Exploration

On the Possible Existence of Strong Elementary Charge & Its Applications

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Abstract

We review the basics of nuclear binding energy scheme and neutron-proton stability scheme assumed to be associated with the existence of a new strong nuclear elementary charge associated with square root of reciprocal of the strong coupling constant. An attempt is made to understand magic proton numbers in terms of their nuclear binding energy.

Keywords: Strong coupling constant, strong nuclear elementary charge, nuclear binding energy, magic numbers.

1. Introduction

As strong interaction [1] is mostly hidden at low energy scales in the form of 'residual nuclear force' and Liquid drop model and Fermi gas model [2-5] are failing in understanding nuclear binding energy with 'strong coupling constant', in our earlier published paper [6] and recent submitted papers [7,8] we suggested that, by considering 'square root' of reciprocal of the strong coupling constant' $(\alpha_s \cong 0.1186)$, as an index of strength of nuclear elementary charge, nuclear binding energy and nuclear stability can be understood. Our model [6-11] seems to be simple and realistic compared to the new integrated model [12,13]. In this paper we review sections 6 and 7 with much better semi empirical relations and in section 8, an attempt is made to understand magic proton numbers in terms of their nuclear binding energy.

2. About the semi-empirical mass formula

Let A be the total number of nucleons, Z the number of protons and N the number of neutrons. According to the semi-empirical mass formula [2,3,4], nuclear binding energy:

$$B \cong a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_a \frac{(A-2Z)^2}{A} \pm \frac{a_p}{\sqrt{A}} \qquad (1)$$

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Here a_v = volume energy coefficient, a_s is the surface energy coefficient, a_c is the coulomb energy coefficient, a_a is the asymmetry energy coefficient and a_p is the pairing energy coefficient. By maximizing $B(A,Z)$ with respect to Z,

$$Z \approx \frac{A}{2 + \left(a_c/2a_a\right)A^{2/3}} \text{ and } A - 2Z \approx \frac{0.4A^2}{A + 200} \tag{2}$$

Maximizing $B(A)/A$ with respect to A gives the nucleus which is most strongly bound or most stable.

3. New concepts & semi-empirical relations of nuclear binding energy & stability

We would like to suggest that,

1) There exists a strong nuclear charge, $e_s \cong \dfrac{e}{\sqrt{\alpha_s}} \cong 4.6523 \times 10^{-19}$ C

2) Proton magnetic moment [1] can be addressed with $\mu_p \cong \dfrac{e_s \hbar}{2m_p} \cong 1.467 \times 10^{-26}$ J.T^{-1}

3) Neutron magnetic moment [1] can be addressed with $\mu_n \cong \left(e_s - e\right)\dfrac{\hbar}{2m_n} \cong 9.602 \times 10^{-27}$ J.T^{-1}.

4) Characteristic nuclear radius can be expressed as,

$$R_0 \cong \left(\frac{1}{\sqrt{\alpha_s}}\right)\left\{\frac{\hbar}{m_p c} + \frac{\hbar}{m_n c}\right\} \cong \left(\frac{e_s}{e}\right)\left(\frac{2\hbar}{m_p c}\right) \cong 1.22 \text{ fm} \text{ where } m_p \approx m_n.$$

5) Nuclear beta stability line [4] can be addressed with a relation of the form, $A_s \cong 2Z + s\left(2Z\right)^2$

where $s \cong \left\{\left(\dfrac{e_s}{m_p}\right) \div \left(\dfrac{e}{m_e}\right)\right\} \cong 0.00158143$.

6) Nuclear binding energy can be understood with a single energy coefficient of magnitude $\left(\dfrac{e_s e}{8\pi\varepsilon_0\left(\hbar/m_p c\right)}\right) \cong 10.0$ MeV.

7) In deuteron, there exists no strong interaction in between neutron and proton.

8) Close to magic and semi magic proton numbers, nuclear binding energy seems to approach $\left[2.531\left(n + \dfrac{1}{2}\right)\right]^2 10.0$ MeV where $\left(\dfrac{m_n - m_p}{m_e}\right) = 2.531$ and n = 0,1,2,3,...

