Cosmological Models in Lyra Geometry with Linearly Varying Deceleration Parameter

Kalyani Desikan[*]

Dept. of Math., School of Advanced Sciences, VIT Chennai. Chennai 600127, India

Abstract

Cosmological models with linearly varying deceleration parameter in the cosmological theory based on Lyra's geometry have been discussed. Exact solutions have been obtained for a spatially flat FRW model by considering a time dependent displacement field. We have also obtained the time periods during which the universe undergoes decelerated and accelerated expansions for a matter-dominated universe.

Keywords: Cosmology, Lyra geometry, linearly varying, deceleration parameter.

1. INTRODUCTION

Einstein gave a geometric description for Gravitation in his general theory of relativity. In the absence of the cosmological term, Einstein's equations allow only non-static cosmological models when the energy density is nonzero. In order to have a static model of the universe as per the cosmological principle, Einstein introduced the cosmological constant into his equations. Weyl [1] put forth a more general theory that was able to provide a geometric description of electromagnetism also. His theory, however, was not well received as it had some unacceptable features such as non-integrability of length of a vector under parallel transport.

Later Lyra [2] came up with a variation of Riemannian geometry. Lyra's geometry may also be considered as a modification of Weyl's geometry. Lyra introduced a gauge function into the structureless manifold. This led to the incorporation of the cosmological constant, in a more natural way, from the geometry. It also overcame the problem of non-integrability of length of a vector under parallel transport that was plaguing Weyl's geometry. Subsequently, a new scalar tensor theory of gravitation was proposed by Sen [3] and Sen and Dunn [4]. Sen's field equations based on Lyra's geometry, in normal gauge can be written as

$$R_{ij} - \frac{1}{2} g_{ij} R + \frac{3}{2} \phi_i \phi_j - \frac{3}{4} \phi_k \phi^k = -8\pi G T_{ij} \qquad (1)$$

where ϕ_i is the displacement vector and the other notations have the usual meaning as in Riemannian geometry.

[*]Correspondence: Kalyani Desikan, Dept. of Math., School of Advanced Sciences, VIT Chennai. Chennai 600127, India.
Email: kalyanidesikan@vit.ac.in

Halford [5, 6] showed that the role that the cosmological term is played by the displacement vector field ϕ_i in Lyra geometry. Many researchers have explored cosmological models in Lyra geometry by considering the displacement field vector to be constant. Nevertheless, there is no apriori reason to consider the displacement field vector to be a constant.

Singh and Singh [7-10] have studied Bianchi Type I, III, Kantowski- Sachs models based on Lyra's geometry with a time dependent displacement field. Singh and Desikan [11] have discussed a class of models in Lyra's geometry based on Einstein's theory by considering a time varying displacement field and a constant deceleration parameter. Desikan and Das [12] have taken the deceleration parameter to be constant and studied the behaviour of cosmological models in Lyra's geometry in the presence of creation of matter.

Type Ia Supernova observations indicate that our present universe besides expanding is also accelerating. This behavior of the universe has been confirmed by various independent observational data. Akarsu and Dereli [13] proposed a linearly varying deceleration parameter to obtain accelerating cosmological solutions. This linearly varying deceleration parameter reduces, as a special case, to the constant deceleration parameter law proposed by Berman [14, 15].

In this paper we have discussed FRW cosmological models with linearly varying deceleration parameter in Lyra geometry for time varying displacement field vector. The field equations are given in section 2. Section 3 deals with the solutions of the field equations and their discussion.

2. FIELD EQUATIONS

In equation (1), the time-like displacement vector is given by

$$\phi_i = (\beta(t),0,0,0) \tag{2}$$

Assuming a perfect fluid, the energy momentum tensor is given by

$$T_{ij} = (\rho + p)u_i u_j - pg_{ij} \tag{3}$$

where ρ and p are the energy density and pressure respectively, u_i the fluid-four velocity, and g_{ij} is the metric tensor.

The field equations (1) for the FRW metric

$$ds^2 = dt^2 - R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + sin^2\theta d\phi^2)\right]$$

where $k = 1, 0, -1$, with equations (2) and (3) become

$$3H^2 + \frac{3k}{R^2} - \frac{3\beta^2}{4} = \chi\rho \tag{4}$$

$$2\dot{H} + 3H^2 + \frac{k}{R^2} + \frac{3\beta^2}{4} = -\chi p \qquad (5)$$

where $\chi = 8\pi G$ and $H = \dfrac{\dot{R}}{R}$ is the Hubble's function.

From equations (4) and (5) we get the following continuity equation

$$\chi\dot{\rho} + \frac{3}{2}\beta\dot{\beta} + 3\left[\chi(\rho + p) + \frac{3\beta^2}{2}\right]H = 0 \qquad (6)$$

By considering a barotropic equation of state

$$p = \gamma\rho, -1 \le \gamma \le 1 \qquad (7)$$

and eliminating the energy density ρ from equations (4) and (5) we have

$$2\dot{H} + 3(1+\gamma)H^2 + (1+3\gamma)\frac{k}{R^2} + (1-\gamma)\frac{3\beta^2}{4} = 0 \qquad (8)$$

Here β^2 plays the role of a time varying cosmological term. We have three unknowns viz., $R(t)$, $\rho(t)$ and $\beta(t)$ and two independent equations. In order to find a unique solution, we need one more relation between the variables. Therefore, we have considered a linearly varying deceleration parameter.

