions of velocity. I recently proposed a concrete physical interpretation for v_0 [?] (see `http://tinyurl.com/yclefxb2`). The value $v_0 = 2^{-12}$ is suggestive on basis of the proposed applications but the parameter can in principle depend on the system considered.

2. Could one consider the possibility that twistor sphere radius for M^4 has CP_2 radius R: $l_P = R$ after all? This would allow to circumvent introduction of Planck length as new fundamental length and would mean a partial return to the original picture. One would $l_P = R$ and $G = R^2/\hbar_{eff}$. $\hbar_{eff}/\hbar$ would be of $10^7 - 10^8$!

The problem is that $\hbar_{eff}$ varies in large limits so that also G would vary. This does not seem to make sense at all. Or does it?!

To get some perspective, consider first the phase transition replacing $\hbar$ and more generally $\hbar_{eff,i}$ with $\hbar_{eff,f} = h_{gr}$.

1. Fine structure constant is what matters in electrodynamics. For a pair of interacting systems with charges Z_1 and Z_2 one has coupling strength $Z_1Z_2e^2/4\pi\hbar = Z_1Z_2\alpha$, $\alpha \simeq 1/137$.

2. As shown in [5, 4, 13, 12] one can also define gravitational fine structure constant α_{gr}. Only α_{gr} should matter in quantum gravitational scattering amplitudes. α_{gr} wold be given by

$$\alpha_{gr} = \frac{GMm}{4\pi\hbar_{gr}} = \frac{v_0}{4\pi} \quad . \tag{2.7}$$

   $v_0/4\pi$ would appear as a small expansion parameter in the scattering amplitudes. This in fact suggests that v_0 is analogous to α and a universal coupling constant which could however be subject to discrete number theoretic coupling constant evolution.

3. The proposed physical interpretation is that a phase transition $\hbar_{eff,i} \to \hbar_{eff,f} = h_{gr}$ at the flux tubes mediating gravitational interaction between M and m occurs if the perturbation series in $\alpha_{gr} = GMm/4\pi/\hbar$ fails to converge ($Mm \sim m_{Pl}^2$ is the naive first guess for this value). Nature would be theoretician friendly and increase h_{eff} and reducing α_{gr} so that perturbation series converges again.

   Number theoretically this means the increase of algebraic complexity as the dimension $n = h_{eff}/h_0$ of the extension of rationals involved increases fron n_i to n_f [?] and the number n sheets in the covering defined by space-time surfaces increases correspondingly. Also the scale of the sheets would increase by the ratio n_f/n_i.

   This phase transition can also occur for gauge interactions. For electromagnetism the criterion is that $Z_1Z_2\alpha$ is so large that perturbation theory fails. The replacement $\hbar \to Z_1Z_2e^2/v_0$ makes $v_0/4\pi$ the coupling constant strength. The phase transition could occur for atoms having $Z \geq 137$, which are indeed problematic for Dirac equation. For color interactions the criterion would mean that $v_0/4\pi$ becomes coupling strength of color interactions when α_s is above some critical value. Hadronization would naturally correspond to the emergence of this phase.

   One can raise interesting questions. Is v_0 (presumably depending on the extension of rationals) a completely universal coupling strength characterizing any quantum critical system independent of the interaction making it critical? Can for instance gravitation and electromagnetism are mediated by the same flux tubes? I have assumed that this is not the case. It it could be the case, one could have for $GMm < m_{Pl}^2$ a situtation in which effective coupling strength is of form $(GmMm/Z_1Z_2e^2)(v_0/4\pi)$.

The possibility of the proposed phase transition has rather dramatic implications for both quantum and classical gravitation.

1. Consider first quantum gravitation. v_0 does not depend on the value of G at all! The dependence of G on $\hbar_{eff}$ could be therefore allowed and one could have $l_P = R$. At quantum level scattering

amplitudes would not depend on G but on v_0. I was of course very happy after having found the small expansion parameter v_0 but did not realize the enormous importance of the independence on G! Quantum gravitation would be like any gauge interaction with dimensionless coupling, which is even small! This might relate closely to the speculated TGD counterpart of AdS/CFT duality between gauge theories and gravitational theories.

2. What about classical gravitation? Here G should appear. What could the proportionality of classical gravitational force on $1/\hbar_{eff}$ mean? The invariance of Newton's equation

$$\frac{d\overline{v}}{dt} = -\frac{GM\overline{r}}{r^3} \tag{2.8}$$

under $h_{eff} \rightarrow xh_{eff}$ would be achieved by scaling $\overline{r} \rightarrow \overline{r}/x$ and $t \rightarrow t/x$. Note that these transformations have general coordinate invariant meaning as scalings of Minkowski coordinates of M^4 in $M^4 \times CP_2$. This scaling means the zooming up of size of space-time sheet by x, which indeed is expected to happen in $h_{eff} \rightarrow xh_{eff}$!