4. Beta stability line with respect to strong coupling constant

If $\alpha_s \cong 0.1186,$ for Z >8, close to the line of beta stability,

$$A_s \cong \left[Z + \left(\frac{e_s}{e} \right) \right]^{6/5} \cong \left(Z + 2.904 \right)^{1.2} \tag{3}$$

See table 1, column-2. For Z >16, close to the line of beta stability,

$$\left. \begin{array}{l} \left(A_s - 2Z \right) \cong \left(Z\beta + 1 \right)^2 - 4 \\ \text{where } \beta = \left(\frac{3}{5} \right) \alpha_s \cong 0.07116. \end{array} \right\} \tag{4}$$

See table 1, column-3.

5. Beta stability line with respect to nucleon mass difference

With reference to nucleon and electron rest masses [1], we noticed that,

$$\exp \left(\frac{\left(m_n - m_p \right) c^2}{m_e c^2} \right) \cong 12.5659102 \cong 4\pi \tag{5}$$

$$\left\{ \begin{array}{l} \text{where, } m_n c^2 \cong 939.565413 \text{ MeV,} \\ m_p c^2 \cong 938.272081 \text{ MeV; } m_e c^2 \cong 0.51099895 \text{ MeV} \end{array} \right.$$

Based on this observation, beta stability line can be understood with the following empirical relations.

$$\text{Let, } k \cong \left(1/4\pi \right)^2 \cong 0.006333 \tag{6}$$

$$\left. \begin{array}{l} A_s \cong 2Z + \left(Z/4\pi \right)^2 \cong 2Z + kZ^2 \\ N_s \cong Z + \left(Z/4\pi \right)^2 \cong Z + kZ^2 \\ \left(Z/\sqrt{A_s - 2Z} \right) \cong 4\pi \end{array} \right\} \tag{7}$$

See table 1, column-4. Based on these relations,

$$\left. \begin{array}{l} \text{A) } \dfrac{\left(A_s - 2Z \right)^2}{A_s} \cong k^2 A_s N_s \sqrt{Z} \\ \text{B) } \dfrac{A_s^{1/2} N_s^{1/4} Z^{1/8}}{\sqrt{A_s - 2Z}} \approx \dfrac{1}{\sqrt{k}} \approx 4\pi \end{array} \right\} \tag{8}$$

Table 1: Estimated stable mass numbers

Proton number Z	Stable mass number A_s		
	Relation (3)	Relation (4)	Relation (7)
2			4
5			10
8			16
11	24		23
14	30	28	29
17	36	35	36
20	43	42	43
23	50	49	49
26	57	56	56
29	64	63	63
32	71	71	70
35	78	78	78
38	86	86	85
41	94	93	93
44	101	101	100
47	109	109	108
50	117	117	116
53	125	125	124
56	133	133	132
59	141	141	140
62	150	149	148
65	158	158	157
68	166	166	165
71	175	175	174
74	183	183	183
77	192	192	192
80	201	201	201
83	209	210	210
86	218	219	219
89	227	228	228
92	236	237	238
95	245	246	247
98	254	256	257
101	263	265	267
107	281	284	287
110	291	294	297
113	300	304	307
116	309	314	317

6. Semi empirical relation for nuclear binding energy

Based on the new integrated model proposed by N. Ghahramany et al [12,13],

$$B(Z,N) = \left\{ A - \left(\frac{\left(N^2 - Z^2\right) + \delta\left(N - Z\right)}{3Z} + 3 \right) \right\} \frac{m_n c^2}{\gamma} \tag{9}$$

where, $\gamma =$ Adjusting coefficient $\approx (90 \text{ to } 100)$. if $N \neq Z$, $\delta\left(N - Z\right) = 0$ and if $N = Z$, $\delta\left(N - Z\right) = 1$. We noticed that,

$$\left. \begin{aligned} \frac{m_n c^2}{\gamma} &\cong \frac{m_n c^2}{\left(90 \text{ to } 100\right)} \\ &\cong \left(\frac{e_s e}{8\pi\varepsilon_0 \left(\hbar/m_p c\right)} \right) \cong 10.0 \text{ MeV} \end{aligned} \right\} \tag{10}$$

And with reference to relation (7), it is also possible to show that, for $Z \cong \left(40 \text{ to } 83\right)$, close to the beta stability line [7],

$$\left[\frac{N_s^2 - Z^2}{Z} \right] \cong kA_s Z \tag{11}$$

Based on the above relations and proposed concepts, and with reference to the first four terms of the semi empirical mass formula, close to the beta stability line [8], if $\alpha_s \approx 0.1186$ and $R_0 \approx 1.22$fm, semi empirically, we developed the following relations.