3. EXACT SOLUTIONS OF THE FIELD EQUATIONS

We make use of the following linearly varying deceleration parameter ansatz proposed by Akarsu and Dereli [13] to obtain the solutions of the field equations:

$$q = \frac{-R\ddot{R}}{(\dot{R})^2} = -k_1 t + m - 1 \qquad (9)$$

where $k_1 \ge 0$ and $m \ge 0$ are constants. Here k_1 is a constant with the dimension of time inverse and m is a dimension free constant. This ansatz covers the rule for constant deceleration parameter presented by Berman [14, 15], as a special case. Equation (9) reduces to the law of Berman when $k_1 = 0$.

On integrating, equation (9) with $k_1 > 0$, $m > 1$ we get the following solution [13]:

$$R(t) = R_1 e^{\frac{2}{m} tanh^{-1}\left(\frac{k_1}{m} t - 1\right)} \tag{10}$$

where R_1 is a constant of integration. Now, the Hubble's parameter is given by

$$H = \frac{\dot{R}}{R} = -\frac{2}{t(k_1 t - 2m)} \tag{11}$$

Using (11) in (8) for a flat universe leads to

$$\beta^2 = \frac{4[2(k_1 t - m) + 3(1+\gamma)]}{3(\gamma-1)} H^2 \tag{12}$$

Using (11) in (12) yields

$$\beta^2 = \frac{16[2(k_1 t - m) + 3(1+\gamma)]}{3(\gamma-1)t^2(k_1 t - m)^2} \tag{13}$$

From (13) it can be seen that β^2 decreases with time. Also, since the denominator is always negative, we observe that for $k_1 > 0$

$$\beta^2 > 0 \ \text{if} \ t > \frac{2m - 3(1+\gamma)}{2k_1} \tag{14}$$

and

$$\beta^2 < 0 \ \text{if} \ t < \frac{2m - 3(1+\gamma)}{2k_1} \tag{15}$$

We know that the displacement vector field ϕ_i behaves like a cosmological term. The attraction due to gravity of matter is resisted by a positive cosmological constant and it drives the rapid expansion of the universe because of its negative pressure.

From observations we know that our current matter-dominated universe $(\gamma = 0)$ is undergoing accelerated expansion. Using $\gamma = 0$ in (14) we get

$$\beta^2 > 0 \ \text{when} \ t > \frac{m - 3/2}{k_1} \tag{16}$$

From equation (9) we see that the expansion of the universe will accelerate when

$$t > \frac{m-1}{k_1} \tag{17}$$

From equations (16) and (17) we observe that $\beta^2 > 0$ and the universe will decelerate when

$$\frac{m-3/2}{k_1} < t < \frac{m-1}{k_1}$$

and accelerate when $t > \frac{m-1}{k_1}$.

Now using (12) in (4) for a flat universe yields

$$\chi\rho = -\frac{(2(k_1 t - m) + 6)}{(\gamma - 1)} H^2 \tag{18}$$

From (18) we see that $\rho \geq 0$ if $t \geq \frac{m-3}{k_1}$ as the denominator is always negative. Also, we see that the universe will undergo decelerated expansion when

$$\frac{m-3}{k_1} < t < \frac{m-3/2}{k_1}$$

During this time period β^2 will be negative.

4. CONCLUSION

We have considered a linearly varying deceleration parameter and obtained solutions for a flat FRW cosmological model in Lyra geometry. Explicit expressions have been obtained for both energy density and the displacement vector field. The behaviour of both these parameters has been discussed for different time periods. Also, the time periods during which the matter-dominated, flat universe undergoes decelerated/accelerated expansions have been identified. The period of accelerated expansion is of particular interest in view of the observations that indicate an accelerating universe at present times.

References

1. Weyl, H. 1918, Sber Preuss. Akad.Wiss.465.
2. Lyra, G. 1951, Math. Z. 54, 52.
3. Sen, D.K. 1957, Z. Phys., 149, 311.
4. Sen, D.K. and Dunn, K.A. 1971, J. Math. Phys., 12 578.
5. Halford, W.D. 1970, Austr. J. Phys. 1970, 23, 863.
6. Halford, W.D. 1972, J. Math. Phys., 13, 1399.
7. Singh, T. and Singh G.P. 1991a, J. Math. Phys., 32, 2456.
8. Singh, T. and Singh G.P. 1991a, Nuovo Cimento, B106, 617.
9. Singh, T. and Singh G.P. 1992, Int. J. Theor. Phys., 31, 1433.
10. Singh, T. and Singh G.P. 1993, Fortschr. Phys., 41, 737.
11. Singh G.P. and Kalyani Desikan 1997, Pramana – J. of Phys., 49, 205.
12. Desikan, K. and Das S., 2017, Int. J. of Pure and Applied Physics, 13(4), 303.
13. Akarsu, O. and Dereli T, 2012, Int. J. Theor. Phys., 51, 612.
14. Berman, M.S., 1983, Nuovo Cimento B, 74, 182.
15. Berman, M.S. and Gomide, F.M., 1988, General Relativity and Gravitation, 20, 191.