What is so intriguing that this connects to an old problem that I pondered a lot during the period 1980-1990 as I attempted to construct to the field equations for Kähler action approximate spherically symmetric stationary solutions [3]. The naive arguments based on the asymptotic behavior of the solution ansatz suggested that the one should have $G = R^2/\hbar$. For a long time indeed assumed $R = l_P$ but p-adic mass calculations [7] and work with cosmic strings [6] forced to conclude that this cannot be the case. The mystery was how $G = R^2/\hbar$ could be normalized to $G = l_P^2/\hbar$: the solution of the mystery is $\hbar \rightarrow \hbar_{eff}$ as I have now - decades later - realized!

Received August 11, 2018; Revised August 24, 2018; Accepted September 22, 2018

References

[1] Nottale L Da Rocha D. Gravitational Structure Formation in Scale Relativity. Available at: http://arxiv.org/abs/astro-ph/0310036, 2003.

[2] Pitkänen M. Expanding Earth Model and Pre-Cambrian Evolution of Continents, Climate, and Life. In *Genes and Memes*. Online book. Available at: http://www.tgdtheory.fi/public_html/genememe/genememe.html#expearth, 2006.

[3] Pitkänen M. The Relationship Between TGD and GRT. In *Physics in Many-Sheeted Space-Time*. Online book. Available at: http://www.tgdtheory.fi/public_html/tgdclass/tgdclass.html#tgdgrt, 2006.

[4] Pitkänen M. Quantum Astrophysics. In *Physics in Many-Sheeted Space-Time*. Online book. Available at: http://www.tgdtheory.fi/public_html/tgdclass/tgdclass.html#qastro, 2006.

[5] Pitkänen M. TGD and Astrophysics. In *Physics in Many-Sheeted Space-Time*. Online book. Available at: http://www.tgdtheory.fi/public_html/tgdclass/tgdclass.html#astro, 2006.

[6] Pitkänen M. Cosmic Strings. In *Physics in Many-Sheeted Space-Time*. Online book. Available at: http://www.tgdtheory.fi/public_html/tgdclass/tgdclass.html#cstrings, 2006.

[7] Pitkänen M. Massless states and particle massivation. In *p-Adic Physics*. Online book. Available at: http://www.tgdtheory.fi/public_html/padphys/padphys.html#mless, 2006.

[8] Pitkänen M. The classical part of the twistor story. In *Towards M-Matrix*. Online book. Available at: http://www.tgdtheory.fi/public_html/tgdquantum/tgdquantum.html#twistorstory, 2006.

[9] Pitkänen M. About twistor lift of TGD? In *Towards M-Matrix*. Online book. Available at: http://www.tgdtheory.fi/public_html/tgdquantum/tgdquantum.html#hgrtwistor, 2016.

[10] Pitkänen M. Questions related to the twistor lift of TGD. In *Towards M-Matrix*. Online book. Available at: http://www.tgdtheory.fi/public_html/tgdquantum/tgdquantum.html#twistquestions, 2016.

[11] Pitkänen M. The Recent View about Twistorialization in TGD Framework. In *Towards M-Matrix*. Online book. Available at: http://www.tgdtheory.fi/public_html/tgdquantum/tgdquantum.html#smatrix, 2018.

[12] Pitkänen M. Criticality and dark matter. In *Hyper-finite Factors and Dark Matter Hierarchy*. Online book. Available at: http://www.tgdtheory.fi/public_html/neuplanck/neuplanck.html#qcritdark, 2014.

[13] Pitkänen M. Quantum gravity, dark matter, and prebiotic evolution. In *Genes and Memes*. Online book. Available at: http://www.tgdtheory.fi/public_html/genememe/genememe.html#hgrprebio, 2014.

[14] Pitkänen M. Hydrinos again. Available at: http://tgdtheory.fi/public_html/articles/Millsagain.pdf, 2016.

[15] Pitkänen M. Does valence bond theory relate to the hierarchy of Planck constants? Available at: http://tgdtheory.fi/public_html/articles/valenceheff.pdf, 2017.

[16] Pitkänen M. Re-examination of the basic notions of TGD inspired theory of consciousness. Available at: http://tgdtheory.fi/public_html/articles/conscrit.pdf, 2017.

[17] Pitkänen M. Clustering of RNA polymerase molecules and Comorosan effect. Available at: http://tgdtheory.fi/public_html/articles/clusterRNA.pdf, 2018.

[18] Pitkänen M. Dark valence electrons and color vision. Available at: http://tgdtheory.fi/public_html/articles/colorvision.pdf, 2018.

[19] Pitkänen M. Expanding Earth hypothesis, Platonic solids, and plate tectonics as a symplectic flow. Available at: http://tgdtheory.fi/public_html/articles/platoplate.pdf, 2018.

[20] Pitkänen M. Expanding Earth Model and Pre-Cambrian Evolution of Continents, Climate, and Life. Available at: http://tgdtheory.fi/public_html/articles/expearth.pdf, 2018.

[21] Pitkänen M. New support for the view about Cambrian explosion being caused by rapid increase of Earth radius. Available at: http://tgdtheory.fi/public_html/articles/HIFs.pdf, 2018.