a) Starting from Z=3, <u>close to the beta stability line,</u>

$$\left. \begin{aligned} (B)_{A_s} &\approx \left\{ (Z-1)\left(\frac{e_s e}{4\pi\varepsilon_0 \left(\hbar/m_p c\right)} \right) \right\} \mp 10.0 \text{ MeV} \\ &\approx \left\{ (Z-1) \times 20.0 \text{ MeV} \right\} \mp 10.0 \text{ MeV} \end{aligned} \right\} \tag{12}$$

For example, binding energy of Oxygen (O) close to its stable atomic nuclides can be estimated to be $\left[(8-1) \times 20 \right] \mp 10.0 \text{ MeV} \cong (130 \text{ to } 150) \text{ MeV}$.

Binding energy of Iron (Fe) close to its stable atomic nuclides can be estimated to be $\left[(26-1) \times 20 \right] \mp 10.0 \text{ MeV} \cong (490 \text{ to } 510) \text{ MeV}$.

Binding energy of Tin (Sn) close to its stable atomic nuclides can be estimated to be $\left[(50-1) \times 20 \right] \mp 10.0 \text{ MeV} \cong (970 \text{ to } 990) \text{ MeV}$.

Binding energy of Lead (Pb) close to its stable atomic nuclides can be estimated to be $\left[(82-1)\times 20\right]\mp 10.0 \text{ MeV} \cong (1610 \text{ to } 1630) \text{ MeV}$.

b) For $A \approx (4 \text{ to } 80)$, <u>close to the beta stability line,</u>

$$
\begin{aligned}
(B)_A &\approx \left\{\left(A-\sqrt{A}\right)\left(\frac{e_s e}{8\pi\varepsilon_0\left(\hbar/m_p c\right)}\right)\right\} \\
&\approx \left\{\left(A-\sqrt{A}\right)\times 10.0 \text{ MeV}\right\}
\end{aligned}
\tag{13}
$$

Binding energy of Helium-4 can be approximately estimated to be
$\left(4-\sqrt{4}\right)\times 10.0 \text{ MeV} \approx 20.0 \text{ MeV}$. Actual binding energy is 28.296 MeV.

Binding energy of Carbon-12 can be approximately estimated to be
$\left(12-\sqrt{12}\right)\times 10.0 \text{ MeV} \approx 95.5 \text{ MeV}$. Actual binding energy is 92.162 MeV.

Binding energy of Scandium-45 can be approximately estimated to be
$\left(45-\sqrt{45}\right)\times 10.0 \text{ MeV} \approx 382.92 \text{MeV}$. Actual binding energy is 387.848 MeV.

Binding energy of Manganese-55 approximately can be estimated to be,
$\left(55-\sqrt{55}\right)\times 10.0 \text{ MeV} \approx 475.84 \text{ MeV}$. Actual binding energy is 482.072 MeV.

Binding energy of Kripton-80 approximately can be estimated to be,
$\left(80-\sqrt{80}\right)\times 10.0 \text{ MeV} \approx 710.56 \text{ MeV}$. Actual binding energy is 695.434 MeV.

c) For $(Z \geq 4)$, <u>close to the beta stability line,</u>

$$
\begin{aligned}
B_{(Z,A)} &\cong \left\{A-\left[\left(\frac{kAZ}{2.531}+2.531\right)+1\right]\right\}10.06 \text{ MeV} \\
&\text{where } \left(\frac{\left(m_n-m_p\right)c^2}{m_e c^2}\right) \cong \ln\left(4\pi\right) \cong 2.531 \text{ and} \\
&\left(\frac{kAZ}{2.531}+2.531\right)=\text{"New term" needs explanation}
\end{aligned}
\tag{14}
$$

See table-2 for the isotopic binding energy of Z=50

Table 2: Comparison of estimated and actual binding energy of isotopes of Z=50

Mass number A	Estimated binding energy (MeV) Relation(14)	Actual binding energy (MeV)	Error (MeV)
112	950.2	953.532	3.3
114	967.8	971.574	3.7
115	976.6	979.121	2.5
116	985.4	988.684	3.2
117	994.2	995.627	1.4
118	1003.0	1004.955	1.9
119	1011.8	1011.438	-0.4
120	1020.6	1020.546	-0.1
122	1038.2	1035.53	-2.7
124	1055.9	1049.963	-5.9

We are working on understanding the physical significance of $\left(\dfrac{kAZ}{2.531} + 2.531 \right)$. It needs further study at basic level. See the following figure-1. Green curve represents the binding energy per nucleon estimated with the first four terms of SEMF relation (1) and (7). Dashed red curve represents the binding energy per nucleon estimated with relations (7) and (14).
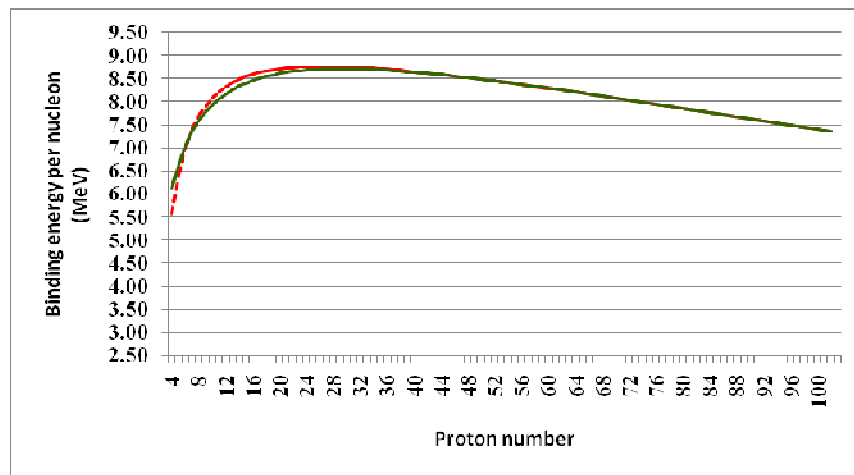

Figure 1: Comparison of estimated and SEMF binding energy per nucleon

7. Understanding the binding energies of Deuteron, Triton and $^{4}_{2}He$.

If it is assumed that there exists no strong interaction in between neutron and proton, above relation (13) can be expressed as follows.

$$(B)_A \approx \left\{ \left(A - \sqrt{A} \right) \left(\frac{e^2}{8\pi\varepsilon_0 \left(\hbar / m_p c \right)} \right) \right\} \approx \left\{ \left(A - \sqrt{A} \right) \times 3.443 \text{ MeV} \right\} \qquad (15)$$

Based on this relation (15), Deuteron $\left({}_1^2 H \right)$ binding energy can be estimated to be 2.02 MeV and actual binding energy is 2.225MeV.

From relation (15), Triton $\left({}_1^3 H \right)$ binding energy can be estimated to be 4.37 MeV. From relation (13), Triton $\left({}_1^3 H \right)$ binding energy can be estimated to be 12.68 MeV. Actual binding energy (8.482 MeV) seems to be close to the average of (4.37 and 12.68) MeV = 8.525 MeV. Clearly speaking, binding energy of $\left({}_1^3 H \right)$ seems to follow electromagnetic interaction as well as strong interaction and needs further study.

In the similar way, $\left({}_2^3 He \right)$ binding energy can be understood in terms of the combined effect of electromagnetic and strong interactions.

8. Understanding magic proton numbers

Basic idea: Atoms are exceptionally stable when their nuclear binding energy approaches $\left[2.531 \left(n + \frac{1}{2} \right) \right]^2 10.0$ MeV where n = 0,1,2,3,...

$$B_A \approx \left\{ (Z-1) \left(\frac{e_s e}{4\pi\varepsilon_0 \left(\hbar / m_p c \right)} \right) \right\}$$
$$\approx \left[2.531 \left(n + \frac{1}{2} \right) \right]^2 10.0 \text{ MeV} \qquad (16)$$

Based on relations, (10), (12), (14) and (16), magic and semi magic proton numbers [14] can be understood with the following relation.

$$M_n \cong \left\{ \frac{1}{2} \left[2.531 \left(n + \frac{1}{2} \right) \right]^2 + 1 \right\} + \Delta$$
$$\cong \left[3.203 \left(n + \frac{1}{2} \right)^2 + 1 \right] + \Delta$$

After rounding off,

if, $\left\{ \frac{1}{2} \left[2.531 \left(n + \frac{1}{2} \right) \right]^2 + 1 \right\}$ is Odd, $\Delta = \mp 1$

if, $\left\{ \frac{1}{2} \left[2.531 \left(n + \frac{1}{2} \right) \right]^2 + 1 \right\}$ is Even, $\Delta = \mp 2$

$$(17)$$

See the following table-3. It is possible to say that,

1) Magic proton numbers 2, (6), (14), 28, 50, 82, 114,.. etc can be shown to be n^{th} levels [15,16].

2) Magic proton numbers 2, 8, 20, 40,… can be shown to be $\left(n+\dfrac{1}{2}\right)$ levels.

Table-3 : To understand the magic proton numbers

$\left(n+\dfrac{1}{2}\right)$	Round off $\left[3.203\left(n+\dfrac{1}{2}\right)^{2}+1\right]$	M_n
0	1	1,2
0.5	2	2,4
1	4	2,4,6
1.5	8	6,8,10
2	14	12,14,16
2.5	21	20,21,22
3	30	28,30,32
3.5	40	38,40,42
4	52	50,52,54
4.5	66	64,66,68
5	81	80,81,82
5.5	98	96,98,100
6	116	114,116,118
6.5	136	134,136,138
7	158	156,158,160
7.5	181	180,181,182
8	206	204, 206, 208

9. Conclusion

Nowadays, estimating and understanding nuclear binding energy with 'strong interaction' seems to attract many nuclear physicists. In this context, by considering the proposed semi empirical relations, existence of the 'strong elementary charge' can be confirmed. With further study and research, a realistic nuclear model pertaining to strong interaction can be developed.

References

[1] C. Patrignani et al. (Particle Data Group), Chin. Phys. C, 40, 100001 (2016) and 2017 update

[2] Weizsäcker, Carl Friedrich von, On the theory of nuclear masses; Journal of Physics **96** (1935) pages 431- 458.

[3] W. D. Myers et al. Table of Nuclear Masses according to the 1994 Thomas-Fermi Model.(from nsdssd.lbl.gov)

[4] P. Roy Chowdhury et al. Modified Bethe-Weizsacker mass formula with isotonic shift and new driplines. Mod.Phys.Lett. A20 (2005) 1605-1618.

[5] J.A. Maruhn et al., Simple Models of Many-Fermion Systems, Springer-Verlag Berlin Heidelberg 2010. Chapter 2, page:45-70.

[6] Seshavatharam U. V. S, Lakshminarayana, S., A Virtual Model of Microscopic Quantum Gravity. Prespacetime Journal, Volume 9, Issue 1, pp. 58-82 (2018)

[7] U. V. S. Seshavatharam, Lakshminarayana S. Is 'reciprocal' of the strong coupling constant – a representation of upper limit of strength of nuclear binding energy? Submitted to QNP2018 conference, Nov.13-17, 2018, Tsukuba, Japan.

[8]   U. V. S. Seshavatharam, Lakshminarayana S. On the possible existence of strong elementary charge and its applications. Submitted to ICNPAP-2018 conference, October 23-26, 2018, Centre for Applied Physics, Central University of Jharkhand, Ranchi, India.

[9] Seshavatharam U. V. S, Lakshminarayana, S., On the role of strong coupling constant and nucleons in understanding nuclear stability and binding energy. Journal of Nuclear Sciences, Vol. 4, No.1, 7-18, (2017)

[10] Seshavatharam U. V. S, Lakshminarayana, S., A Review on Nuclear Binding Energy Connected with Strong Interaction. Prespacetime Journal, Volume 8, Issue 10, pp. 1255-1271 (2018)

[11]   U. V. S. Seshavatharam, Lakshminarayana S. To unite nuclear and sub-nuclear strong interactions. International Journal of Physical Research, 5 (2) 104-108 (2017)

[12] Ghahramany et al. New approach to nuclear binding energy in integrated nuclear model. Journal of Theoretical and Applied Physics 2012, 6:3

[13] N. Ghahramany et al. Stability and Mass Parabola in Integrated Nuclear Model. Universal Journal of Physics and Application 1(1): 18-25, (2013).

[14] U. V. S. Seshavatharam et al. Simple relations for estimating and predicting light, medium and heavy magic numbers. Journal of Applied Physical Science International. 6(2): 88-91, 2016

[15] Tran, D. T. et al, Evidence for prevalent Z = 6 magic number in neutron-rich carbon isotopes. Nature Communications**,** Vol 9, Article number: 1594 (2018)

[16] Fridmann J, et al. Magic nucleus 42Si. Nature. 2005;435:922-924.